

History of Mathematics

MA13309

Week 2

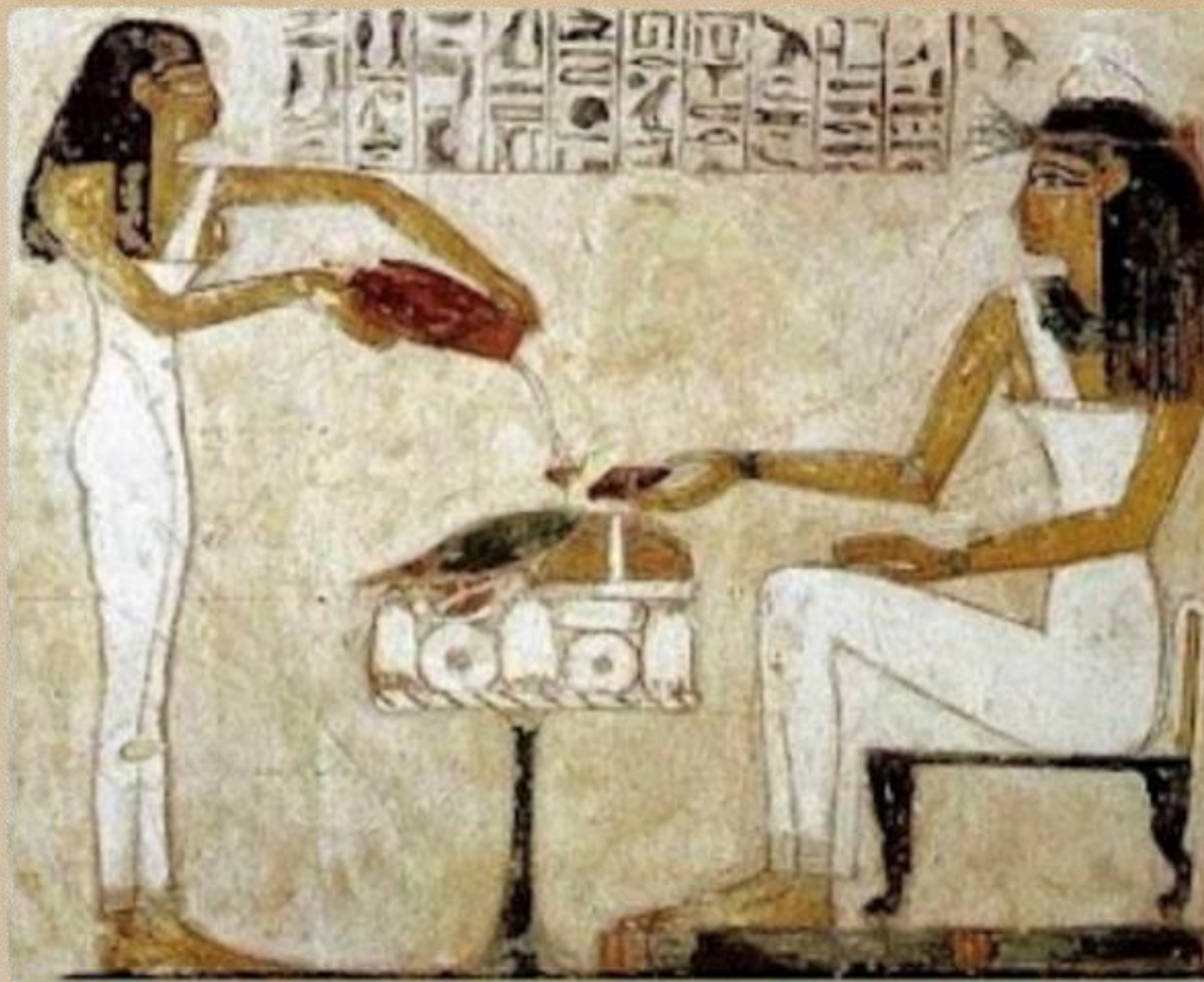


Chapter 2

Mathematics in Early Civilizations

Contents

- General view
- China
- India
- Babylon
- Egypt



1. General view

Advent of Writing



Barter System



Systematic Taxation

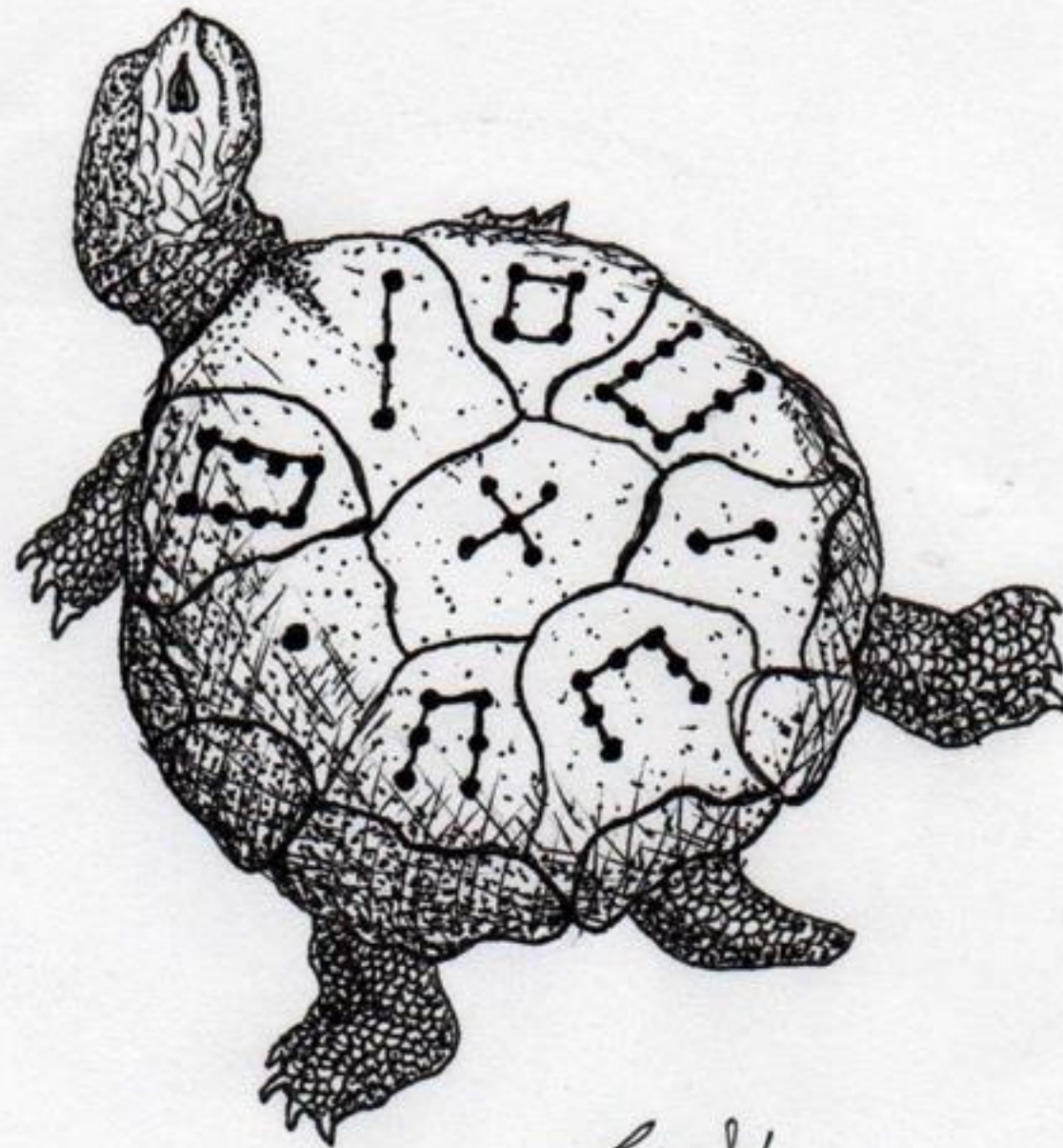


Stone masonry



Seagoing





Lo Shu

2. China

Reign of Huang-ti



Reign of Huang-ti. In the year 2704 B. C.¹ Huang-ti, the Yellow Emperor, began his reign. Under his patronage it is said that Li Shu wrote on astronomy and that Ta-nao established the *Chia-tsū*, or sexagesimal system, both of these statements being supported by copies (possibly altered) of ancient records.² Even the emperor himself is said to have taken such an interest in mathematics as to write upon astronomy and arithmetic, and in his reign an eclipse of the sun was observed and recorded. Tradition assigns to this period even the decimal system of counting, although it is more likely that some popular work on the subject was written at this time.

East and West



It is this emperor Yau and his successor, the emperor Shun, who, it is said, carried farther to the eastward the dominion established by the Bak tribes which had come from western Asia. These tribes had been under the civilizing influences of the people of Susiana, who in turn had received their civilization from Babylon.¹ If this theory proves to be correct, the similarity between certain early forms of astronomy and mathematics in the East and the West is more easily explained.

Early China Mathematics

1600 B.C. - beginning of Shang Dynasty; the chief of Shang tribe, Tang, led his army and overthrew the Xia Dynasty. Tang established the Shang Dynasty and made Bo the capital city.

1525 B.C. - moved kingdom from Xiang to Geng, then from Geng to Xing

1523 B.C. - first records of Chinese characters

1395 B.C. - the Shang move their capital to Yin (near Anyang)

1200 B.C. - earliest inscribed oracle bones; prosper of Shang oracle bones

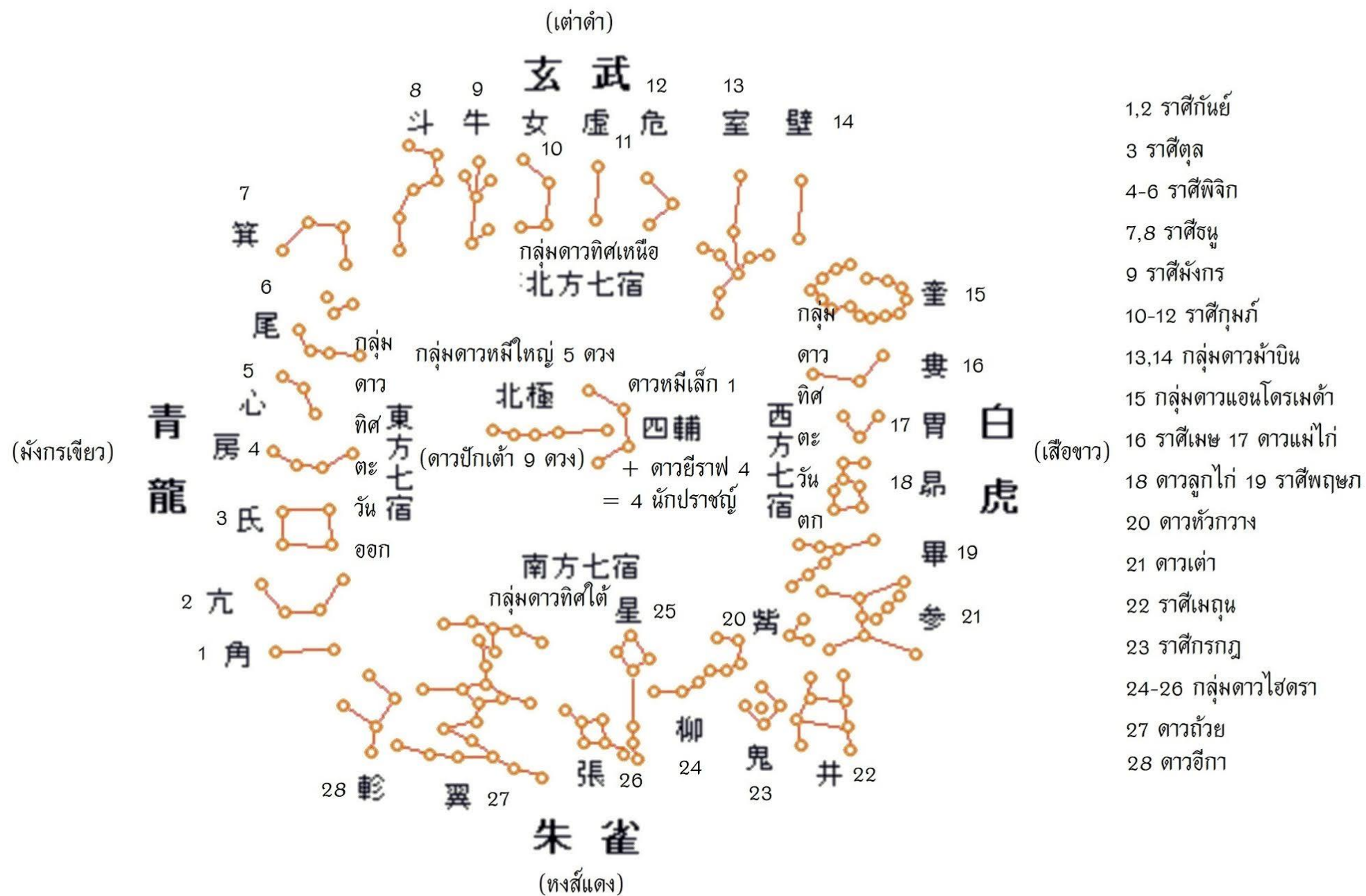
1122 B.C. - fall of Shang Dynasty; Battle of Mu Yu; Wu Wang, the "Martial King" of the Zhou defeated the Shang through the use of over 300 chariots

While such claims are generally doubted by competent Sinologists, it is quite likely that the Chinese developed some acquaintance with **descriptive astronomy** at an early period, and that this development necessitated such knowledge of mathematics as the **measure of time and angles** and the use of fairly **large numbers**. Reasonably well-founded tradition gives the probable dates of Fuh-hi,¹ the reputed first emperor of China, as 2852–2738 B.C.,² and in his reign there were **extensive astronomical observations**. In this general period the Chinese are believed to have **changed their zodiac** into one of twenty-eight animals.



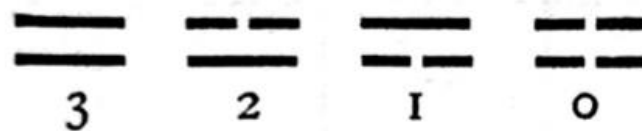
Wu Wang
1122
B.C.

28 Zodiac



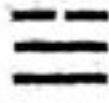
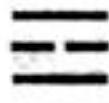
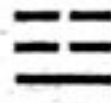
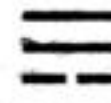
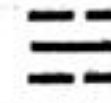
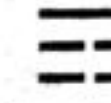
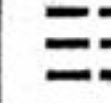
Book of permutation

I-king. Of the "Five Canons" (*Wu-king*) of the Chinese probably the third in point of antiquity is the *I-king*, or *Book of Permutations*.² In this appear the *Liang I*, or "two principles" (the male, *yang*, —; and the female, *ying*, — —) and from these were formed the *Sz' Siang*, or "four figures,"



and the *Pa-kua* (eight-kua) or eight trigrams, the eight permutations of two forms taken three at a time, repetitions being allowed. These *Pa-kua* had various virtues assigned to them and have been used from a very early period until the present for purposes of divination. It was probably *Wön-wang* (1182–1135 B.C.) who wrote the *I-king*; at any rate it was he who extended the *Pa-kua* into the sixty-four hexagrams now found in this classic.³

Eight Trigrams

							
<i>k'ien</i> heaven	<i>tui</i> steam	<i>li</i> fire	<i>ch'ün</i> thunder	<i>sün</i> wind	<i>k'an</i> water	<i>k'ün</i> mountain	<i>k'un</i> earth
7	6	5	4	3	2	1	0
HEAVEN SKY	COLLECTED WATER	FIRE	THUNDER	WIND WOOD	WATER AS IN RAIN MOON	HILLS	EARTH
S.	S.E.	E.	N.E.	S.W.	W.	N.W.	N.

THE PA-KUA, OR EIGHT TRIGRAMS

From the *I-king*, or *Book of Permutations*. On the ordinary diviner's compass these directions are reversed

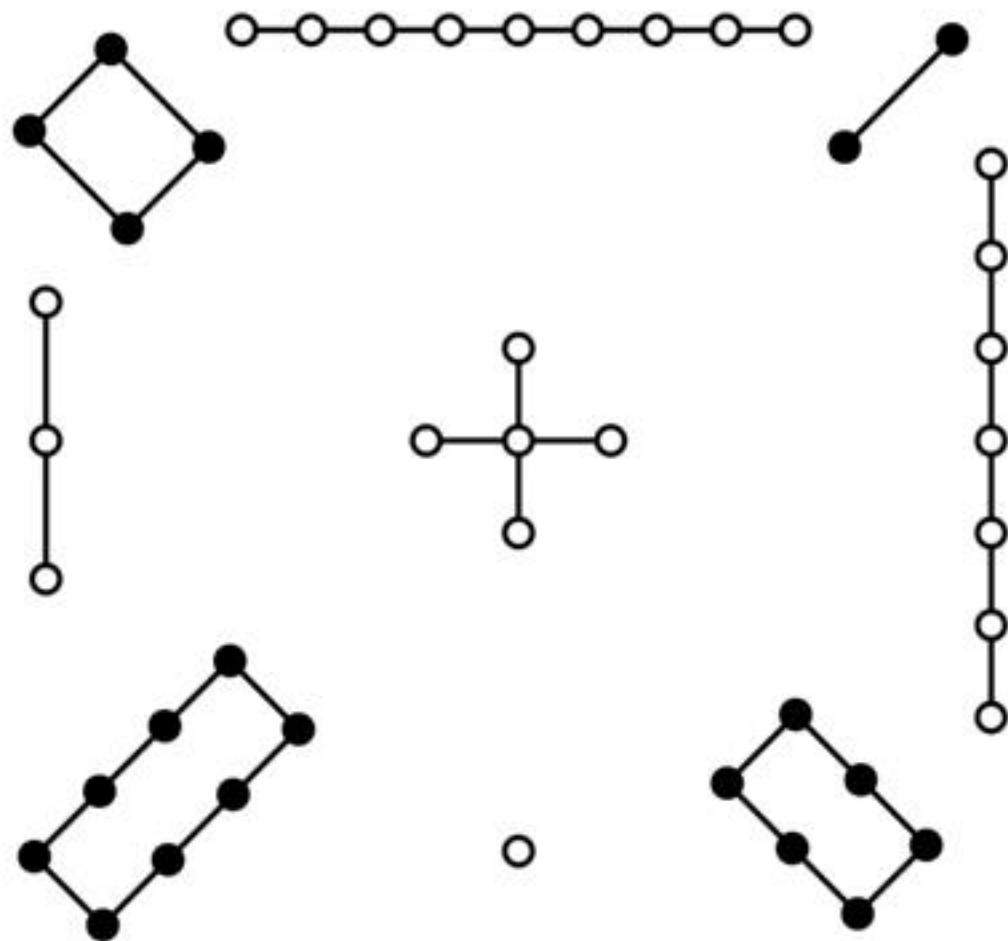
Although there is no historical evidence that the Chinese looked upon the *Pa-kua* as numerals, based upon the scale of two, it is true that if we take  for one and  for zero, the successive trigrams, beginning at the right, have values which we may represent by our numerals as 000, 001, 010, 011, 100, 101, 110, and 111. If these are considered as numbers written on the scale of two, their respective values are 0, 1, 2, 3, 4, 5, 6, and 7.



THIBETAN "WHEEL OF LIFE"

From a sheet of block printing done at Lhassa. This portion represents the signs of the zodiac, the *Pa-kua*, and, in the center, a magic square

Wheel of life

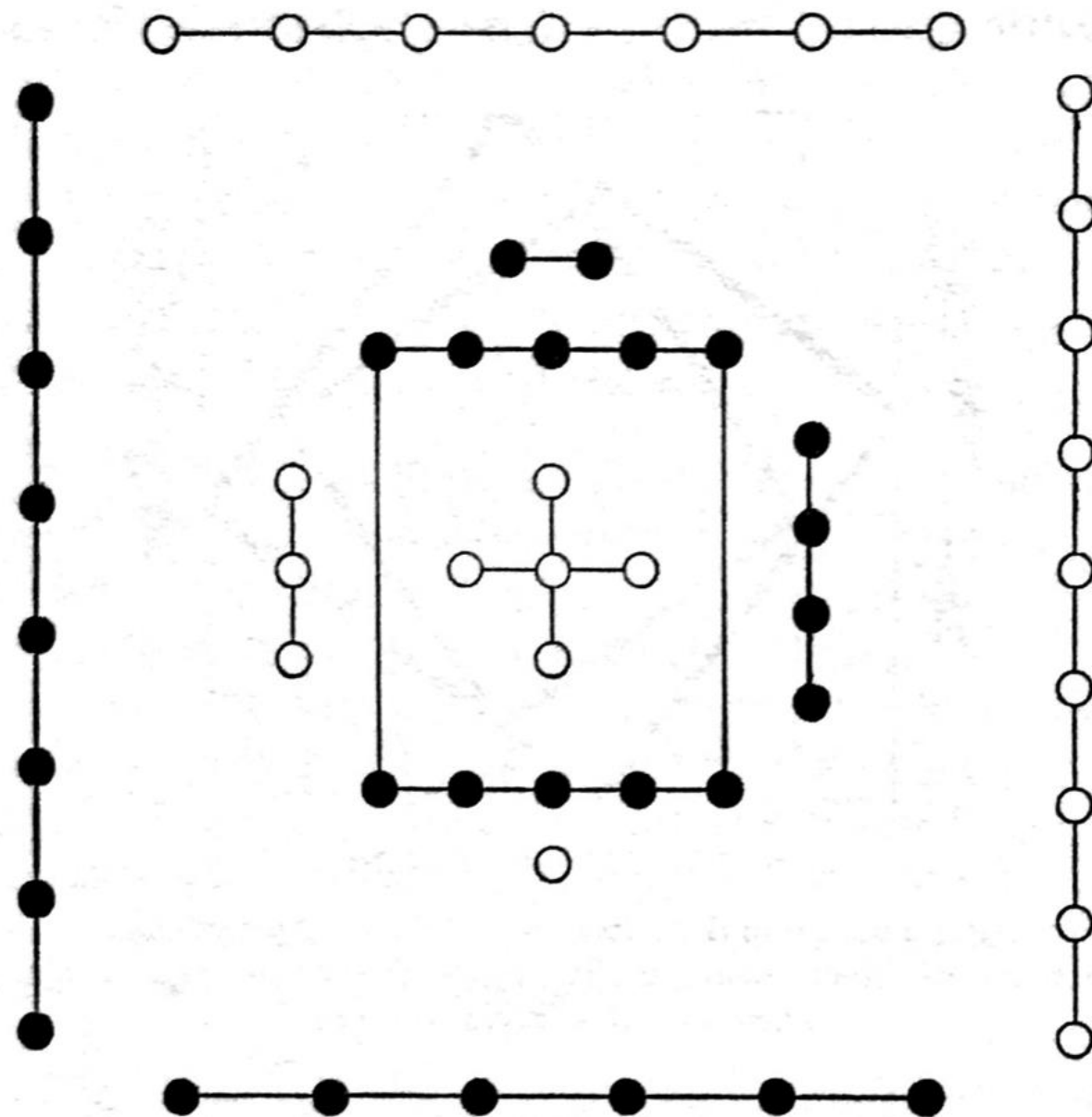


The Lo-shu

4	9	2
3	5	7
8	1	6

Magic square

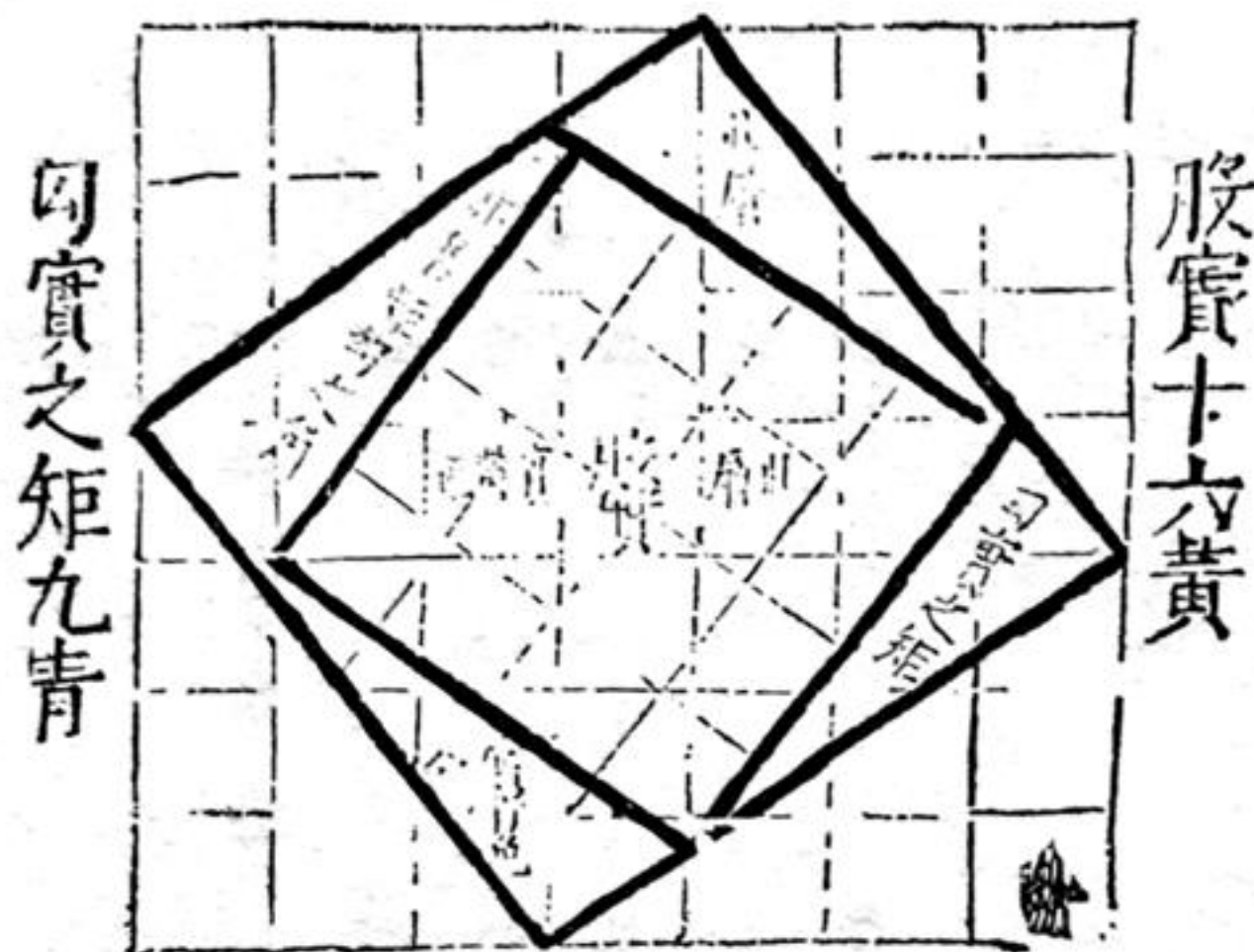
The ho-t'u



THE HO-T'U FROM THE I-KING

This was never considered so important as the *lo-shu*, lacking as it does the interest of the magic square

The chou-peí



A work written in the second millenium B.C. This illustration is from a very early specimen of block printing. It shows the figure of the Pythagorean Theorem, but gives no proof

K'iu-ch'ang Suan-shu

1. *Fang-t'ien* (*Squaring the farm*), relating to surveying, with correct rules for the area of the triangle, trapezium (trapezoid),² and circle ($\frac{1}{2}c \cdot \frac{1}{2}d$ and $\frac{1}{4}cd$), and with the circle approximations $\frac{3}{4}d^2$ and $\frac{1}{12}c^2$, where π is taken as 3.

2. *Su-mi* (*Calculating the cereals*), relating to percentage and proportion.

3. *Shuai-fen* (*Calculating the shares*), relating to partnership and the Rule of Three.³

4. *Shao-Kuang* (*Finding length*), relating to the finding of the sides of figures, and including square and cube roots.

5. *Shang-kung* (*Finding volumes*), relating to volumes.

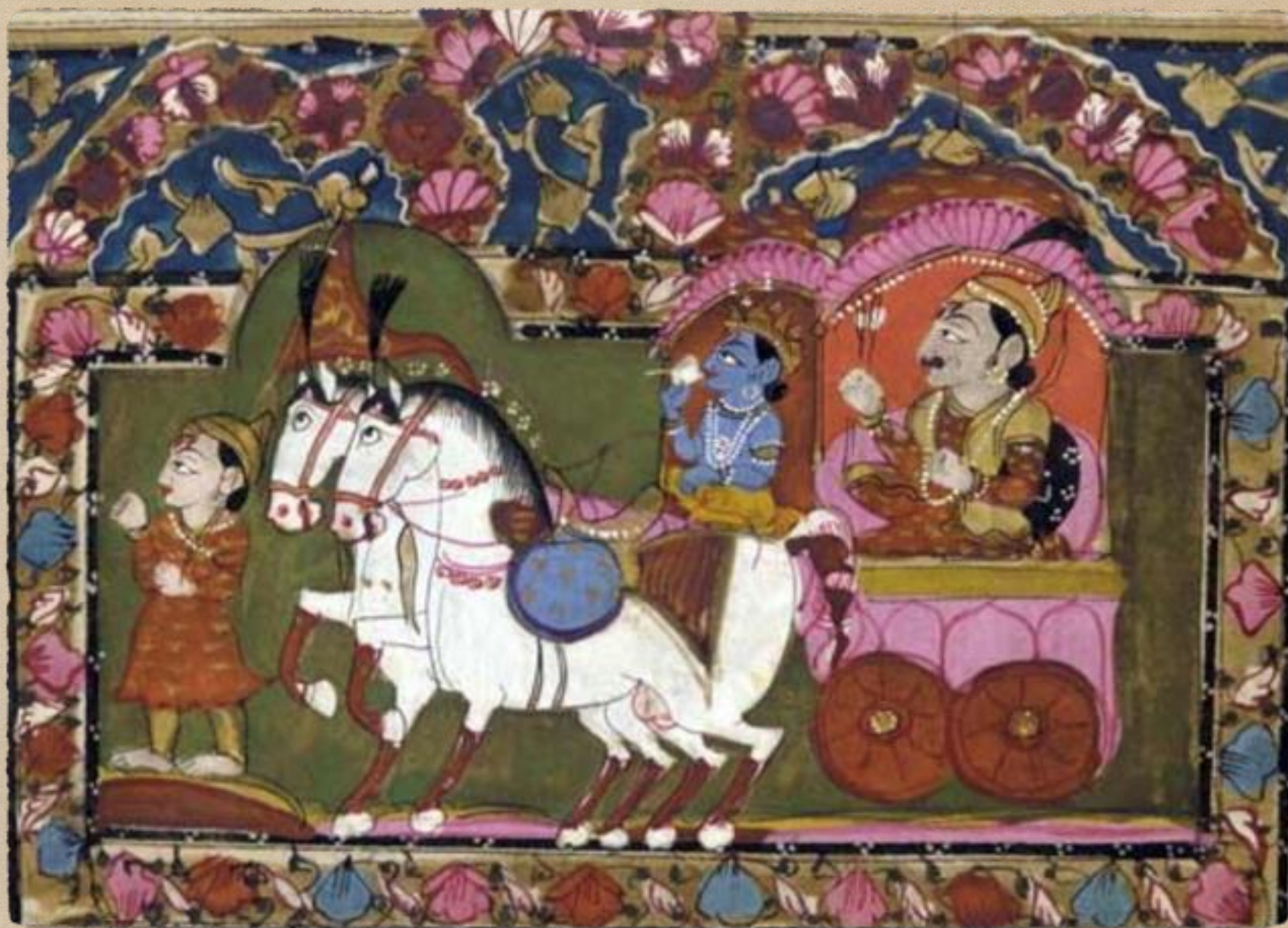
6. *Chün-shu*, or *Kin-shu* (*Alligation*), relating to motion problems (couriers, hare and hound) and alligation.

7. *Ying-pu-tsu*, or *Ying-nu* (*Excess and deficiency*), relating to the Rule of False Position,¹ the terms "excess" and "deficiency" relating to two concepts that are used in this rule.

8. *Fang-ch'êng* (*Equation*), relating to simultaneous linear equations, with some idea of determinants.

9. *Kou-ku* (*Right triangle*), relating to the Pythagorean Triangle.

Arithmetic in Nine Section



3. India

Astronomy

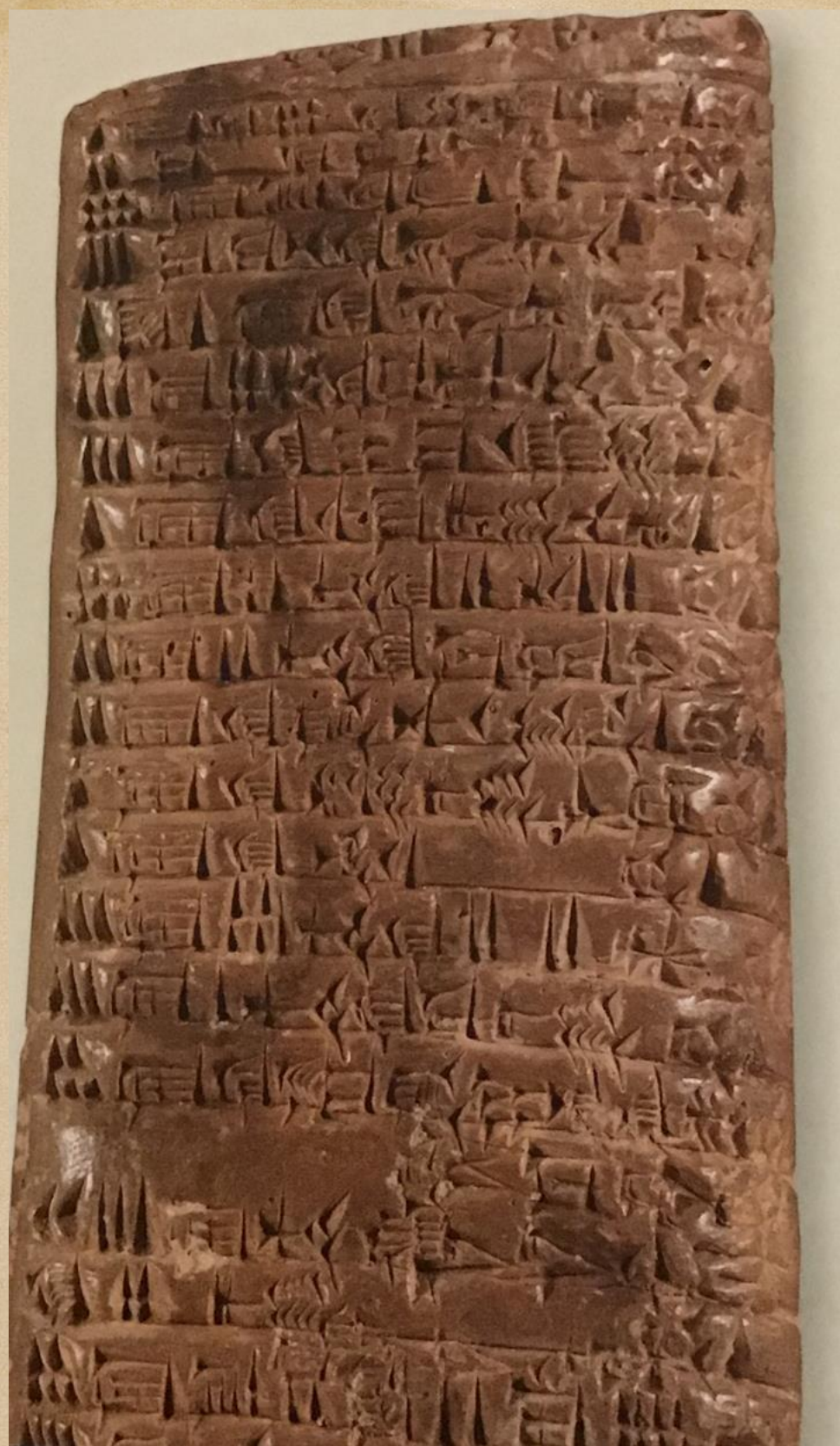


The snake sacrifice of Janamejaya

sand years before our era. The Vedas, the sacred writings of India, lead us to understand that in this period some attention was given to astronomy, as was the case in contemporary China, Mesopotamia, and Egypt.

All that we can say, therefore, about this period of Hindu mathematics is that there is some evidence from ancient literature that in very early times India paid attention to astronomy and calculation, just as was the case with other advanced peoples of that period.²



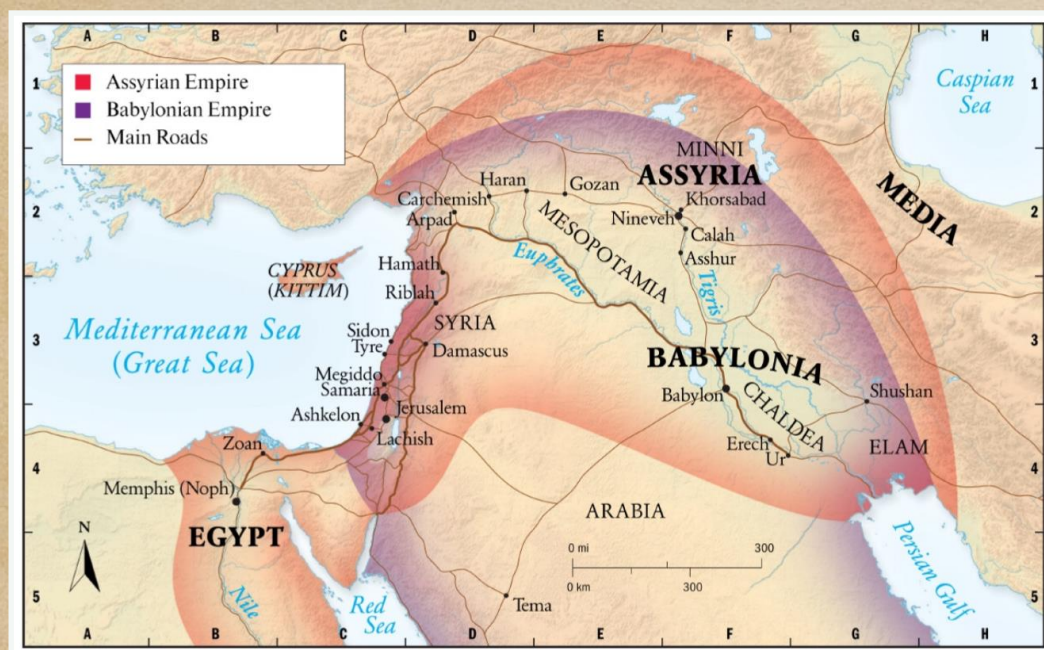


4. Babylon

Map of ancient Assyria



Early Babylonian Mathematics. For our purposes Chaldea and Babylonia are synonymous, each name referring to the land extending from the delta of the Tigris and Euphrates northward to Assyria, the hilly, forest-covered district originally surrounding the ancient capital of Assur (Asshur). Indeed, it is convenient at present to consider as one large group all those Semitic peoples descended from the wanderers from the Southern Grasslands who settled in Assyria, in the region about Nineveh, in Asia Minor, and along the Phoenician coast. We shall also find it convenient to include a non-Semitic tribe, the Sumerians, who dwelt in the land of Sumer at the head of the Persian Gulf, directly in one of the chief paths of world



Sumarian Tablet

2800 B.C.



Numerals of Babylon

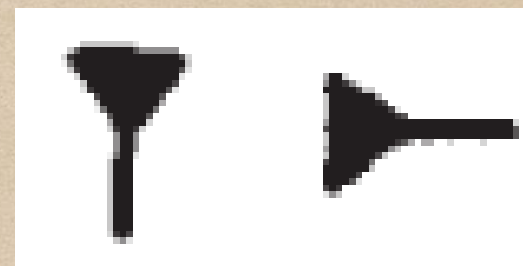
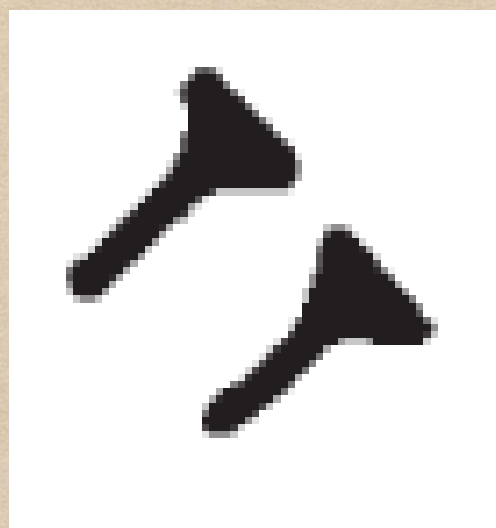
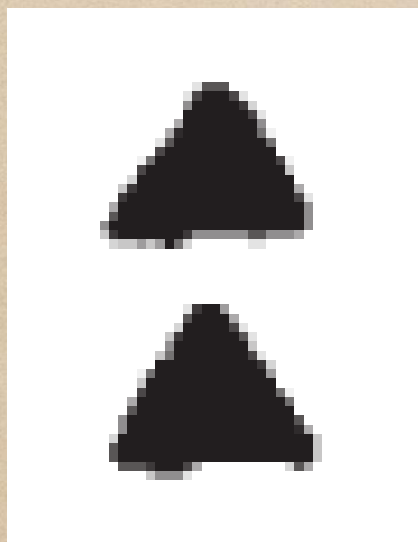
1		11		21		31		41		51	
2		12		22		32		42		52	
3		13		23		33		43		53	
4		14		24		34		44		54	
5		15		25		35		45		55	
6		16		26		36		46		56	
7		17		27		37		47		57	
8		18		28		38		48		58	
9		19		29		39		49		59	
10		20		30		40		50			

Numerals of Babylon

ตารางเลขยกกำลังสองจากยุคบาบิโลเนีย

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Numerals of Babylon



Scale of Sixty



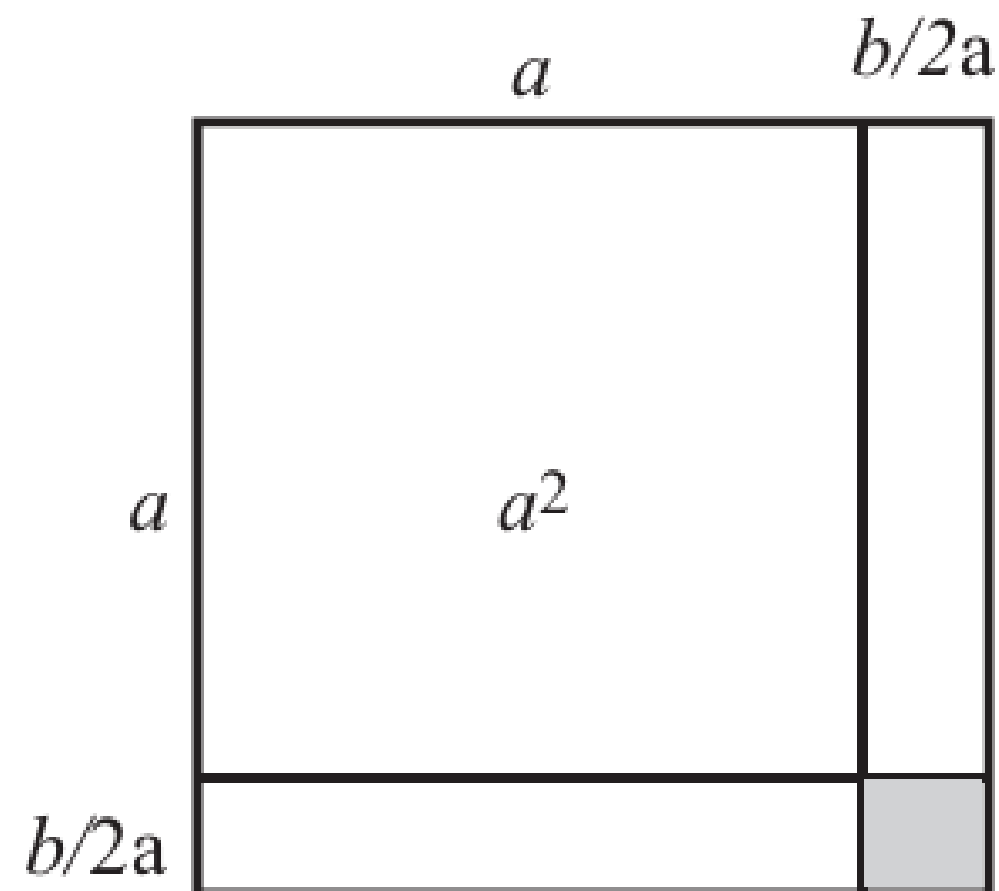
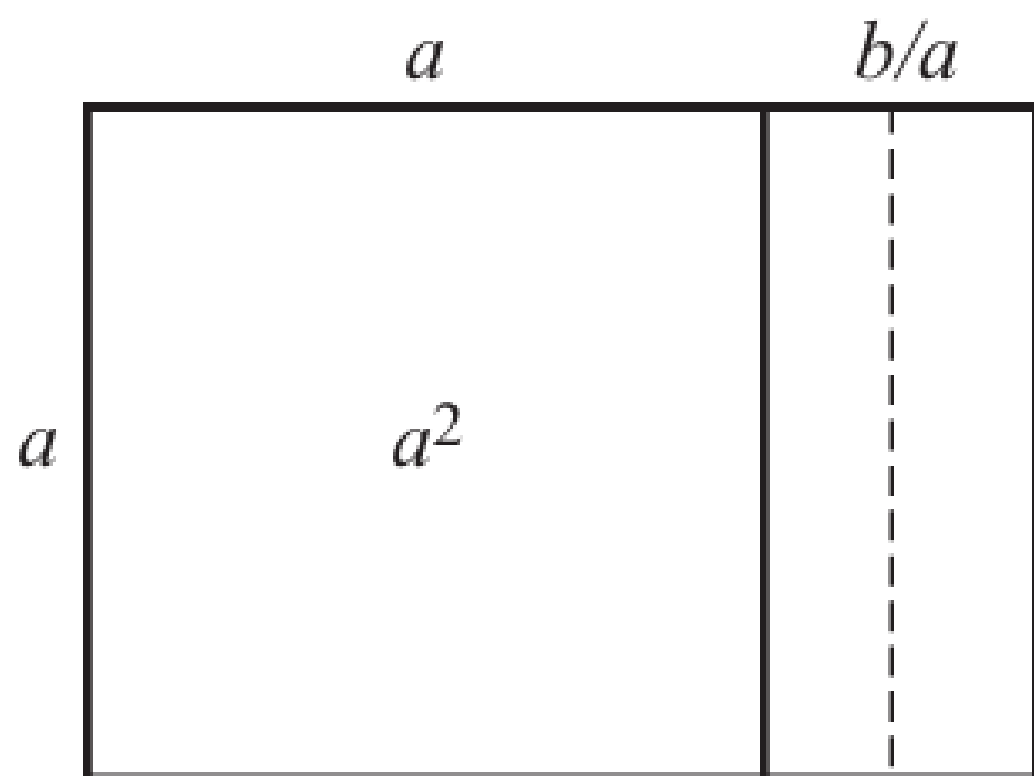
Scale of Sixty. One peculiarity of Babylonian arithmetic is the constant use of the number 60,—a use which finally suggested the development of sexagesimal fractions and which still survives in our division of degrees, hours, and minutes into sixty sub-units. It is generally thought that the Babylonians, interested as they were in watching the stars, early came to believe that the circle of the year consisted of 360 days. It is also thought that they knew that the side of the regular inscribed hexagon is equal to the radius of the circle, this property suggesting the division of 360 into six equal parts, and 60 being thus looked upon as a kind of mystic number. This may, indeed, be the origin of this use of 60, but we find other nations using 40, 20, and even 15 in somewhat the same way, with no apparent reason, so that all such customs may have developed from racial notions which were started by some leader or sect with no particular reason in mind. It is more probable that 60 was chosen because of its integral divisors 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30, thus rendering work with its fractional parts very simple.

Babylonian Geometry

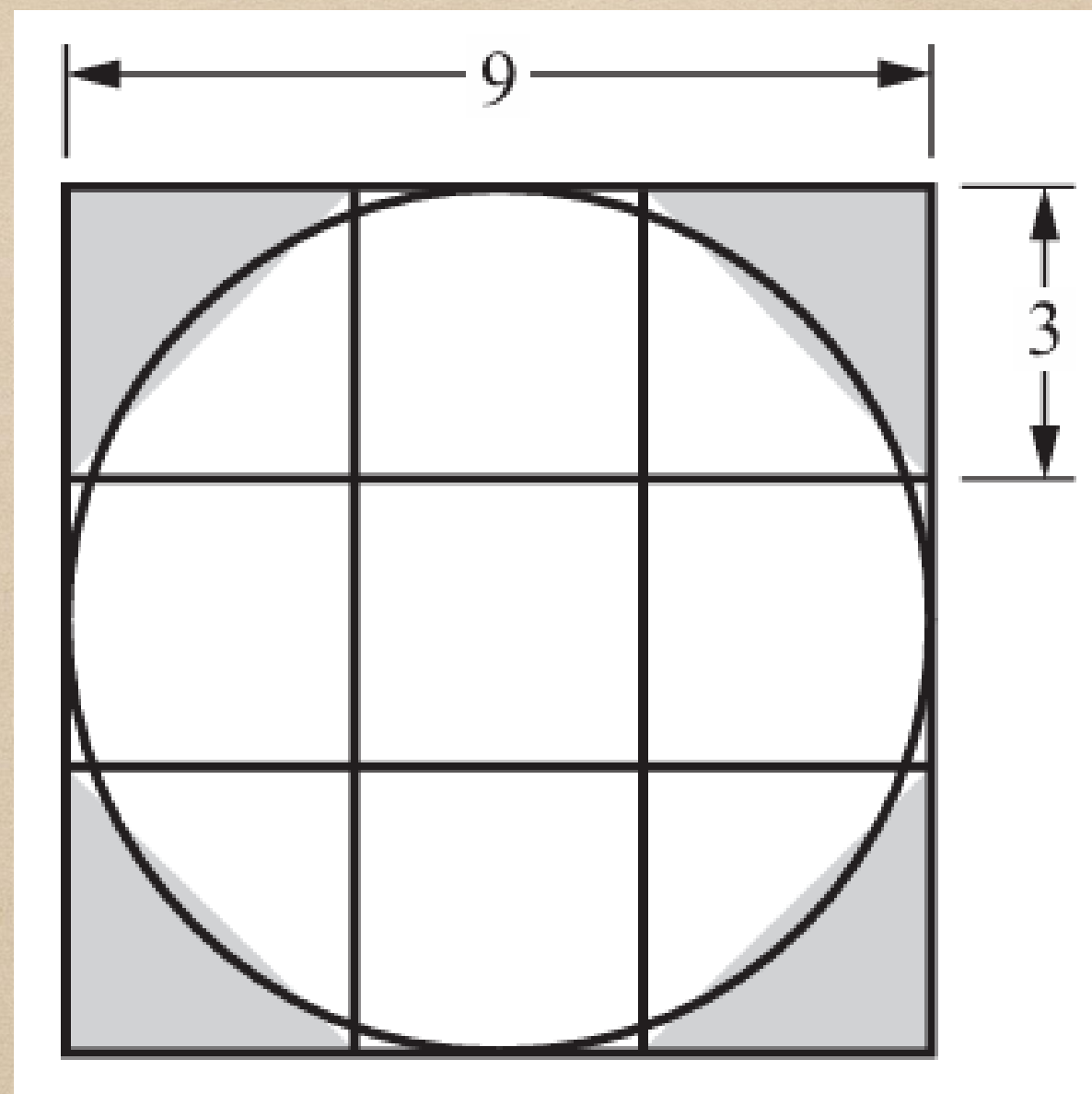


Babylonian Geometry. The tablets found at Nippur and elsewhere also give us some knowledge of the Babylonian geometry. From these it seems that as early as 1500 B.C. the Babylonians could find the area of a rectangle, including that of a square; the area of a right-angled triangle; the area of a trapezium (trapezoid); and possibly the area of a circle, the volume of a parallelepiped, and the volume of a cylinder. There is ground for the belief that they knew the law of expansion of $(a + b)^2$, although we have no knowledge as to whether this was inferred from a geometric figure or from their extensive study of square numbers. There is also some reason to believe that they knew the abacus, since it has been suggested that one of their signs (SID) may have been derived from a pictograph of such an instrument.

Babylonian Geometry



Babylonian Geometry



Babylonian Formula

Thus, the Babylonian instructions amount to using a formula equivalent to the familiar rule

$$x = \sqrt{\left(\frac{a}{2}\right)^2 + b} - \frac{a}{2}$$

for solving the quadratic equation $x^2 + ax = b$. Although the Babylonian mathematician had no “quadratic formula” that would solve all quadratic equations, the instructions in these concrete examples are so systematic that we can be pretty certain they were intended to illustrate a general procedure.



5. Egypt

The Great Pyramid of Gízeh
After 2900 B.C.



Egyptian Engineers

Accuracy of Early Engineers. Such was the degree of accuracy secured by these early surveyors that Petrie found the maximum error in fixing the length of the sides of the Great Pyramid to be only 0.63 of an inch, or less than $\frac{1}{14000}$ of the total length, and the angle error at the corners to be 12", or only $\frac{1}{27000}$ of a right angle.

Egyptian Engineers

The Great Pyramid has down to the present red adventurous minds to the wildest speculations. These pyramid mystics (or as they are sometimes uncharitably called, pyramidiots) have ascribed to the ancient builders all sorts of metaphysical intentions and esoteric knowledge. Among the extraordinary things claimed is that the pyramid was built so that half the perimeter of the base divided by the height should be exactly equal to π . While the difference between the two values

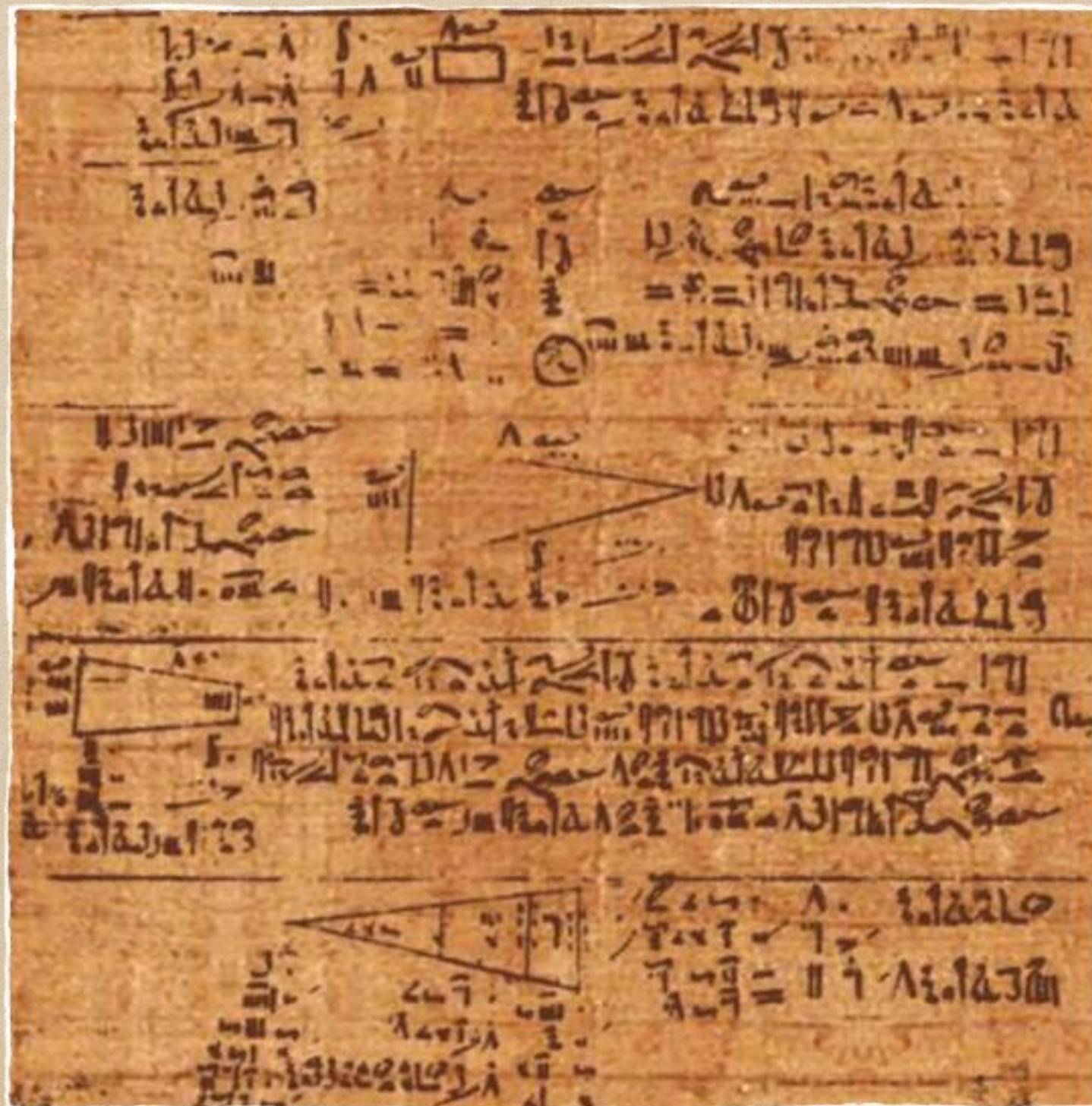
$$\pi = 3.1415926 \dots \quad \text{and} \quad \frac{2(755.78)}{481.2} = 3.14123 \dots$$

Mathematics of the Feudal Age

Amenemhat III

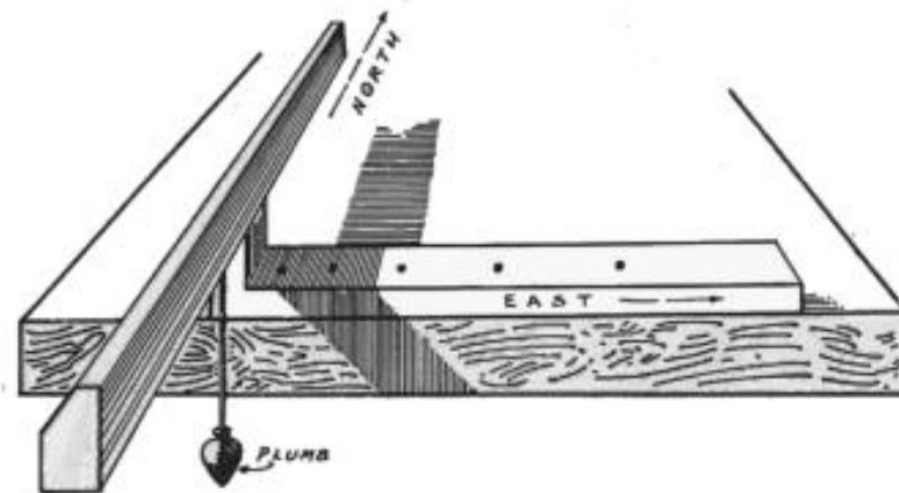
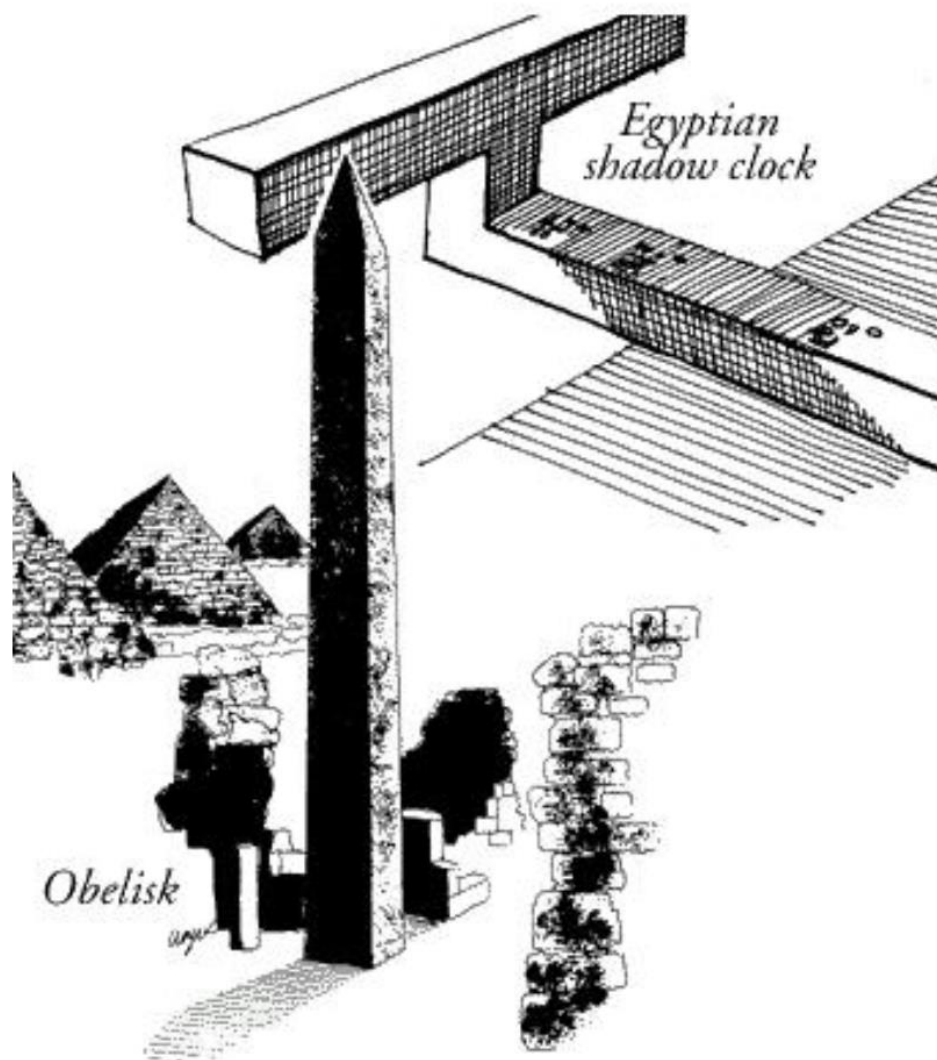


Ahmes Papyrus : 1650 B.C



The oldest sundial : 1500 B.C

The oldest dial known c. 1500 B.C. The sundial is considered to be the first scientific instrument.



Rameses II

The division of
the day into
twelve parts



Harris Papyrus : 1167 B.C

Rameses IV



Commercial System

Numerals of Egypt

1	2	3	4	5	6	7	8	9	10
	∩	∩∩	—	>	⌌	⌌⌌	⌌⌌⌌	⌌⌌⌌⌌	⌌⌌⌌⌌⌌
20	30	40	50	60	70	80	90	100	1000
⌌⌌	⌌⌌⌌	⌌⌌⌌⌌	⌌⌌⌌⌌⌌	⌌⌌⌌⌌⌌⌌	⌌⌌⌌⌌⌌⌌⌌	⌌⌌⌌⌌⌌⌌⌌⌌	⌌⌌⌌⌌⌌⌌⌌⌌⌌	⌌⌌⌌⌌⌌⌌⌌⌌⌌⌌	⌌⌌⌌⌌⌌⌌⌌⌌⌌⌌⌌

1	10	100	1000	10,000	100,000	1,000,000	10,000,000
	∩	∩∩	⌌⌌⌌	⌌	⌌⌌⌌	⌌⌌⌌⌌⌌⌌ or ⌌⌌⌌⌌⌌⌌	⌌⌌⌌⌌⌌⌌⌌⌌⌌⌌

Seven old women were on the road to Rome;
Each woman had seven donkeys;
Each donkey carried seven sacks;
Each sack contained seven loaves of bread;
With each loaf were seven knives;
Each knife was in seven sheaths.
What is the total?

$$x + \frac{x}{7} = 19$$



พื้นที่ 60 ตร.หน่วย เส้นทแยงมุมยาว 13 หน่วย
จงหาความยาวด้านของรูปสี่เหลี่ยม

Discussion 2

สำคัญ มโนทัศน์ที่ได้จากการเรียนรู้

1. ยุคโบราณเริ่มต้นในช่วงเวลาประมาณก่อน 3,000 ปีก่อนคริสต์ศักราช ซึ่งเป็นช่วงที่มนุษย์มีการใช้ภาษาเขียน
2. คัมภีร์ฮีบรูถึงการนำเส้นมาซ้อนกันสามเส้นได้ 8 รูปแบบเรียกว่า ปากัว
3. ตำราโจวปี่เป็นตำราคณิตศาสตร์ที่เก่าแก่ที่สุดของจีน
4. ตำราคณิตศาสตร์เก้าบทได้ค่าพายประมาณ 3
5. คัมภีร์แรกของอินเดียที่ถูกบันทึกด้วยตัวอักษรคือ ฤคเวท
6. 60 เป็นเลขสัณฐานของชาวบาบิโลนจากการค้นพบทฤษฎี รูปหกเหลี่ยมด้านเท่ามุมเท่าแบบในวงกลมจะมีความยาวของด้านเท่ากับรัศมีของวงกลม
7. ค่าพายของชาวบาบิโลนมีค่าประมาณ 3.16



คำถามท้ายบทที่ 2:

1. ระบบแลกเปลี่ยนโดยมีสื่อกลางในยุคโบราณเป็นอย่างไร
2. ระบบการจัดเก็บภาษีในยุคโบราณเป็นอย่างไร
3. จตุรัสกลคืออะไร
4. ฤคเวทเป็นรากฐานให้กับวรรณกรรมชื่อดัง 2 เรื่องของอินเดียคือเรื่องใด
5. ทำไมนักประวัติศาสตร์จึงเชื่อว่าการนับจำนวนด้วยอักษรลัมเป็นเลขฐาน 60
6. ทำไมนักประวัติศาสตร์จึงเชื่อว่าอียิปต์มีความรู้คณิตศาสตร์ที่พัฒนาอย่างมาก
ทั้ง ๆ ที่มีการค้นพบหลักฐานที่เป็นลายลักษณ์อักษรน้อยมาก

There are only 25 problems in the Moscow Papyrus, but one of them contains the masterpiece of ancient geometry. Problem 14 shows that the Egyptians of about 1850 B.C. were familiar with the correct formula for the volume of a truncated square pyramid (or frustum). In our notation, this is

$$V = \frac{h}{3}(a^2 + ab + b^2),$$

where h is the altitude and a and b are the lengths of the sides of the square base and square top, respectively.

