แบบฝึกหัด 5.6

1. $\int \sin^2 x \cos x \, dx$

2. $\int \cot^3 x \csc^4 x \, dx$

3. $\int \sin^{1/3} x \cos x \, dx$

4. $\int \cot^3 x \, dx$

5. $\int \cos^5 x \sin x \, dx$

6. $\int \csc^4 x \, dx$

7. $\int \sin^2 2x \cos^2 2x \, dx$

8. $\int \sin x \cos 2x \, dx$

9. $\int \tan^5 x \sec^4 x \, dx$

10. $\int \sin 3t \cos 2t \, dt$

11. $\int \tan^4 x \sec^4 x \, dx$

12. $\int \sin x \cos\left(\dfrac{x}{2}\right) dx$

13. $\int \tan 3t \sec^3 3t \, dt$

14. $\int \cos 5t \cos\left(\dfrac{3t}{4}\right) dt$

15. $\int \sec^4 x \, dx$

16. $\int x \tan^2 x^2 \sec^2 x^2 \, dx$

17. $\int \tan^4 5x \, dx$

18. $\int \dfrac{\sec \sqrt{x}}{\sqrt{x}} \, dx$

19. $\int \tan x \sec^4 x \, dx$

20. $\int \sin 4x \sin 7x \, dx$

1. $\dfrac{1}{3}\sin^3 x + C$

2. $\dfrac{1}{6}\dfrac{\cos^4 x}{\sin^6 x} - \dfrac{1}{12}\dfrac{\cos^4 x}{\sin^4 x} + C$

3. $\dfrac{3}{4}\sin^{4/3} x + C$

4. $-\dfrac{1}{2}\cot^2 x + \dfrac{1}{2}\ln(\cot^2 x + 1) + C$

5. $-\dfrac{1}{6}\cos^6 x + C$

6. $-\dfrac{1}{3}\dfrac{\cos x}{\sin^3 x} - \dfrac{2}{3}\dfrac{\cos x}{\sin x} + C$

7. $-\dfrac{1}{10}\sin^2 2x\cos^3 2x - \dfrac{1}{15}\cos^3 2x + C$

8. $-\dfrac{1}{6}\cos 3x + \dfrac{1}{2}\cos x + C$

9. $\dfrac{1}{8}\dfrac{\sin^6 x}{\cos^8 x} + \dfrac{1}{24}\dfrac{\sin^6 x}{\cos^6 x} + C$

10. $-\dfrac{1}{10}\cos 3t - \dfrac{1}{2}\cos t + C$

11. $\dfrac{1}{7}\dfrac{\sin^5 x}{\cos^7 x} + \dfrac{2}{35}\dfrac{\sin^5 x}{\cos^3 x} + C$

12. $-\dfrac{1}{3}\cos\left(\dfrac{3x}{2}\right) - \cos\left(\dfrac{x}{2}\right) + C$

13. $\dfrac{1}{9}\sec^3 3t + C$

14. $\dfrac{2}{17}\sin\left(\dfrac{17t}{4}\right) + \dfrac{2}{23}\sin\left(\dfrac{23t}{4}\right) + C$

15. $\dfrac{1}{3}\dfrac{\sin x}{\cos^3 x} + \dfrac{2}{3}\dfrac{\sin x}{\cos x} + C$

16. $\dfrac{1}{6}\dfrac{\sin^3 x^2}{\cos^3 x^2} + C$

17. $\dfrac{1}{15}\tan^3 5x - \dfrac{1}{5}\tan 5x + x + C$

18. $2\ln(\sec\sqrt{x} + \tan\sqrt{x}) + C$

19. $\dfrac{2}{21}\dfrac{(4\cos^2 x + 3)\sqrt{\dfrac{\sin x}{\cos x}}\sin x}{\cos^3 x} + C$

20. $\dfrac{1}{6}\sin 3x - \dfrac{1}{22}\sin 11x + C$

น การอินทิเกรตโดยการเปลี่ยนตัวแปร
ให้อยู่ในรูปของฟังก์ชันตรีโกณมิติ "

แบบฝึกหัด 5.7

จงหาอินทิกรัลของฟังก์ชันต่อไปนี้

1. $\displaystyle\int \frac{\sqrt{9-x^2}}{x^2}\,dx$

2. $\displaystyle\int \frac{x^2}{\sqrt{x^2-1}}\,dx$

3. $\displaystyle\int \frac{1}{\left(x^2+4\right)^{3/2}}\,dx$

4. $\displaystyle\int \frac{1}{x^2\left(x^2+4\right)^{1/2}}\,dx$

5. $\displaystyle\int \sqrt{16-x^2}\,dx$

6. $\displaystyle\int \frac{1}{\left(4-x^2\right)^{3/2}}\,dx$

7. $\displaystyle\int \frac{x^3}{\sqrt{9+x^2}}\,dx$

8. $\displaystyle\int \frac{1}{\sqrt{9+x^2}}\,dx$

9. $\displaystyle\int \frac{x+1}{\sqrt{9-x^2}}\,dx$

10. $\displaystyle\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$

11. $\displaystyle\int x^3\sqrt{9-x^2}\,dx$

12. $\displaystyle\int \frac{x^2}{\sqrt{1-4x^2}}\,dx$

คำตอบแบบฝึกหัด **5.7**

1. $-\dfrac{\sqrt{9-x^2}}{x} - \arcsin\dfrac{x}{3} + c$

2. $\dfrac{x}{2}\sqrt{x^2-1} + \dfrac{1}{2}\ln\left| x + \sqrt{x^2+1} \right| + c$

3. $\dfrac{x}{4\sqrt{x^2+4}} + c$

4. $-\dfrac{\sqrt{x^2+4}}{4x} + c$

5. $\dfrac{1}{2}\left[x\sqrt{16-x^2} + 16\arcsin\left(\dfrac{x}{4}\right) \right] + c$

6. $\dfrac{x}{4\sqrt{4-x^2}} + c$

7. $\dfrac{\left(9+x^2\right)^{3/2}}{3} - 9\sqrt{9+x^2} + c$

8. $\ln\left(x + \sqrt{9+x^2} \right) + c$

9. $-\sqrt{9-x^2} + \arcsin\left(\dfrac{x}{3}\right) + c$

10. $-\dfrac{\sqrt{9-x^2}}{9x} + c$

11. $\dfrac{\left(9-x^2\right)^{5/2}}{5} - 3\left(9-x^2\right)^{5/2} + c$

12. $\dfrac{1}{16}\arcsin 2x - \dfrac{x\sqrt{1-4x^2}}{8} + c$

" การอินทิเกรตฟังก์ชันตรรกยะโดยการทำเป็นเศษส่วนย่อย "

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แบบฝึกหัด 5.8

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จงหาค่าอินทิกรัลของฟังก์ชันต่อไปนี้

1. $\int \dfrac{1}{(x-2)(x+1)}\,dx$

2. $\int \dfrac{x+4}{x^2+5x-6}\,dx$

3. $\int \dfrac{1}{x^2+x}\,dx$

4. $\int \dfrac{x^2-9x-9}{x^3-9x}\,dx$

5. $\int \dfrac{3x^2-10}{x^2-4x+4}\,dx$

6. $\int \dfrac{x^5+2x^2+1}{x^3-x}\,dx$

7. $\int \dfrac{x^2+x-16}{(x+1)(x-3)^2}\,dx$

8. $\int \dfrac{2x^2-1}{(4x-1)(x^2+1)}\,dx$

9. $\int \dfrac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)}\,dx$

10. $\int \dfrac{x^2+1}{(x^2+2x+3)^2}\,dx$

11. $\int \dfrac{1-\sqrt{x}}{1+\sqrt{x}}\,dx$

12. $\int \dfrac{1}{1+e^x}\,dx$

13. $\int \dfrac{2x+1}{(x-1)(x-2)}\,dx$

14. $\int \dfrac{x^2+2}{x^3-3x^2+3x-1}\,dx$

15. $\int \dfrac{x}{x^4+6x^2+5}\,dx$

16. $\int \dfrac{x^4-x^3+5x^2+x+3}{x^3+1}\,dx$

17. $\int \dfrac{8x^3+13x}{(x^2+2)^2}\,dx$

18. $\int \dfrac{x^3-2x}{x^2+3x+2}\,dx$

19. $\int \dfrac{12}{x^4-x^3-2x^2}\,dx$

20. $\int \dfrac{2x^3+10x}{(x^2+1)^2}\,dx$

คำตอบแบบฝึกหัด 5.8

1. $-\dfrac{1}{3}\ln(x+1)+\dfrac{1}{3}\ln(x-2)$

2. $\dfrac{5}{7}\ln(x-1)+\dfrac{2}{7}\ln(x+6)$

3. $-\ln(x+1)+\ln x$

4. $-\dfrac{3}{2}\ln(x-3)+\dfrac{3}{2}\ln(x+3)+\ln x$

5. $3x-\dfrac{2}{x-2}+12\ln(x-2)$

6. $\dfrac{x^3}{3}+x+\ln(x+1)+2\ln(x-1)-\ln x$

7. $x-2\ln(x-3)+7\ln(x-1)$

8. $\dfrac{6}{17}\ln(x^2+1)+\dfrac{3}{17}\arctan x-\dfrac{7}{34}\ln(4x-1)$

9. $3\arctan x+\dfrac{1}{2}\ln(x^2+3)$

10. $\dfrac{1}{2}\sqrt{2}\arctan\left(\dfrac{(2x+2)\sqrt{2}}{4}\right)+\dfrac{1}{x^2+2x+3}$

11. $-x+4\sqrt{x}-4\ln(1+\sqrt{x})$

12. $-\dfrac{\ln(1+e^x)}{\ln e}+\dfrac{\ln e^x}{\ln e}$

13. $-3\ln(x-1)+5\ln(x-2)$

14. $-\dfrac{2}{x-1}-\dfrac{3}{2(x-1)^2}+\ln(x-1)$

15. $-\dfrac{1}{8}\ln(x^2+5)+\dfrac{1}{8}\ln(x^2+1)$

16. $\dfrac{x^2}{2}-x+3\ln(x+1)+\ln(x^2-x+1)+\dfrac{4}{3}\sqrt{3}\arctan\left(\dfrac{(2x-1)\sqrt{3}}{3}\right)$

17. $\dfrac{3}{2(x^2+2)}+4\ln(x^2+2)$

18. $\dfrac{x^2}{2}-3x+\ln(x+1)+4\ln(x+2)$

19. $\dfrac{6}{x}-4\ln(x+1)+\ln(x-2)+3\ln x$

20. $\ln(x^2+1)-\dfrac{4}{x^2+1}$

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แบบฝึกหัด 5.9

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จงหาค่าอินทิกรัลของฟังก์ชันต่อไปนี้

1. $\int \dfrac{1}{x(1-\sqrt[4]{x})}\,dx$

2. $\int \dfrac{1}{2\sqrt{x}-\sqrt[4]{x}}\,dx$

3. $\int x\sqrt[3]{3x+2}\,dx$

4. $\int \dfrac{z^5}{\sqrt{1+z^3}}\,dz$

5. $\int \dfrac{1}{\sqrt{1+\sqrt{x}}}\,dx$

6. $\int \dfrac{1}{(1+6x)^{\frac{1}{2}}+(1+6x)^{\frac{1}{3}}}\,dx$

คำตอบแบบฝึกหัด 5.9

1. $4\ln\left|\dfrac{\sqrt[4]{x}}{1-\sqrt[4]{x}}\right|+c$

2. $\sqrt{x}+\sqrt[4]{x}+\dfrac{1}{2}\ln\left|2\sqrt[4]{x}-1\right|+c$

3. $\dfrac{(3x+2)^{\frac{7}{3}}}{21}-\dfrac{(3x+2)^{\frac{4}{3}}}{6}+c$

4. $\dfrac{2}{9}(z^3-2)\sqrt{z^3+1}+c$

5. $\dfrac{4}{3}(\sqrt{x}-2)\sqrt{1+\sqrt{x}}+c$

6. $\ln\left|(1+6x)^{\frac{1}{6}}+1\right|+c$