An Introduction to Control Charts

(Hong Qin, 2024)

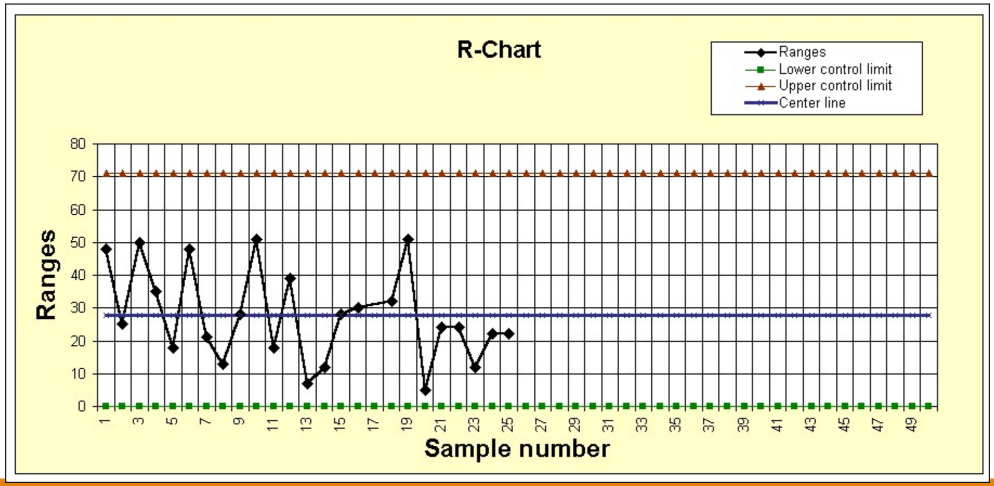
Basic Conceptions

What is a control chart?

The control chart is a graph used to study how a process changes over time. Data are plotted in time order.

A control chart always has a central line for the average, an upper line for the upper control limit and a lower line for the lower control limit.

Lines are determined from historical data. By comparing current data to these lines, you can draw conclusions about whether the process variation is consistent (in control) or is unpredictable (out of control, affected by special causes of variation).



Basic Conceptions

When to use a control chart?

- Controlling ongoing processes by finding and correcting problems as they occur.
- Predicting the expected range of outcomes from a process.
- Determining whether a process is stable (in statistical control).
- Analyzing patterns of process variation from special causes (non-routine events) or common causes (built into the process).
- Determining whether the quality improvement project should aim to prevent specific problems or to make fundamental changes to the process.

(http://www.asq.org/learn-about-quality/data-collection-analysis-tools/overview/control-chart.html)

Basic Conceptions

Control Chart Basic Procedure

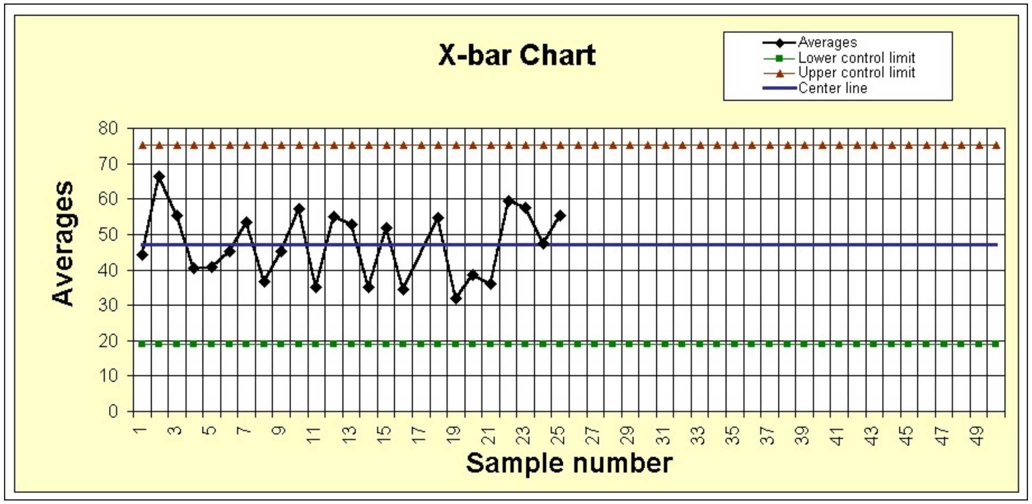
- Choose the appropriate control chart for the data.
- Determine the appropriate time period for collecting and plotting data.
- Collect data, construct the chart and analyze the data.
- Look for “out-of-control signals” on the control chart. When one is identified, mark it on the chart and investigate the cause. Document how you investigated, what you learned, the cause and how it was corrected.
- Continue to plot data as they are generated. As each new data point is plotted, check for new out-of-control signals.

(http://www.asq.org/learn-about-quality/data-collection-analysis-tools/overview/control-chart.html)

Basic Principles

Basic components of control charts

- A centerline, usually the mathematical average of all the samples plotted;
- Lower and upper control limits defining the constraints of common cause variations;
- Performance data plotted over time.



Basic Principles

General model for a control chart

- UCL = $\mu + k\sigma$
- CL = μ
- LCL = $\mu - k\sigma$

where μ is the mean of the variable, and σ is the standard deviation of the variable. UCL=upper control limit; LCL = lower control limit; CL = center line. where k is the distance of the control limits from the center line, expressed in terms of standard deviation units. When k is set to 3, we speak of 3-sigma control charts. Historically, k = 3 has become an accepted standard in industry.

(Proposed by Walter A. Shewhart in 1920's)

Basic Principles of Control Charts

Types of the control charts

Variables control charts

- Variable data are measured on a continuous scale. For example: time, weight, distance or temperature can be measured in fractions or decimals.
- Applied to data with continuous distribution

Attributes control charts

- Attribute data are counted and cannot have fractions or decimals. Attribute data arise when you are determining only the presence or absence of something: success or failure, accept or reject, correct or not correct. For example, a report can have four errors or five errors, but it cannot have four and a half errors.
- Applied to data following discrete distribution

(http://www.asq.org/learn-about-quality/data-collection-analysis-tools/overview/control-chart.html)

Basic Principles of Control Charts

Variables control charts

- X-bar and R chart (also called averages and range chart)
- X-bar and s chart
- moving average–moving range chart (also called MA–MR chart)
- target charts (also called difference charts, deviation charts and nominal charts)
- CUSUM (cumulative sum chart)
- EWMA (exponentially weighted moving average chart)
- multivariate chart

http://www.statgraphics.com/control_charts.htm#variables

Basic Principles of Control Charts

Attributes control charts

- p chart (proportion chart)
- n_p chart
- c chart (count chart)
- u chart

R-Chart

Always look at the Range chart first. The control limits on the X-bar chart are derived from the average range, so if the Range chart is out of control, then the control limits on the X-bar chart are meaningless.

Look for out of control points. If there are any, then the special causes must be eliminated..

There should be more than five distinct values plotted, and no one value should appear more than 25% of the time. If there are values repeated too often, then you have inadequate resolution of your measurements, which will adversely affect your control limit calculations. In this case, you'll have to look at how you measure the variable, and try to measure it more precisely.

Once the effect of the out of control points from the Range chart is removed, look at the X-bar Chart.

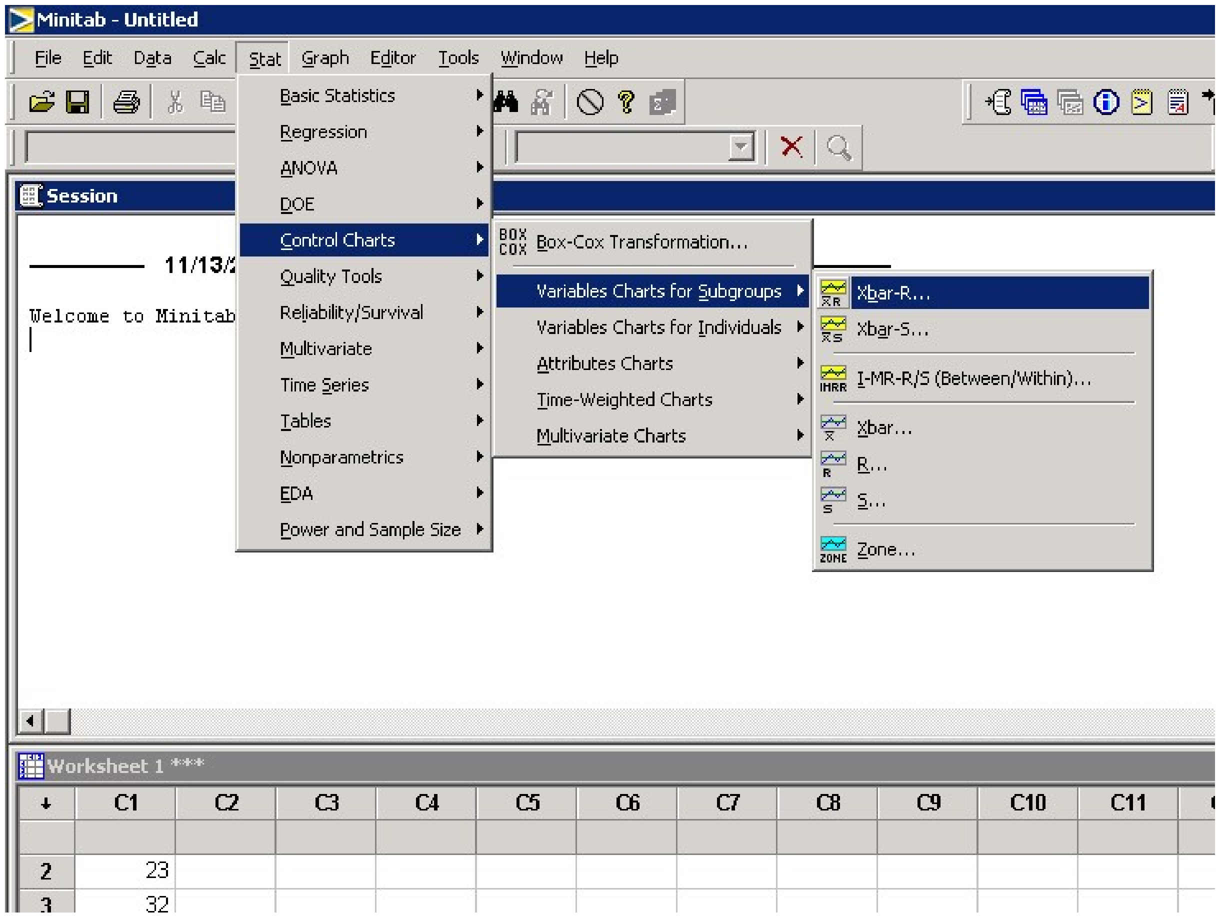
$$UCL = \overline{R}D_4$$

$$LCL = \overline{R}D_3$$

Example: R Control Chart

In the manufacturing of a certain machine part, the percentage of aluminum in the finished part is especially critical. For each production day, the aluminum percentage of five parts is measured. The table below consists of the average aluminum percentage of ten consecutive production days, along with the minimum and maximum sample values (aluminum percentage) for each day. The sum of the 10 samples means (below) is 258.8.

Day	1	2	3	4	5	6	7	8	9	10
Sample Mean	25.2	26.0	25.2	25.2	26.0	25.6	26.0	26.0	24.6	29.0
Maximum Value	26.6	27.6	27.7	27.4	27.6	27.4	27.5	27.9	26.8	31.6
Minimum Value	23.5	24.4	24.6	23.2	23.3	23.3	24.1	23.8	23.5	27.4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Basic Statistics
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Control Charts
Quality Tools
Reliability/Survival
Multivariate
Time Series
Tables
Nonparametrics
EDA
Power and Sample Size
Box-Cox Transformation...
Variables Charts for Subgroups
Variables Charts for Individuals
Attributes Charts
Time-Weighted Charts
Multivariate Charts
Xbar-R...
Xbar-S...
I-MR-R/S (Between/Within)...
Xbar...
R...
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Worksheet 1 ***
C1 C2 C3 C4 C5 C6 C7 C8 C9 C10 C11
2 23
3 32

X-bar Chart

$$UCL = \bar{\bar{x}} + A_3 \bar{s} (\bar{x} and S chart)$$

$$LCL = \bar{\bar{x}} - A_3 \bar{s} (\bar{x} and S chart)$$

$$A_2 \bar{R} for \bar{x} and R chart$$

The X-bar chart monitors the process location over time, based on the average of a series of observations, called a subgroup.

X-bar / Range charts are used when you can rationally collect measurements in groups (subgroups) of between two and ten observations. Each subgroup represents a "snapshot" of the process at a given point in time. The charts' x-axes are time based, so that the charts show a history of the process. For this reason, data should be time-ordered; that is, entered in the sequence from which it was generated. If this is not the case, then trends or shifts in the process may not be detected, but instead attributed to random (common cause) variation.

For subgroup sizes greater than ten, use X-bar / Sigma charts, since the range statistic is a poor estimator of process sigma for large subgroups. In fact, the subgroup sigma is always a better estimate of subgroup variation than subgroup range. The popularity of the Range chart is only due to its ease of calculation, dating to its use before the advent of computers.

For **subgroup sizes equal to one**, an Individual-X / Moving Range chart can be used, as well as EWMA or CuSum charts.

X-bar Charts are efficient at detecting relatively large shifts in the process average, typically shifts of +-1.5 sigma or larger. The larger the subgroup, the more sensitive the chart will be to shifts, providing a Rational Subgroup can be formed. For more sensitivity to smaller process shifts, use an EWMA or CuSum chart.

S Chart

The sample standard deviations are plotted in order to control the variability of a variable.

For sample size (n>10), the S-chart is more efficient than R-chart.

For situations where sample size exceeds 10, the X-bar chart and the S-chart should be used.

$$s_i = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\bar{s} = \frac{\sum s_i}{k}$$

$$UCL = \bar{s}B_4$$

$$LCL = \bar{s}B_3$$

(http://www.statsoft.com/textbook/stquacon.html)

S^2 Chart

In this chart, the sample variances are plotted in order to control the variability of a variable.

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

Moving Average (MA)/Range Chart

Moving Average / Range Charts are a set of control charts for variables data.

The Moving Average chart monitors the process location over time, based on the average of the current subgroup and one or more prior subgroups. The Range chart monitors the process variation over time.

Moving Average Charts are generally used for detecting **small shifts** in the process mean. **They will detect shifts of .5 sigma to 2 sigma much faster. They are, however, slower in detecting large shifts in the process mean.**

Always look at the Range chart first. The control limits on the Moving Average chart are derived from the average range, so if the Range chart is out of control, then the control limits on the Moving Average chart are meaningless.

(http://www.statsoft.com/textbook/stquacon.html)

Cumulative Sum (CUSUM) Chart

The CUSUM chart reacts more sensitively than the X-bar chart to a shifting of the mean value in the range of 0.5-2s; therefore, it is sited for monitoring processes with a high degree of imprecision.

If one plots the cumulative sum of deviations of successive sample means from a target specification, even minor, permanent shifts in the process mean will eventually lead to a sizable cumulative sum of deviations. Thus, this chart is particularly well-suited for detecting such small permanent shifts that may go undetected when using the X-bar chart.

(http://www.statsoft.com/textbook/stquacon.html)

Exponentially-weighted Moving Average (EWMA) Chart

The idea of moving averages of successive (adjacent) samples can be generalized. In principle, in order to detect a trend we need to weight successive samples to form a moving average; however, instead of a simple arithmetic moving average, we could compute a geometric moving average. It is also called Geometric Moving Average chart, see Montgomery, 1985, 1991).

EWMA Charts are generally used for detecting small shifts in the process mean. They will detect shifts of .5 sigma to 2 sigma much faster. They are, however, slower in detecting large shifts in the process mean. In addition, typical run tests cannot be used because of the inherent dependence of data points.

EWMA Charts may also be preferred when the subgroups are of size n=1.

$$EWMA_{(t+1)} = \lambda Y_t + (1-\lambda) EWMA_t$$

$$UCL = EWMA_1 + ks\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL = EWMA_1 - ks\sqrt{\frac{\lambda}{2-\lambda}}$$

where λ is the weighting factor. The factor k is chosen generally to be 2 or 3.

Attributes Control Charts

An example of a common quality characteristic classification would be designating units as "conforming units" or "nonconforming units". Another quality characteristic criteria would be sorting units into "non defective" and "defective" categories. Quality characteristics of that type are called attributes.

Examples of quality characteristics that are attributes are the number of failures in a production run, the proportion of malfunctioning wafers in a lot, the number of people eating in the cafeteria on a given day, etc.

Control charts dealing with the number of defects or nonconformities are called c charts (for count).

Control charts dealing with the proportion or fraction of defective product are called p chart (for proportion).

(http://www.itl.nist.gov/div898/handbook/pmc/section3/pmc33.htm)

P-Chart

To evaluate process stability when counting the fraction defective.

It is used when the sample size varies: the total number of circuit boards, meals, or bills delivered varies from one sampling period to the next.

Repeated samples of 150 coffee cans are inspected to determine whether a can is out of round or whether it contains leaks due to improper construction. Such a can is said to be nonconforming. Following is the data.

Sample	1	2	3	4	5	6	7	8	9	10
Nonconforming#	19	10	4	6	8	9	3	1	0	4

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n_j}}$$

$$LCL = \max[0, \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n_j}}]$$

P-Chart (cont.)

Evaluating process stability when counting the fraction defective.

The n_p chart is useful when it's easy to count the number of defective items and the sample size is always the same. Examples might include: the number of defective circuit boards, meals in a restaurant, teller interactions in a bank, invoices, or bills.

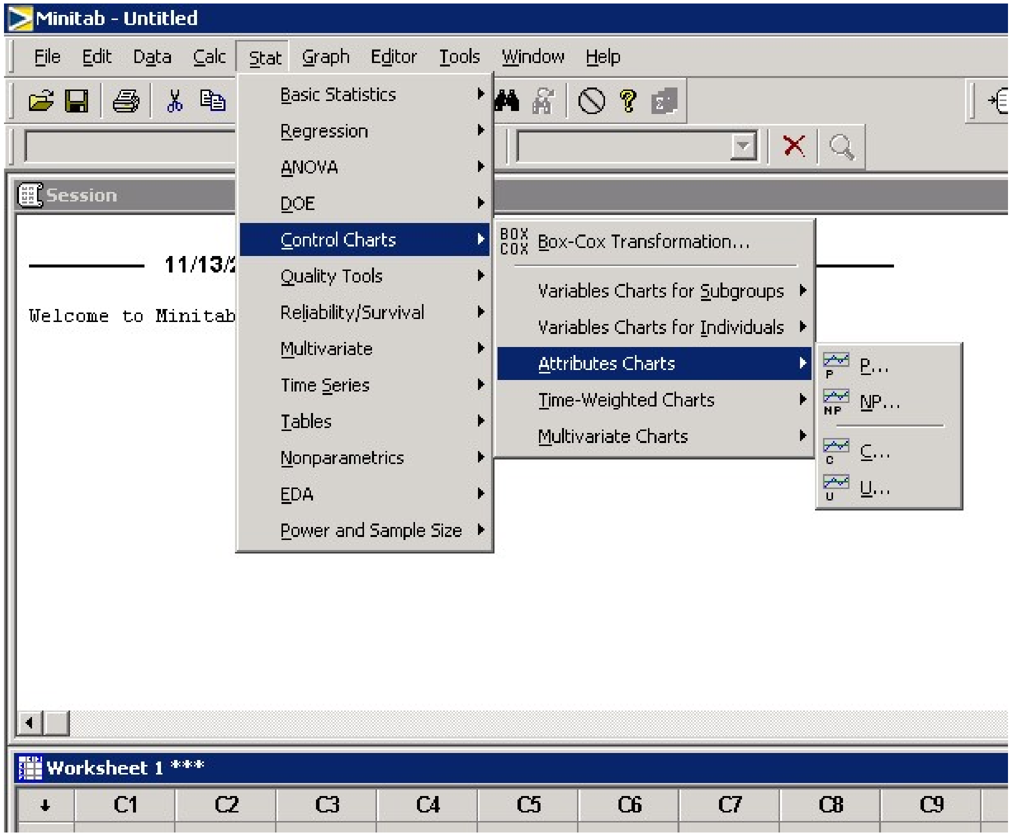
A fully capable process delivers zero defects. Although this may be difficult to achieve, it should still be our goal. Once we resolve the out-of-control point, we could use the quality problem solving process to begin to eliminate the common causes of defective paychecks. What are the most common types of paycheck errors? Why do they occur? What are the root causes of these paycheck errors?

$$\bar{p} = \frac{\sum_{j=1}^{m} (count)_j}{m \bullet n}$$

$$n\bar{p} = \frac{\sum_{j=1}^{m} (count)_j}{m}$$

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}\left(1-\bar{p}\right)}$$

$$LCL_{np} = MAX\left[0\,,\, n\bar{p} - 3\sqrt{n\bar{p}\left(1-\bar{p}\right)}\right]$$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Box-Cox Transformation...
Variables Charts for Subgroups
Variables Charts for Individuals
Attributes Charts
Time-Weighted Charts
Multivariate Charts
P...
NP...
C...
U...
Worksheet 1 ***
C1 C2 C3 C4 C5 C6 C7 C8 C9

C-Chart

Determining stability of "counted" data (e.g., errors per widget, inquiries per month, etc.)

The c chart will help evaluate process stability when there can be more than one defect per unit. Examples might include: the number of defective elements on a circuit board, the number of defects in a dining experience--order wrong, food too cold, check wrong, or the number of defects in bank statement, invoice, or bill.

This chart is especially useful when you want to know how many defects there are not just how many defective items there are.

The c chart is useful when it's easy to count the number of defects and the sample size is always the same.

(http://www.qimacros.com/qiwizard/cchart.html)

C-Chart (cont.)

An automobile assembly worker is interested in monitoring and controlling the # of minor paint blemishes appearing on the outside door panel on the driver's side of a certain make of automobile. The following data were obtained, using a sample of 25 door panel.

Sample	1	2	3	4	5	6	7	-----	-----	25
# of Paint Blemishes	19	10	4	6	8	9	3	------	-----	4

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$\bar{c} = \frac{\sum_{j=1}^{m} (count)_j}{m}$$

$$LCL = MAX\left[0, \bar{c} - 3\sqrt{\bar{c}}\right]$$

U-Chart

Determining stability of "counted" data (e.g., errors per widget, inquiries per month, etc.) when the sample size varies.

The u chart will help evaluate process stability when there can be more than one defect per unit.

This chart is especially useful when you want to know how many defects there are not just how many defective items there are. It's one thing to know how many defective circuit boards, meals, statements, invoices, or bills there are; it is another thing to know how many defects were found in these defective items.

It is used when the sample size varies: the number of circuit boards, meals, or bills delivered each day varies.

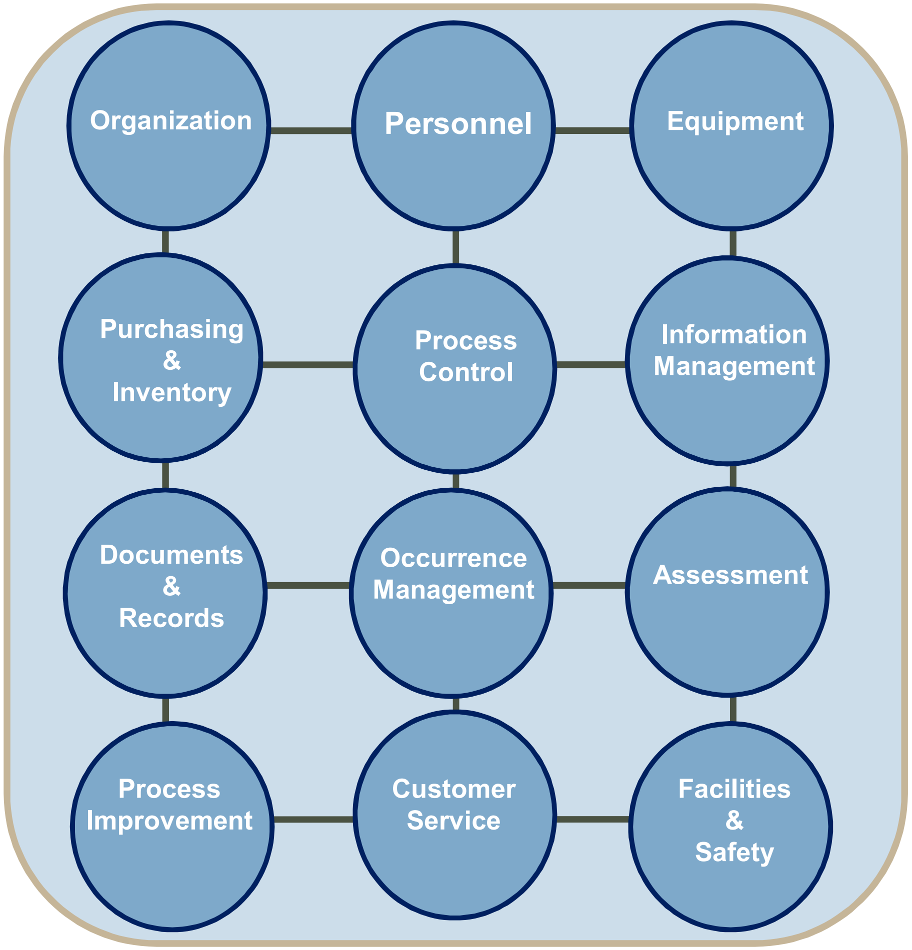
$$u_j = \frac{(count)_j}{n_j} \qquad UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_j}}$$

$$\bar{u} = \frac{\sum_{j=1}^{m}(count)_j}{m} \qquad LCL = MAX\left[0, \bar{u} - 3\sqrt{\frac{\bar{u}}{n_j}}\right]$$

Process Control:
Introduction to Quality Control

(CDC, WHO and CLSI, 2023)

The Quality Management System



Definition

Quality Control (QC) is part of quality management focused on fulfilling quality requirements ISO 9000:2000 (3.4.10)

QC is examining “control” materials of known substances along with patient samples to monitor the accuracy and precision of the complete examination (analytic) process.

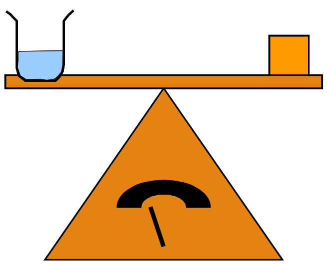
Purpose

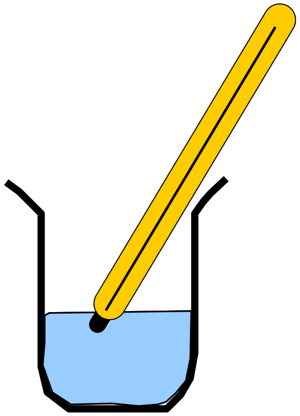
The **goal** of QC is to detect errors and correct them before patients' results are reported

Quantitative Examinations

Measure the quantity of a particular substance in a sample

Measurements should be both accurate and precise





Qualitative Examination Methods

Examinations that do not have numerical results:

- growth or no growth
- positive or negative
- reactive or non-reactive
- color change

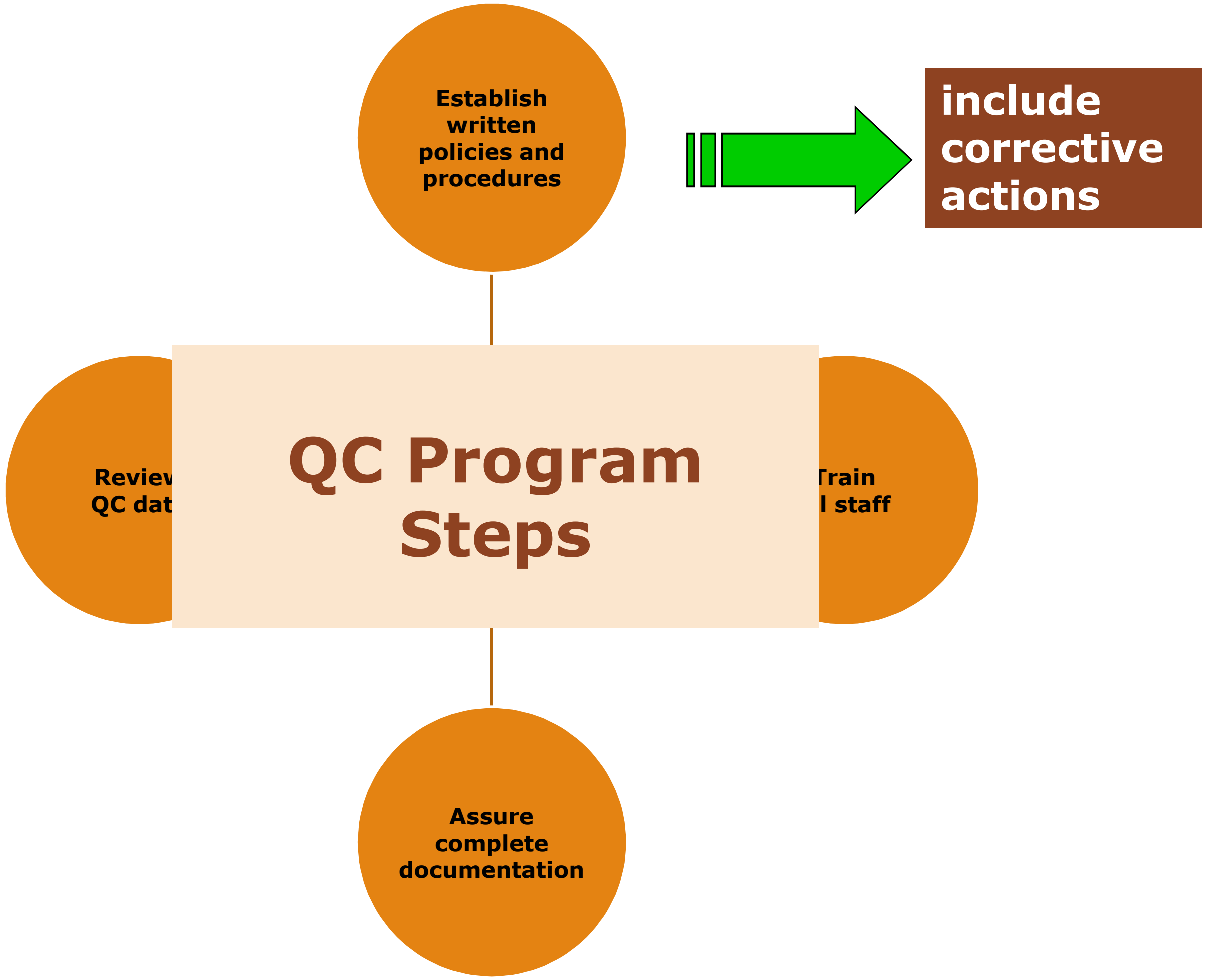
Semi-quantitative Examination Methods

Results are expressed as an estimate of the measured substance:

“trace amount”, “moderate amount,” or “1+, 2+, or 3+”

number of cells per microscopic field

titers and dilutions in serologic tests


Establish written policies and procedures
include corrective actions
Reviev QC dat
QC Program Steps
Train l staff
Assure complete documentation

QC Summary

- important part of quality management system
- goal is to identify errors and eliminate them before reporting patient results
- different methods applied for quantitative, qualitative, and semi-quantitative results

Overview of the Process

Quantitative tests measure the quantity of a substance in a sample, yielding a numeric result. For example, the quantitative test for blood glucose can give a result of 5 mg/dL. Since quantitative tests have numeric values, statistical tests can be applied to the results of QC material to differentiate between test runs that are "in control" and "out of control". This is done by calculating acceptable limits for control material, then testing the control with the patient's samples to see if it falls within established limits.

As a part of the quality management system, the laboratory must establish a QC programme for all quantitative tests. Evaluating each test run in this way allows the laboratory to determine if patient results are accurate and reliable.

Implementation Process

The steps for implementing a QC programme are:

- establish policies and procedures
- assign responsibility for monitoring and reviewing
- train all staff in how to properly follow policies and procedures
- select good QC material
- establish control ranges for the selected material
- develop graphs to plot control values—these are called Levey–Jennings charts
- establish a system for monitoring control values
- take immediate corrective action if needed
- maintain records of QC results and any corrective actions taken.

Control Materials

Defining Control Materials

Controls are substances that contain an established amount of the substance being tested—the analyte. Controls are tested at the same time and in the same way as patient samples. The purpose of the control is to validate the reliability of the test system and evaluate the operator's performance and environmental conditions that might impact results.

Differentiating Controls and Calibrators

It is important not to confuse calibrators and control materials. Calibrators are solutions with a specified defined concentration that are used to set or calibrate an instrument, kit, or system before testing is begun. Calibrators are often provided by the manufacturer of an instrument. They should not be used as controls since they are used to set the instrument. Calibrators are sometimes called standards, but the term calibrator is preferred. They usually do not have the same consistency as patients' samples.

Characteristics of Control Materials

It is critical to select the appropriate control materials. Some important characteristics to consider when making the selection are:

- Controls must be appropriate for the targeted diagnostic test—the substance being measured in the test must be present in the control in a measurable form.
- The amount of the analyte present in the controls should be close to the medical decision points of the test; this means that controls should check both low values and high values.
- Controls should have the same matrix as patient samples; this usually means that the controls are serum based, but they may also be based on plasma, urine or other materials.

Because it is more efficient to have controls that last for some months, it is best to obtain control materials in large quantities.

Types and Sources of Control Material

Control materials are available in a variety of forms. They may be frozen, freeze- dried or chemically preserved. The freeze-dried or lyophilized materials must be reconstituted, requiring great care in pipetting in order to ensure the correct concentration of the analyte.

Control materials may be purchased, obtained from a central or reference laboratory, or made in-house by pooling sera from different patients.

Purchased controls may be either assayed or unassayed. Assayed controls have a predetermined target value, established by the manufacturer. When using assayed controls, the laboratory must verify the value using its own methods. Assayed controls are more expensive to purchase than unassayed controls.

When using either unassayed or "in-house" controls, the laboratory must establish the target value of the analyte.

Control Charts for Variables

Variation

- There is no two natural items in any category are the same.
- Variation may be quite large or very small.
- If variation very small, it may appear that items are identical, but precision instruments will show differences.

3 Categories of variation

WITHIN-PIECE VARIATION

- variation found within a single item or unit measured at different points or portions.

PIECE-TO-PIECE VARIATION

- differences observed between individual units or items that are supposed to be identical.

TIME-TO-TIME VARIATION

- the changes in measurements or outcomes that can occur over time due to temporal fluctuations or external factors impacting the process.

Source **of variation**

EQUIPMENT

- Tool wear, machine vibration, …

MATERIAL

- Raw material quality

ENVIRONMENT

- Temperature, pressure, humidity

OPERATOR

- Operator performs- physical & emotional

Control Chart Viewpoint

- Variation due to
  - Common or chance causes
  - Assignable causes
- Control chart may be used to discover "assignable causes"

Some Terms

- **RUN CHART** - without any upper/lower limits
- **SPECIFICATION/TOLERANCE LIMITS** - not statistical
- **CONTROL LIMITS** - statistical

Control Chart Functions

- Control charts are powerful aids to understanding the performance of a process over time.

Control Charts Identify Variation

- Chance causes - "common cause"
  - inherent to the process or random and not controllable
  - if only common cause present, the process is considered stable or "in control"
- Assignable causes - "special cause"
  - variation due to outside influences
  - if present, the process is "out of control"

Control Charts Help Us Learn More About Processes

- Separate common and special causes of variation
- Determine whether a process is in a state of statistical control or out-of-control
- Estimate the process parameters (mean, variation) and assess the performance of a process or its capability

Control Charts to Monitor Processes

To monitor output, we use **a** control chart

- we check things like the mean, range, standard deviation

To monitor a process, we typically use **two** control charts

- mean (or some other central tendency measure)
- variation (typically using range or standard deviation)

Sources of Variation

Many factors contribute to variation

- **Source of variation -** combination of equipments, materials, environment, operator, etc.
- **Equipment -** tool wear, electrical fluctuations for welding
- **Material -** tensile strength, moisture content (e.g. raw material)
- **Environment -** temperature, light, humidity etc.
- **Operator -** method, SOP followed, motivation level, training
- **Inspection -** inspector, inspection equipment, environment

Causes of Variation - Chance & Assignable

- ✓**Chance or random causes** are unavoidable
- ✓As long as fluctuate in natural/expected/stable pattern of chance causes of variation which are small – it is OK
- ✓This is in ‘state of statistical control’
- ✓When causes of variation large in magnitude; can be identified, classified as assignable causes of variation. If present, process variation is excessive (beyond expected natural variation)
- ✓‘state of out of control’ – **assignable cause**
- ✓Example : Body temperature - 36.5^{o}C ~ 37.5^{o}C

Control Chart Method

- **Control chart** - means of visualizing variations that occur in the central tendency and dispersion of a set of observations
- Graphical record of a particular quality characteristic – hardness, length, etc.

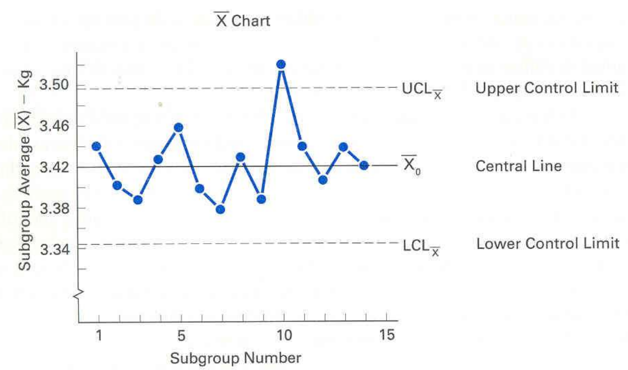


FIGURE 5-1 Example of a control chart.

Control Chart Method

- Control limits are not specification limits
- CL are permissible limits of a quality characteristic
- Evaluate variations in quality subgroup to subgroup
- Limits established at ± 3 standard dev. from central line; for normal distribution – we expect 99.73% of items would lie within the limits

Control Chart Method

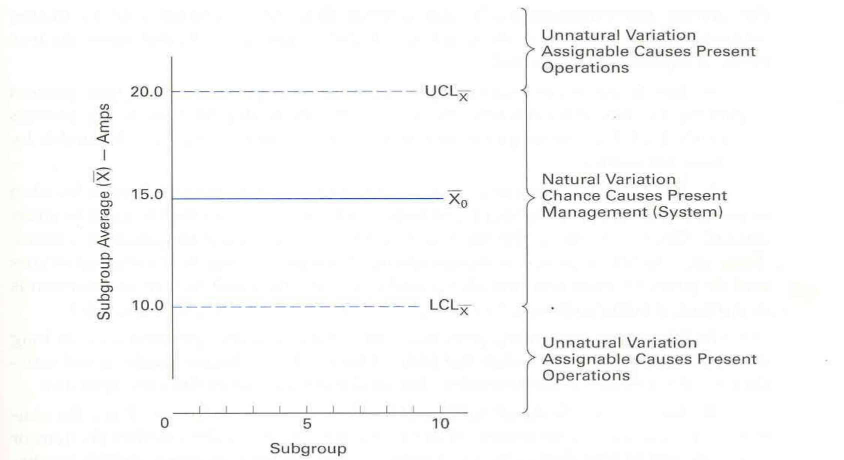


FIGURE 5-3 Natural and unnatural causes of variation.

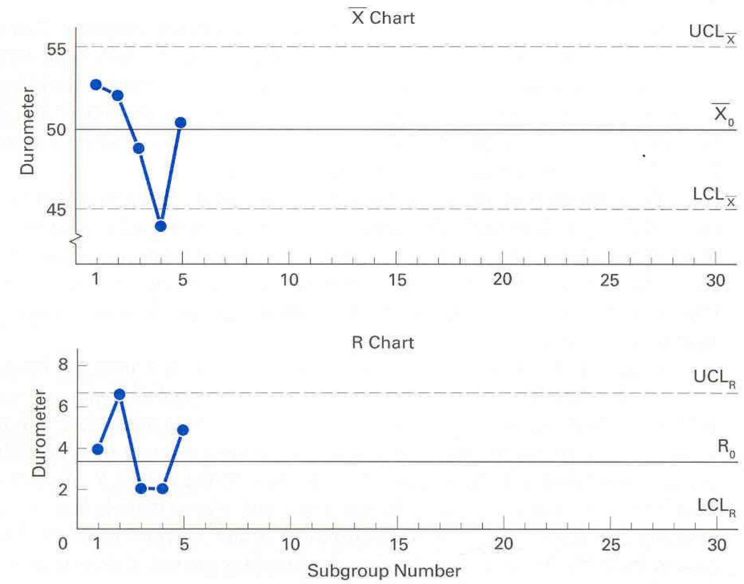


FIGURE 5-2 Example of a method of reporting inspection results.

Types of Data

VARIABLE DATA

- Product characteristic that can be measured
  - Length, size, weight, height, time, velocity

ATTRIBUTE DATA

- Product characteristic evaluated with a discrete choice
  - Good/bad, yes/no

Control Chart for Variables

- Variables are the measurable characteristics of a product or service.
- Measurement data is taken and arrayed on charts.

Control Charts for Variables

X-bar chart

- In this chart the sample *means* are plotted in order to control the mean value of a variable (e.g., size of piston rings, strength of materials, etc.).

R chart

- In this chart, the sample *ranges* are plotted in order to control the variability of a variable.

S chart

- In this chart, the sample *standard deviations* are plotted in order to control the variability of a variable.

S^2 chart

- In this chart, the sample *variances* are plotted in order to control the variability of a variable.

X-bar and R charts

THE X- BAR CHART is developed from the average of each subgroup data.

- used to detect changes in the mean between subgroups.

THE R- CHART is developed from the ranges of each subgroup data

- used to detect changes in variation within subgroups

Control chart components

CENTERLINE

- shows where the process average is centered or the central tendency of the data

UPPER CONTROL LIMIT (UCL) AND LOWER CONTROL LIMIT (LCL)

- describes the process spread

Variable Control Chart – $\bar{x}$ (average)- R chart

1. Select quality characteristic

- Measurable data (basic units, length, mass, time, etc.)
- Affecting performance, function of product
- From Pareto analysis – highest % rejects, high production costs
- Impossible to control all characteristics - selective or use attributes chart

Variable Control Chart – $\bar{x}$ (average)- R chart

2. Choose rational subgroup

   Rational subgroup - variation within the group due only to chance causes and can detect between groups changes

   Two ways selecting subgroup samples

   1. Select subgroup samples at one instant of time or as close as possible
   2. Select period of time products are produced

- Rational subgroup from homogeneous lot : same machine, same operator
- Decisions on size of sample empirical judgment + relates to costs

choosen = 4 or 5 $\rightarrow$ use R-chart

when n $\geq$ 10 $\rightarrow$ use s-chart

- frequency of taking subgroups often enough to detect process changes
- Guideline of sample sizes/frequency using

Say, 4000 parts/day

$\therefore$ 75 samples

if n = 4 $\therefore$ 19 subgroups

or n = 5 15 subgroups

TABLE 5-1 Sample Sizes (From ANSI/ASQ Z1.9—1993, Normal Inspection, Level II)

LOT SIZE	SAMPLE SIZE
91–150	10
151–280	15
281–400	20
401–500	25
501–1,200	35
1,201–3,200	50
3,201–10,000	75
10,001–35,000	100
35,001–150,000	150

3. Collect data

- Use form or standard check sheet
- Collect a minimum of 25 subgroups
- Data can be vertically / horizontally arranged

	Subgroup Number								
Measure	1	2	3	4	5	……	…..	….	25
x_1 x_2 x_3 x_4 x_5	35 40 32 37 34								34 40 38 35 38
$\bar{x}$	35.6								37.0
R	8								6

Example Problem

TABLE 5-2 Data on the Depth of the Keyway (millimeters)[a]

SUBGROUP NUMBER	DATE	TIME	MEASUREMENTS X_1	X_2	X_3	X_4	AVERAGE $\bar{X}$	RANGE R	COMMENT
1	12/26	8:50	35	40	32	37	6.36	0.08	
2		11:30	46	37	36	41	6.40	0.10	
3		1:45	34	40	34	36	6.36	0.06	
4		3:45	69	64	68	59	6.65	0.10	New, temporary
5		4:20	38	34	44	40	6.39	0.10	operator
6	12/27	8:35	42	41	43	34	6.40	0.09	
7		9:00	44	41	41	46	6.43	0.05	
8		9:40	33	41	38	36	6.37	0.08	
9		1:30	48	44	47	45	6.46	0.04	
10		2:50	47	43	36	42	6.42	0.11	
11	12/28	8:30	38	41	39	38	6.39	0.03	
12		1:35	37	37	41	37	6.38	0.04	
13		2:25	40	38	47	35	6.40	0.12	
14		2:35	38	39	45	42	6.41	0.07	
15		3:55	50	42	43	45	6.45	0.08	
16	12/29	8:25	33	35	29	39	6.34	0.10	
17		9:25	41	40	29	34	6.36	0.12	
18		11:00	38	44	28	58	6.42	0.30	Damaged oil line
19		2.35	35	41	37	38	6.38	0.06	
20		3:15	56	55	45	48	6.51	0.11	Bad material
21	12/30	9:35	38	40	45	37	6.40	0.08	
22		10:20	39	42	35	40	6.39	0.07	
23		11:35	42	39	39	36	6.39	0.06	
24		2:00	43	36	35	38	6.38	0.08	
25		4:25	39	38	43	44	6.41	0.06	
Sum							160.25	2.19	

[a] For simplicity in recording, the individual measurements are coded from 6.00 mm.

4. Determine trial control limits

Calculate Central line

$$\overline{X} = \frac{\sum_{i=1}^{g} \overline{x}_i}{g} \qquad R = \frac{\sum_{i=1}^{g} R_i}{g}$$

$$UCL_{\overline{x}} = \overline{\overline{x}} + 3\sigma_{\overline{x}} = \overline{\overline{x}} + A_2\overline{R}$$

$$LCL_{\overline{x}} = \overline{\overline{x}} - 3\sigma_{\overline{x}} = \overline{\overline{x}} - A_2\overline{R}$$

$$UCL_R = \overline{R} + 3\sigma_{\overline{x}} = D_4\overline{R}$$

$$LCL_R = \overline{R} - 3\sigma_{\overline{x}} = D_3\overline{R}$$

$\overline{X}$ = avg. of subgroup avg.

$\overline{x}_i$ = avg. of ith subgroup

g = no. of subgroups

$\overline{R}$ = avg. of subgroup ranges

R_i = range of ith subgroup

Where A2, D4, D3 are factors - vary according to different n

TABLE 5-2 Data on the Depth of the Keyway (millimeters)[a]

SUBGROUP NUMBER	DATE	TIME	MEASUREMENTS X_1	X_2	X_3	X_4	AVERAGE $\bar{X}$	RANGE R	COMMENT
1	12/26	8:50	35	40	32	37	6.36	0.08	
2		11:30	46	37	36	41	6.40	0.10	
3		1:45	34	40	34	36	6.36	0.06	
4		3:45	69	64	68	59	6.65	0.10	New, temporary
5		4:20	38	34	44	40	6.39	0.10	operator
6	12/27	8:35	42	41	43	34	6.40	0.09	
7		9:00	44	41	41	46	6.43	0.05	
8		9:40	33	41	38	36	6.37	0.08	
9		1:30	48	44	47	45	6.46	0.04	
10		2:50	47	43	36	42	6.42	0.11	
11	12/28	8:30	38	41	39	38	6.39	0.03	
12		1:35	37	37	41	37	6.38	0.04	
13		2:25	40	38	47	35	6.40	0.12	
14		2:35	38	39	45	42	6.41	0.07	
15		3:55	50	42	43	45	6.45	0.08	
16	12/29	8:25	33	35	29	39	6.34	0.10	
17		9:25	41	40	29	34	6.36	0.12	
18		11:00	38	44	28	58	6.42	0.30	Damaged oil line
19		2:35	35	41	37	38	6.38	0.06	
20		3:15	56	55	45	48	6.51	0.11	Bad material
21	12/30	9:35	38	40	45	37	6.40	0.08	
22		10:20	39	42	35	40	6.39	0.07	
23		11:35	42	39	39	36	6.39	0.06	
24		2:00	43	36	35	38	6.38	0.08	
25		4:25	39	38	43	44	6.41	0.06	
Sum							160.25	2.19	

[a] For simplicity in recording, the individual measurements are coded from 6.00 mm.

5. Revised Control Limits

- First plot preliminary data collected using control limits & center lines established in step 4
- Use/adopt standard values, if good control i.e. no out-of-control points

$$\bar{\bar{X}} \rightarrow \bar{X}_o \qquad \bar{R} \rightarrow R_o$$

- If there are points out-of-control discard from data, look at records – if show an assignable cause – don't use

$$\bar{\bar{X}}_{new} = \frac{\Sigma\bar{x} - \Sigma\bar{x}_d}{g - gd}$$

$$\bar{R}_{new} = \frac{\Sigma R - \Sigma Rd}{g - gd}$$

$$\sigma_o = \frac{R_o}{d_2}$$

Control Charts with Limits Established

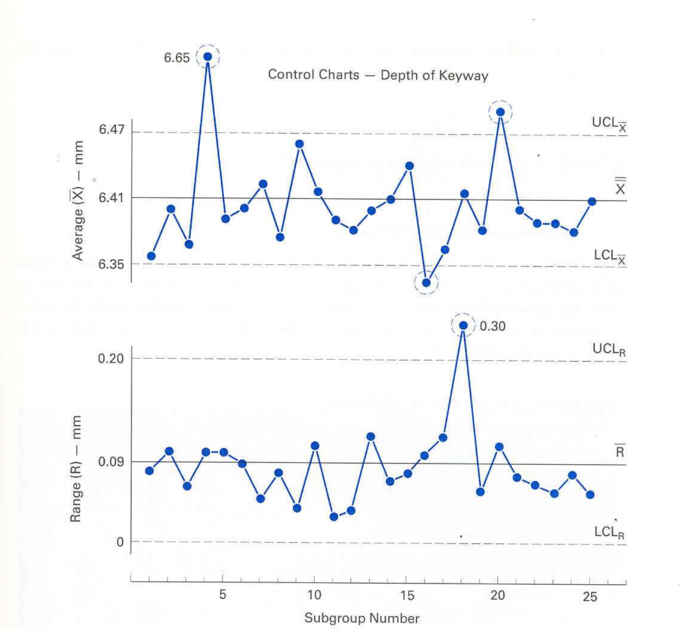


FIGURE 5-5 $\bar{X}$ and R chart for preliminary data with trial control limits.

- Limits for both charts become narrower after revised
- Revised limits used to report / plot future sub-groups
- For effective use – chart must be displayed and easily seen

Comments about CC

1. Some analyst eliminate the revised step - but actually more representative of process
2. Formula mathematically same $\overline{X}_o + A\sigma_o = \overline{\overline{X}}_{new} + A_2\overline{R}_{new}$
3. Initial estimate of process capability known - $6\sigma o$ - true Cp is next
4. If use specification; nominal (target) value = $\overline{X}o$. Range doesn't change
5. Adjustments made to processes while taking data – not necessarily continue making defectives while collecting data
6. Process determines center line and the control limits, not design or manufacturing
7. When population values known easily obtained limits, $\overline{X}_o = \mu \cdot \sigma_o = \sigma$

6. Achieving objective

- Initiate control charts results in quality improvement
- Less variation in sub-group averages
- Reduction in variation of range
- Can reduce frequency of inspection - monitoring purpose – even once/mth.

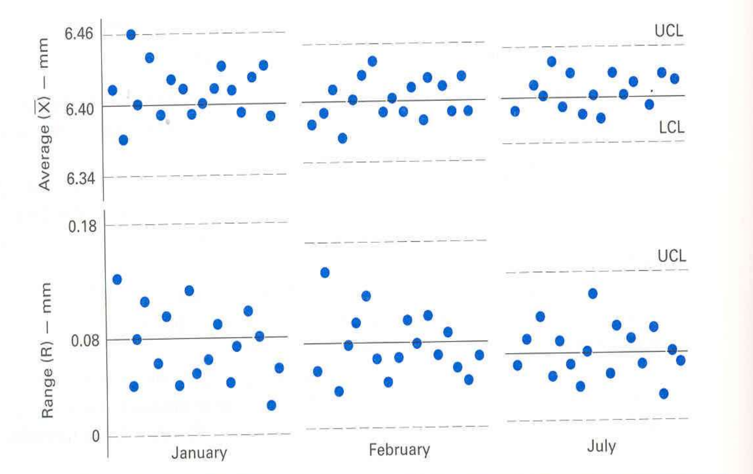


FIGURE 5-7 Continuing use of control charts, showing improved quality.

How Control Chart Helps in QI

- Psychological effect to do better – example - maintenance group helps adjust process center
- Purchasing involved in changing material supplier to ensure consistent quality
- Production – standardize work methods, use/develop new tooling
- Improvements must be from investigation of assignable causes (need technical back up)

Sample Std. Deviation Chart ($\bar{x}$ - s control chart)

Both R and s measure dispersion of data

R chart- simple, only use X_H (highest) and X_L(lowest)

s chart - more calculation
- use ALL xi's

∴ more accurate, need calculate sub-group sample standard deviation

When n < 10

R chart ≅ s chart

n ≥ 10 - s chart better , R not accurate any more

$$UCL_{\bar{x}} = \bar{\bar{X}} + A_3\bar{s}$$

$$LCL_{\bar{x}} = \bar{\bar{X}} + A_3\bar{s}$$

$$UCL_s = B_4\bar{s}$$

$$LCL_s = B_3\bar{s}$$

$$\sigma_o = \frac{s_o}{C_4} = \frac{\bar{s}}{C_4}$$

State of Control

- When assignable causes eliminated and points plotted are within C.L.- process state of control
- Further improvement through changing basic process, system
- What are the characteristics of process in control? (natural pattern of variation)
- 34% within 1σ from Center Line
- 13% between 1σ & 2σ
- 2.5% of plotted points - $2\sigma \rightarrow 3\sigma$
- Points located back & forth across center line random way
- No points out of control

State of Control

- Subgroup averages forms frequency distribution which is normal distribution and limits – established at 3σ from center line.
- Choice of 3σ is economic decision with respect to 2 types of error
- Type I - occurs when looking for assignable cause but in reality chance cause present > FALSE ALARM
- When limits set $\pm 3\sigma$ Type I error probability = 0.27% or 3/1000
- Say point our control $\rightarrow$ due to assignable but 3/1000 of the time can be due to chance cause
- Type II - assume chance cause present, but in fact assignable cause present > TRUE ALARM
- Records indicate 3σ limits balance between 2 errors.

Process in Control

- Individual parts will be more uniform – less variation and fewer rejects
- Cost of inspection will decrease
- Process capability easily attained
- Trouble can be anticipated before it occurs
- Percentage of parts fall between two values can be predicted with highest degree of accuracy, e.g. filling machines
- $\overline{X}$-R charts can be used as statistical evidence for process control

- Predictable and stable process only chance causes present

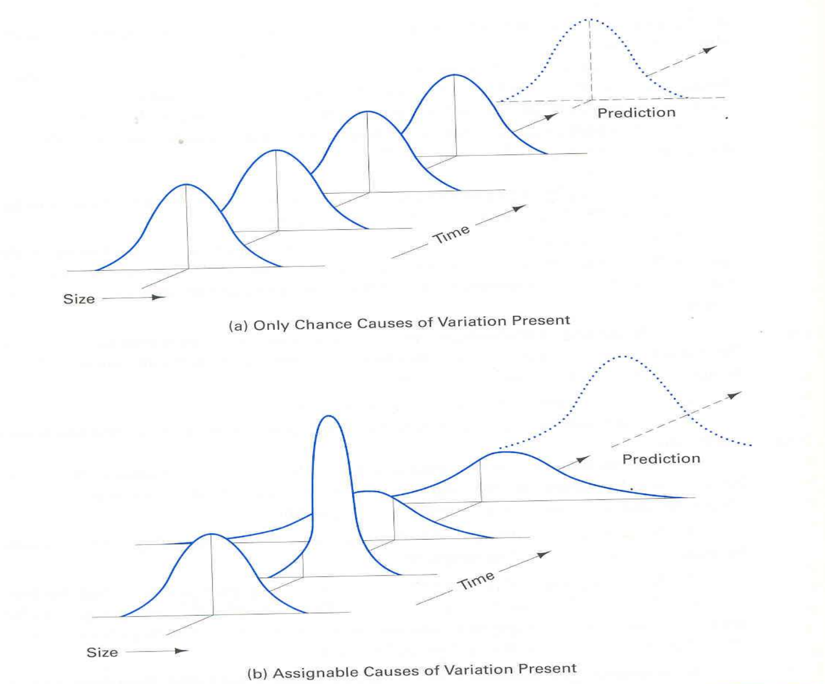


FIGURE 5-10 Stable and unstable variation.

Control Chart for Attributes

Introduction

- Many quality characteristics cannot be conveniently represented numerically.
- In such cases, each item inspected is classified as either **conforming** or **nonconforming** to the specifications on that quality characteristic.
- Quality characteristics of this type are called **attributes**.
- Examples are nonfunctional semiconductor chips, warped connecting rods, etc,.

Types of Control Charts

Control Charts for Variables Data

$\bar{X}$ and R charts: for sample averages and ranges.

X and s charts: for sample means and standard deviations.

$\bar{M}$d and R charts: for sample medians and ranges.

X charts: for individual measures; uses moving ranges.

Control Charts for Attributes Data

p charts: proportion of units nonconforming.

np charts: number of units nonconforming.

c charts: count of nonconformities.

u charts: count of nonconformities per unit.

Attribute

- The term Attribute refers to those quality characteristics that conform to specifications or do not conform to specifications.
- Attributes are used:
  1. Where measurements are not possible.
  2. Where measurements can be made but are not made because of time, cost, or need.

Attribute (cont.)

- A nonconformity is a departure of a quality characteristic from its intended level or state that occurs with a severity sufficient to cause an associated product or service not to meet a specification requirement.
- Defect is concerned with satisfying intended normal, or reasonably foreseeable, usage requirement.

Attribute (cont.)

- Defect is appropriate for use when evaluation is in terms of usage.
- Nonconformity is appropriate for conformance to specifications.
- The term *Nonconforming Unit* is used to describe a unit of product or service containing at least one nonconformity.

Attribute (cont.)

- Defective is analogous to defect and is appropriate for use when unit of product or service is evaluated in terms of usage rather than conformance to specifications.
- Limitations of variable control charts: These charts cannot be used for quality characteristics which are attributes.

Attribute (cont.)

Types of Attribute Charts:

1. Nonconforming Units (based on the Binomial distribution): *p* chart, *np* chart.
2. Nonconformities (based on the Poisson distribution): *c* chart, *u* chart.

Types of Attribute Charts

***p* charts**

This chart shows the fraction of nonconforming or defective product produced by a manufacturing process.

It is also called the control chart for fraction nonconforming.

***np* charts**

This chart shows the number of nonconforming. Almost the same as the *p* chart.

***c* charts**

This shows the number of defects or nonconformities produced by a manufacturing process.

***u* charts**

This chart shows the nonconformities per unit produced by a manufacturing process.

The P Chart

- The *P* Chart is used for data that consist of the proportion of the number of occurrences of an event to the total number of occurrences.
- It is used in quality to report the fraction or percent nonconforming in a product, quality characteristic, or group of quality characteristics.

The p chart

- In this chart, we plot the percent of defectives (per batch, per day, per machine, etc.).
- However, the control limits in this chart are not based on the distribution of rate events but rather on the binomial distribution (of proportions).

The P Chart

- It can be used to control one quality characteristic, as is done with X bar and R chart,
- Or to control a group of quality characteristics of the same type or of the same part,
- Or to control the entire product.
- It can be established to measure the quality produced by a work center, by a department, by a shift, or by an entire plant.

The P Chart

- It is frequently used to report the performance of an operator, group of operators, or management as a means of evaluating their quality performance.
- The subgroup size of the *P* chart can be either variable or constant.

Formula

Fraction nonconforming:

$$p = (np)/n$$

Where

p = proportion or fraction nc in the sample or subgroup,

n = number in the sample or subgroup,

np = number nc in the sample or subgroup.

Example

During the first shift, 450 inspection are made of book-of the month shipments and 5 nc units are found. Production during the shift was 15,000 units. What is the fraction nc?

$$p = (np)/n = 5/450 = 0.011$$

The p, is usually small, say 0.10 or less.

If $p > 0.10$, indicate that the organization is in serious difficulty.

p-Chart construction for constant subgroup size

- Select the quality characteristics.
- Determine the subgroup size and method
- Collect the data.
- Calculate the trial central line and control limits.
- Establish the revised central line and control limits.
- Achieve the objective.3

Select the quality characteristics

The quality characteristic?

- A single quality characteristic
- A group of quality characteristics
- A part
- An entire product, or
- A number of products.

Determine the subgroup size and method

The size of subgroup is a function of the proportion nonconforming.

If $p = 0.001$, and $n = 1000$, then the average number nc, $np = 1$. *Not good, since a large number of values would be zero.*

If $p = 0.15$, and $n = 50$, then $np = 7.5$, would make a good chart.

Therefore, the selection subgroup size requires some preliminary observations to obtain a rough idea of the proportion nonconforming.

Collect the data

- The quality technician will need to collect sufficient data for at least 25 subgroups.
- The data can be plotted as a run chart.
- Since the run chart does not have limits, its is not a control chart.

Calculate the trial central line and control limits

The formula:

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

$$\overline{p} = \frac{\sum np}{\sum n}$$

n = number inspected in a subgroup

The p-chart for Constant Subgroup Sizes

The Centerline and Control Limits

$$\text{Centerline}(p) = \bar{p} = \left[\frac{\text{Total number of defectives in all subgroups under investigation}}{\text{Total number of units examined in all subgroups under investigation}}\right]$$

$$\text{UCL}(p) = p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{LCL}(p) = p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Construction of a p-chart

An Example

- As an illustration, consider the case of an importer of decorative ceramic tiles. Some tiles are cracked or broken before or during transit, rendering them useless scrap. The fraction of cracked or broken tiles is naturally of concern to the firm. Each day a sample of 100 tiles is drawn from the total of all tiles received from each tile vendor.

Daily Cracked Tiles

Day	*Sample Size*	*Number Cracked or Broken*	*Fraction*
1	100	14	0.14
2	100	2	0.02
3	100	11	0.11
4	100	4	0.04
5	100	9	0.09
6	100	7	0.07
7	100	4	0.04
8	100	6	0.06
9	100	3	0.03
10	100	2	0.02
11	100	3	0.03
12	100	8	0.08
13	100	4	0.04
14	100	15	0.15
15	100	5	0.05
16	100	3	0.03
17	100	8	0.08
18	100	4	0.04
19	100	2	0.02
20	100	5	0.05
21	100	5	0.05
22	100	7	0.07
23	100	9	0.09
24	100	1	0.01
25	100	3	0.03
26	100	12	0.12
27	100	9	0.09
28	100	3	0.03
29	100	6	0.06
30	100	9	0.09
Totals	3,000	183	

Centerline (p) = 183/3000 = 0.061

$$UCL(p) = 0.061 + 3\sqrt{\frac{0.061(1-0.061)}{100}} = 0.133$$

$$LCL(p) = 0.061 - 3\sqrt{\frac{0.061(1-0.061)}{100}} = -0.011 = 0.000$$

An Importer of Decorative Ceramic Tiles Example (cont.)

- For a stable process, the probability that any subgroup fraction will be outside the three-sigma limits is small.
- Also, if the process is stable, the probability is small that the data will demonstrate any other indications of the presence of special causes of variation.
- But if the process is not in a state of statistical control, the control chart provides an economical basis upon which to search for and identify indications of this lack of control.

An Importer of Decorative Ceramic Tiles Example (cont.)

- For this p-chart -- or, in fact, for any of the attribute control charts -- the exact probabilities that a stable process will generate points indicating a lack of control are impossible to calculate because even a stable process exhibits variation in its mean, dispersion, and shape.
- Nevertheless, the exact value of these probabilities is not too important for ordinary applications; what is important is the fact that they are small.
- Therefore, if a point does lie beyond the upper or lower control limits, we will infer that it indicates a lack of control.
- Additionally, for p-charts, the six other rules for out-of-control points described in Chapter 6 can all be applied. In order to do so, we need to compute the boundaries for the A, B, and C zones.

7.4.2 Construction of a p-chart

- Boundary between upper zones B and C

$$= \bar{p} + \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

In our example this value is 0.061 + 0.024 = 0.085

- Boundary between lower zones B and C

$$= \bar{p} - \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

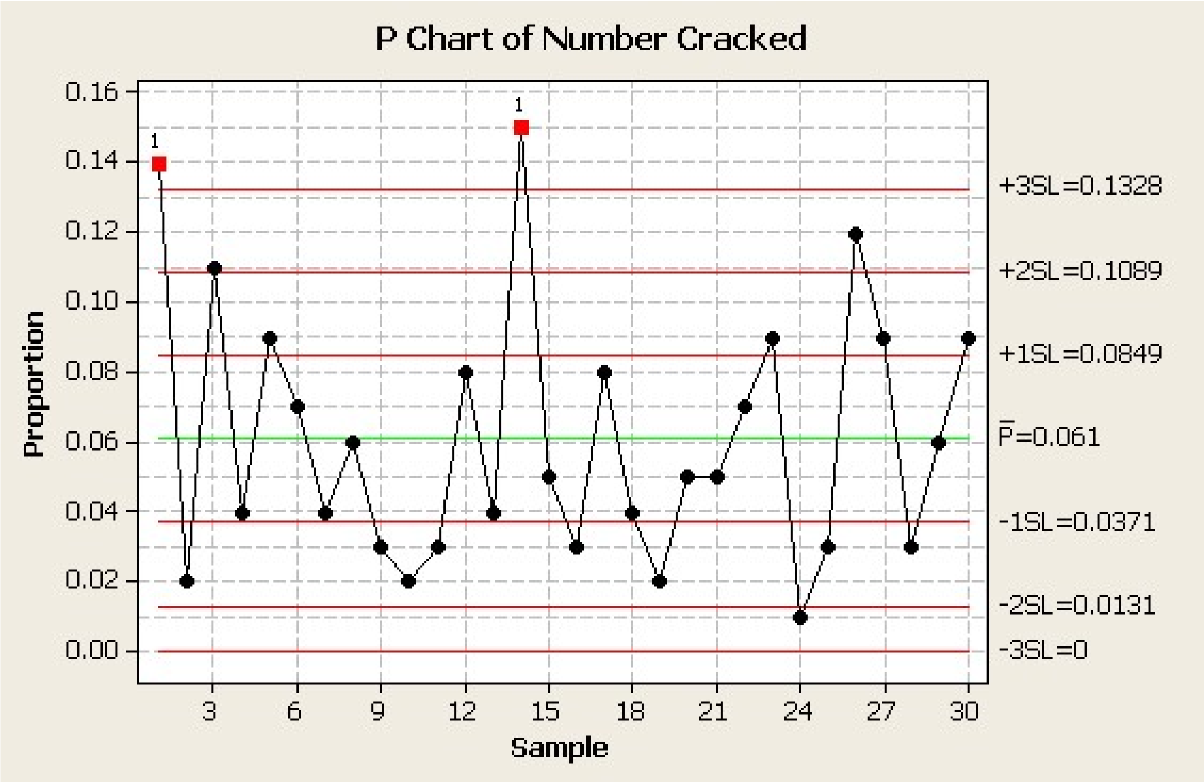
In our example this value is 0.061 - 0.024 = 0.037.

◦ Boundary between upper zones A and B

$$\bar{p} + 2\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.061 + 2(0.024) = 0.109$$

◦ Boundary between lower zones A and B

$$\bar{p} - 2\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.061 - 2(0.024) = 0.013$$


P Chart of Number Cracked
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Proportion
1
1
+3SL=0.1328
+2SL=0.1089
+1SL=0.0849
P̄=0.061
-1SL=0.0371
-2SL=0.0131
-3SL=0
3
6
9
12
15
18
21
24
27
30
Sample

NP Chart

- The *np* chart is almost the same as the *p* chart

Central line = np_o

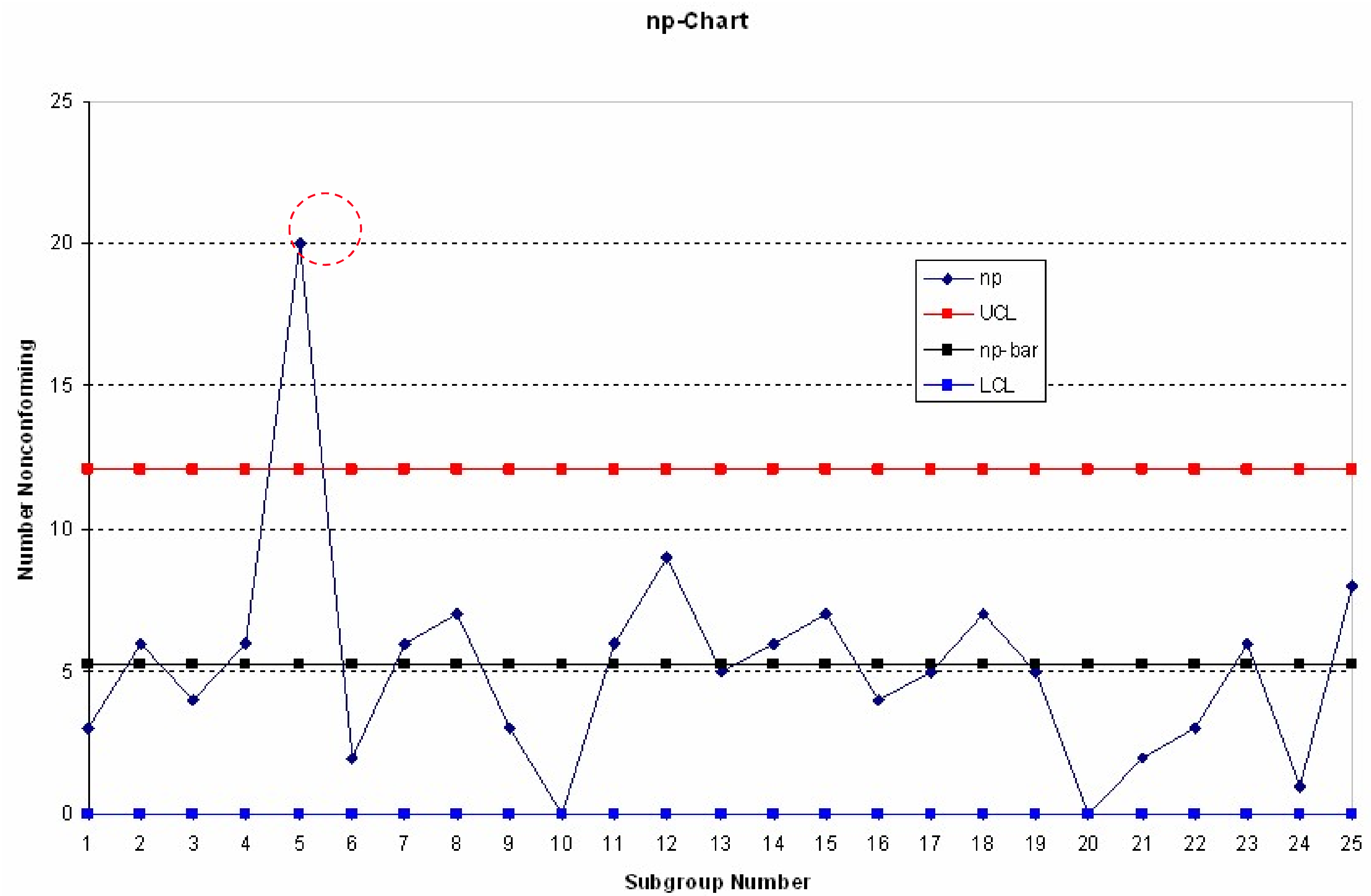
$$UCL = np_o + 3\sqrt{np_o(1 - p_o)}$$

$$LCL = np_o - 3\sqrt{np_o(1 - p_o)}$$

- If p_o is unknown, it must be determined by collecting data, calculating UCL, LCL.

Example

Subgroup	*n*	*np*	UCL	*np*-bar	LCL
1	300	3	12.0	5.24	0.0
2	300	6	12.0	5.24	0.0
3	300	4	12.0	5.24	0.0
4	300	6	12.0	5.24	0.0
5	300	20	12.0	5.24	0.0
21	300	2	12.0	5.24	0.0
22	300	3	12.0	5.24	0.0
23	300	6	12.0	5.24	0.0
24	300	1	12.0	5.24	0.0
25	300	8	12.0	5.24	0.0


np-Chart
25
20
15
10
5
0
Number Nonconforming
np
UCL
np-bar
LCL
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
Subgroup Number

C Chart

- The procedures for c chart are the same a s those for the *p* chart.
- If count of nonconformities, co, is unknown, it must be found by collecting data, calculating UCL & LCL.

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

$$\bar{c} = \frac{\sqrt{c}}{g}$$ = average count of nonconformities

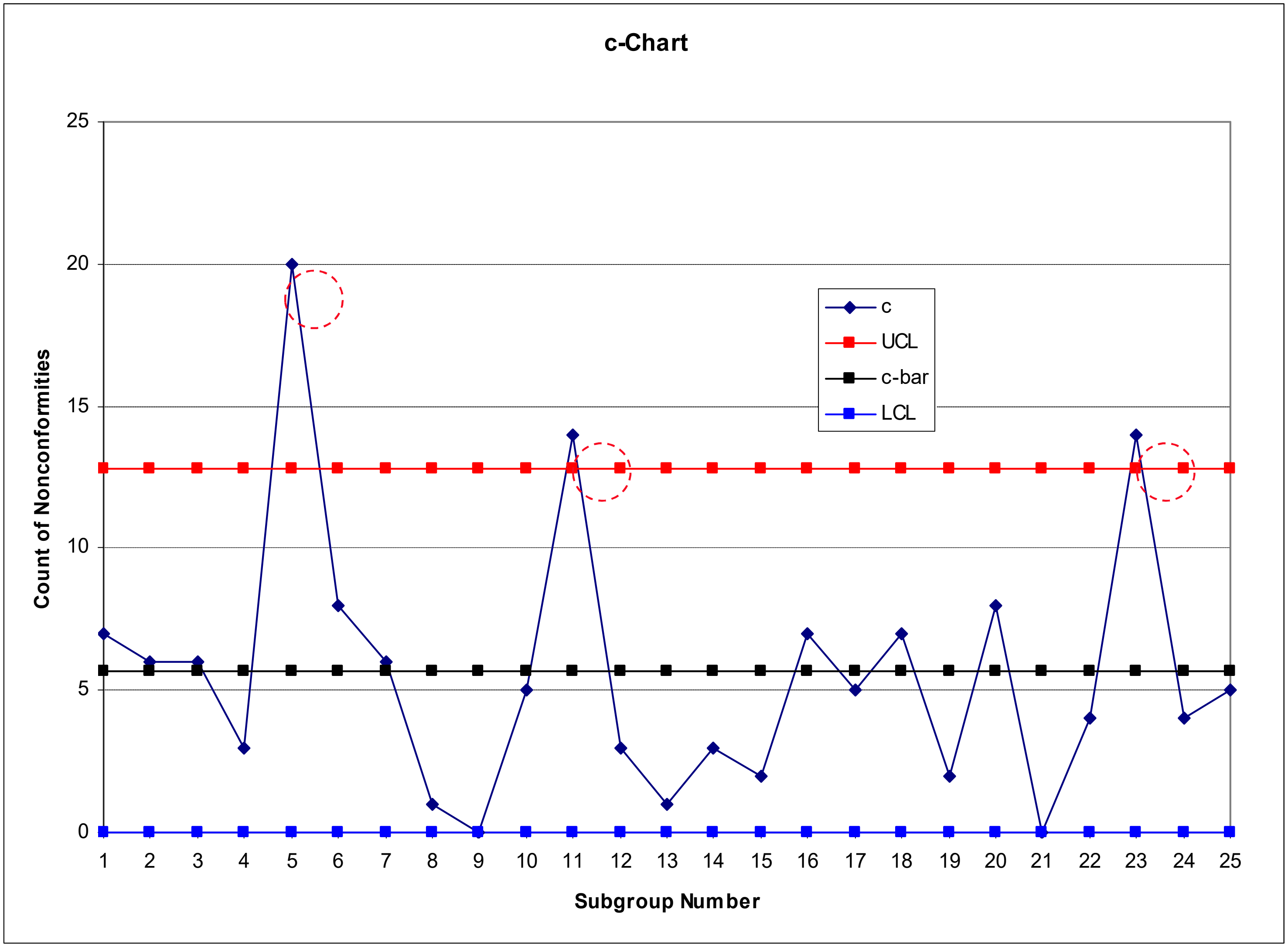
Example

ID Number	Subgroup	*c*	UCL	*c*-bar	LCL
MY102	1	7	12.76	5.64	0
MY113	2	6	12.76	5.64	0
MY121	3	6	12.76	5.64	0
MY125	4	3	12.76	5.64	0
MY132	5	20	12.76	5.64	0
MY143	6	8	12.76	5.64	0
MY150	7	6	12.76	5.64	0
MY152	8	1	12.76	5.64	0
MY164	9	0	12.76	5.64	0
MY166	10	5	12.76	5.64	0
MY172	11	14	12.76	5.64	0
MY267	22	4	12.76	5.64	0
MY278	23	14	12.76	5.64	0
MY281	24	4	12.76	5.64	0
MY288	25	5	12.76	5.64	0

$$UCL = 5.64 + 3\sqrt{5.64} = 12.76$$

$$\bar{c} = \frac{\sqrt{c}}{g} = \frac{141}{25} = 5.64$$

$$LCL = 5.64 - 3\sqrt{5.64}$$
$$= -1.48 = 0$$


c-Chart
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
Count of Nonconformities
Subgroup Number
c
UCL
c-bar
LCL

Revised

Out-of-control: sample no. 5, 11, 23.

$$\bar{c}_{new} = \frac{\sqrt{c} - c_d}{g - g_d} = \frac{141 - 20 - 14 - 14}{25 - 3} = 4.23$$

$$UCL = c_o + 3\sqrt{c_o} = 4.23 + 3\sqrt{4.23} = 10.40$$

$$LCL = c_o - 3\sqrt{c_o} = 4.23 - 3\sqrt{4.23} = -1.94 = 0$$

U Chart

The u chart is mathematically equivalent to the c chart.

$$u = \frac{c}{n}$$

$$\overline{u} = \frac{\sum c}{\sum n}$$

$$UCL = \overline{u} + 3\sqrt{\frac{\overline{u}}{n}}$$

$$LCL = \overline{u} - 3\sqrt{\frac{\overline{u}}{n}}$$

Example

$$\bar{u} = \frac{\sum c}{\sum n} = \frac{3389}{2823} = 1.20$$

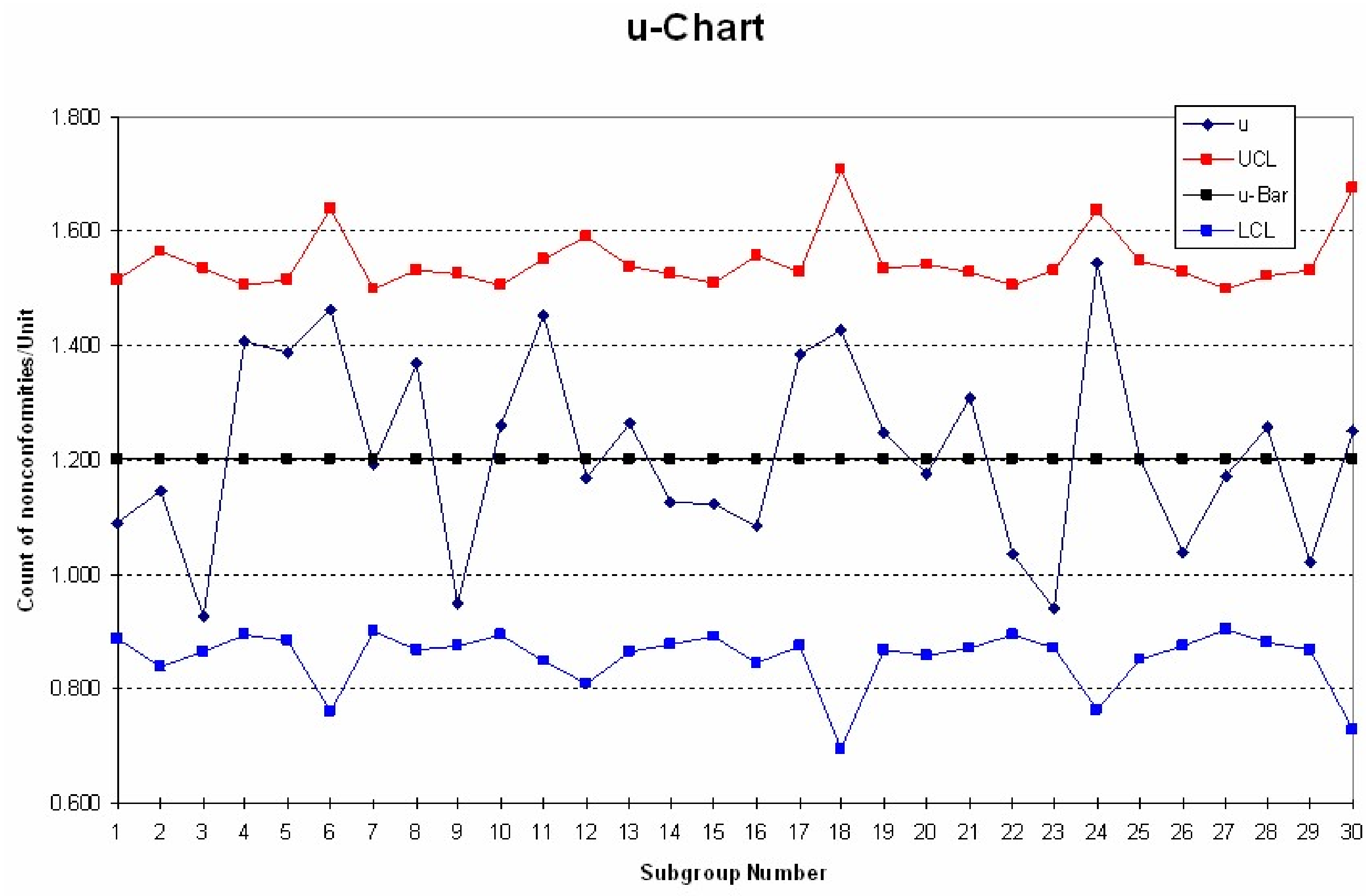
ID Number	Subgroup	*n*	*c*	*u*	UCL	*u*-Bar	LCL
30-Jan	1	110	120	1.091	1.51	1.20	0.89
31-Jan	2	82	94	1.146	1.56	1.20	0.84
01-Feb	3	96	89	0.927	1.54	1.20	0.87
02-Feb	4	115	162	1.409	1.51	1.20	0.89
03-Feb	5	108	150	1.389	1.52	1.20	0.88
04-Feb	6	56	82	1.464	1.64	1.20	0.76
28-Feb	26	101	105	1.040	1.53	1.20	0.87
01-Mar	27	122	143	1.172	1.50	1.20	0.90
02-Mar	28	105	132	1.257	1.52	1.20	0.88
03-Mar	29	98	100	1.020	1.53	1.20	0.87
04-Mar	30	48	60	1.250	1.67	1.20	0.73

For January 30:

$$u_{Jan\ 30} = \frac{c}{n} = \frac{120}{110} = 1.09$$

$$UCL_{Jan\ 30} = 1.20 + 3\sqrt{\frac{1.20}{110}} = 1.51$$

$$LCL_{Jan\ 30} = 1.20 - 3\sqrt{\frac{1.20}{110}} = 0.89$$


u-Chart
u
UCL
u-Bar
LCL
1.800
1.600
1.400
1.200
1.000
0.800
0.600
Count of nonconfomities/Unit
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Subgroup Number