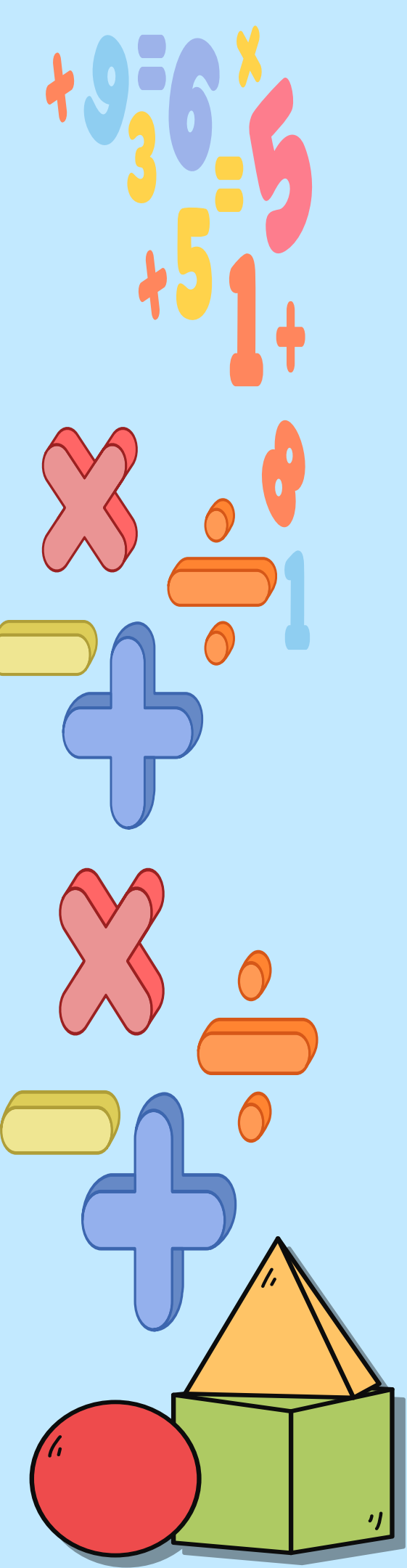
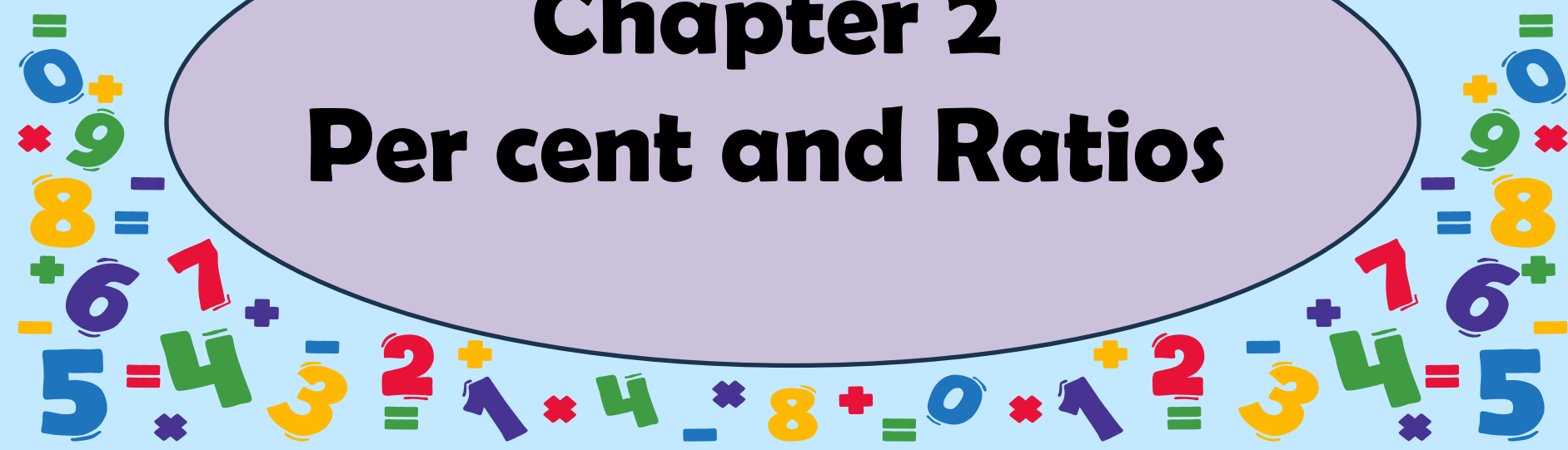
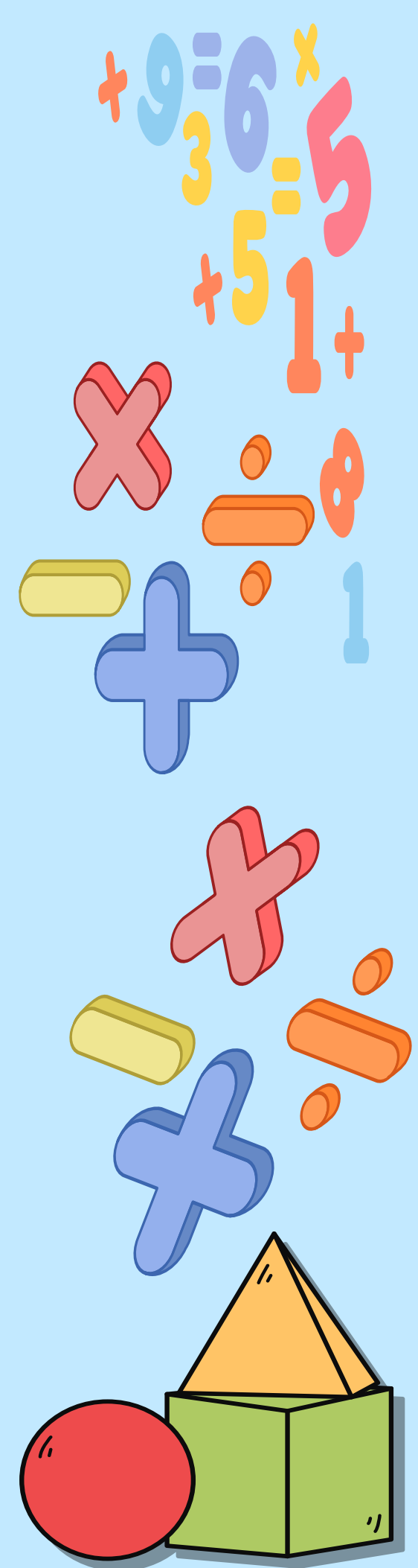




Chapter 2

Per cent and Ratios



Outlines



1. Intro to Per cent
2. Working with Per cent
3. Number Sense and Percent
4. Per cent Increases and Decreases

5. Sequential Per cent Changes
6. Simple and Compound Interest
7. Intro to Ratios
8. Ratios and Rates.

1. Intro to Per cent (%)

What does per cent mean?

Percent comes from the Latin per centum

percent = per 100

45% means
45 out of every 100

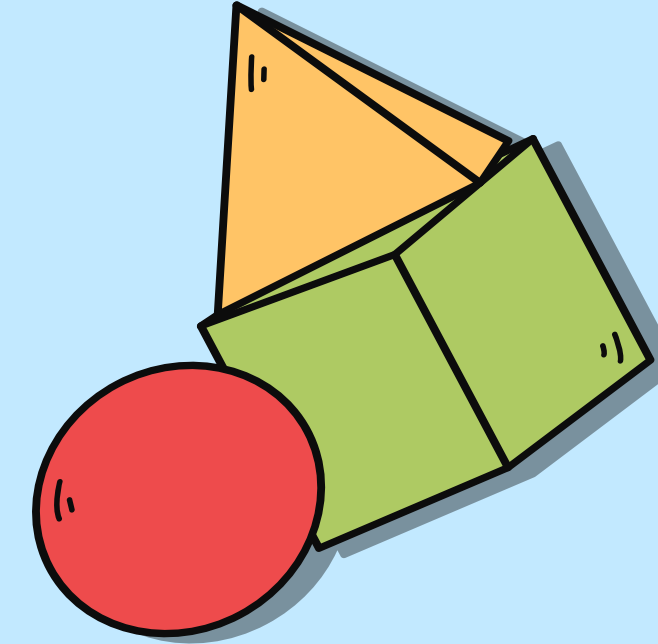
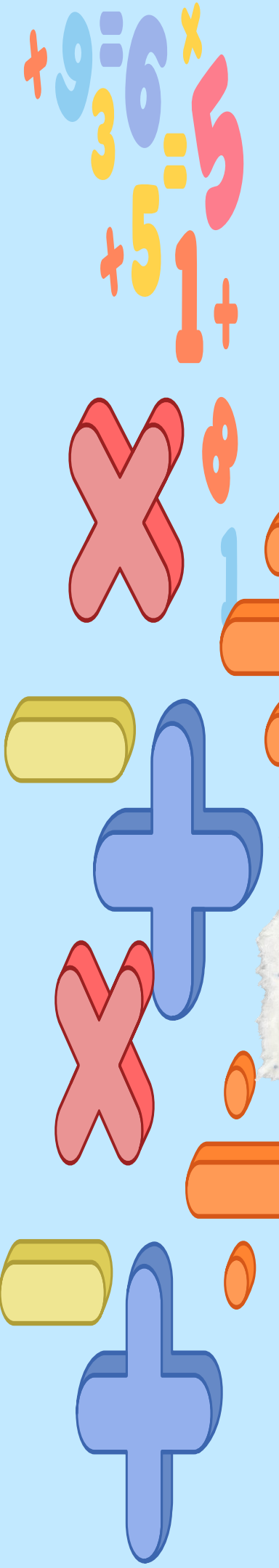
$$X\% \rightarrow \frac{x}{100}$$

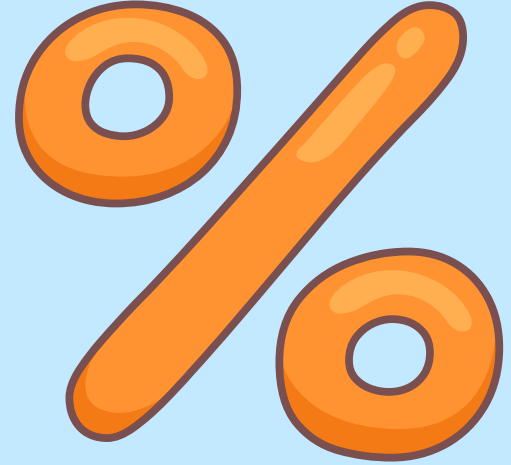
Thus. "per cent" means "divided by 100"

$$\therefore 53\% = \frac{53}{100} \text{ or the decimal } 0.53$$

Similarly 0.09% means the fraction

$$\frac{0.09}{100} = \frac{9}{10000} = 0.0009$$





Percent, fraction, and decimal

Percent

25%

A comparison out of 100.

Fraction

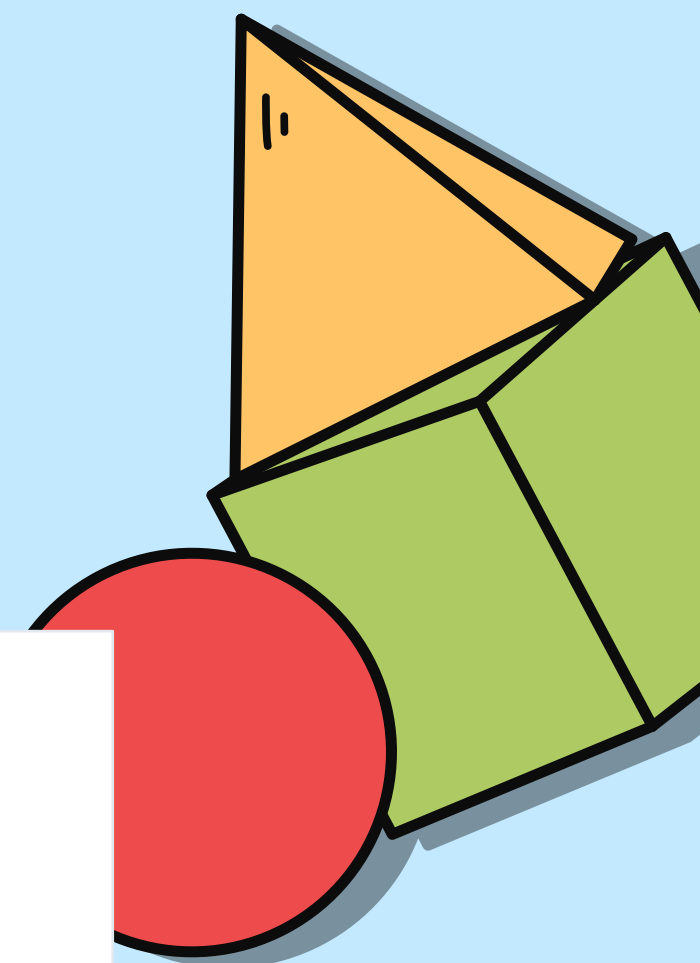
$25/100 = 1/4$

A part-whole relationship.

Decimal

0.25

A place-value form.

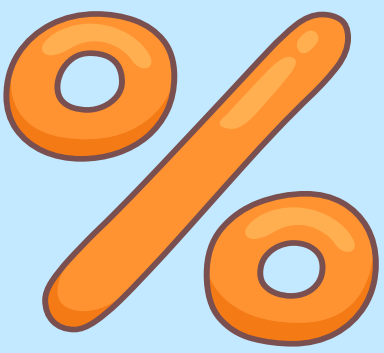


Quick rules

Percent → Decimal: divide by 100

Decimal → Percent: multiply by 100

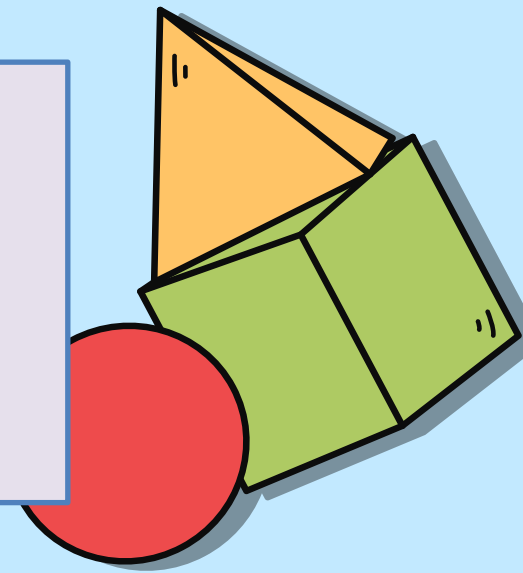
Percent → Fraction: write over 100, then simplify



This is divided by 100, so we move decimal point two to the left

Un-divided by 100 which essentially is multiply by 100. Thus, the decimal point is moved two places to the right.

We just put per cent over 100, after that, we may have to simplify a bit.



$$73.8\% = 0.738$$

$$10\% =$$

$$0.45\% =$$

$$0.032\% =$$

$$0.72 = 72\%$$

$$0.065 =$$

$$5.3 =$$

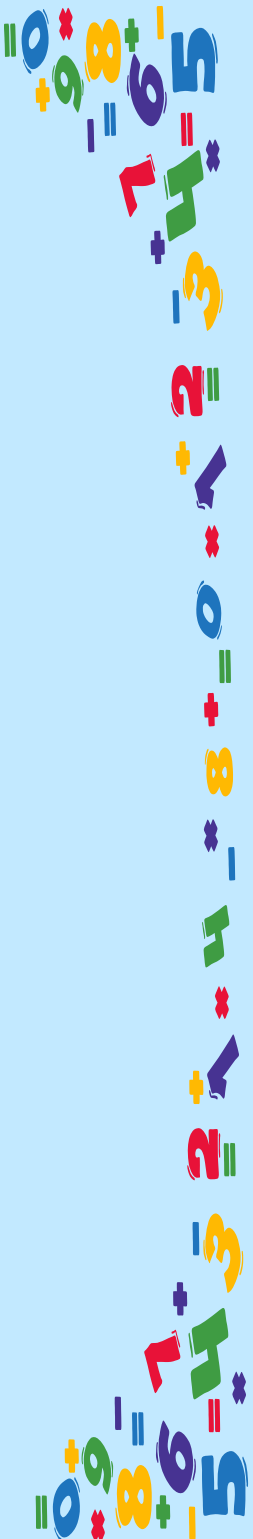
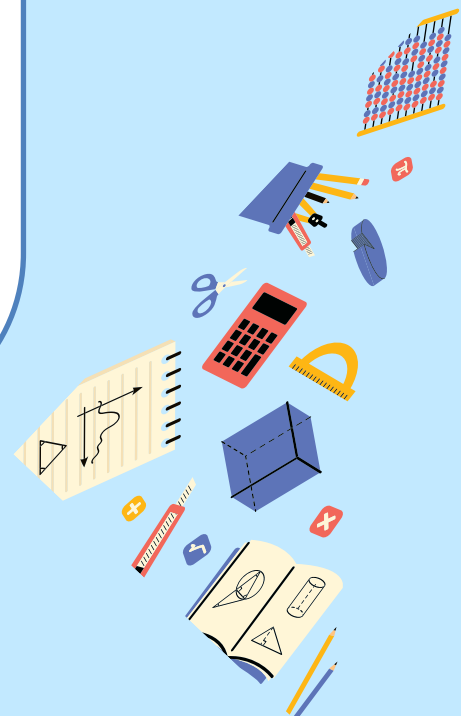
$$2.34 =$$

$$50\% = \frac{50}{100} = \frac{1}{2}$$

$$63\% =$$

$$0.65\% =$$

$$0.8\% =$$



Changing from Fractions to Per cents

Method 1: Change the Denominator to 100

Rewrite the fraction as an equivalent fraction with a denominator of 100.

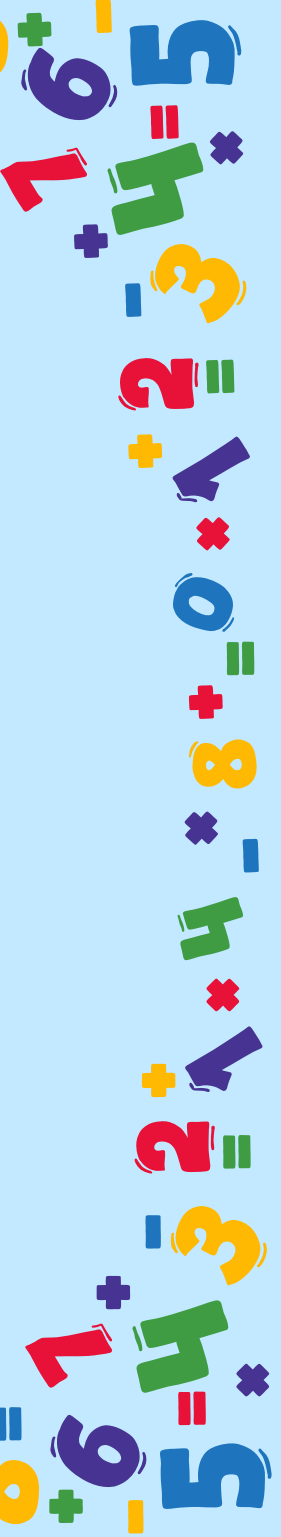
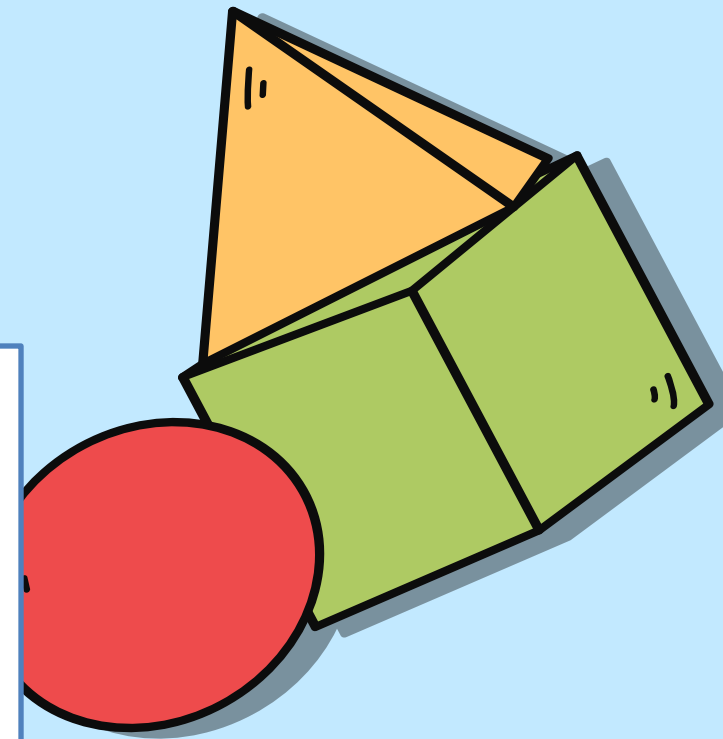
Example 1

Convert $\frac{1}{2}$ to a percent.

$$\frac{1}{2} = \frac{1 \times 50}{2 \times 50} = \frac{50}{100}$$

Therefore:

$$\frac{1}{2} = 50\%$$



Changing from Fractions to Per cents

Method 2: Divide and Multiply by 100

When the denominator cannot easily be changed to 100, divide the numerator by the denominator and multiply the result by 100.

$$\text{Percent} = \frac{\text{Numerator}}{\text{Denominator}} \times 100\%$$

Example 4

Convert $\frac{2}{5}$ to a percent.

$$\begin{aligned}\frac{2}{5} \times 100\% \\ = 0.4 \times 100\% \\ = 40\%\end{aligned}$$

Therefore:

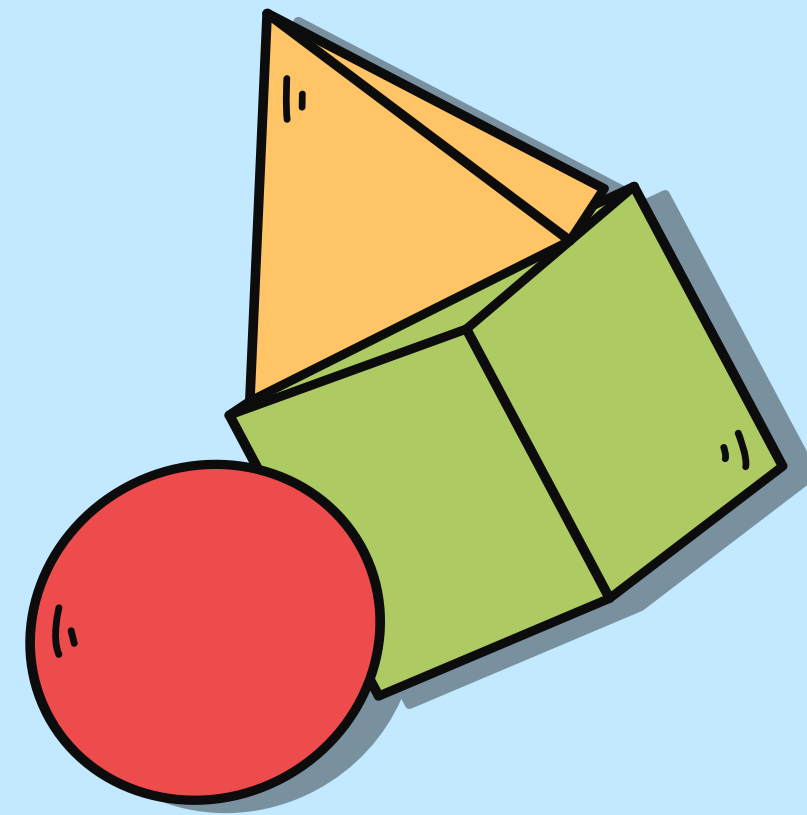
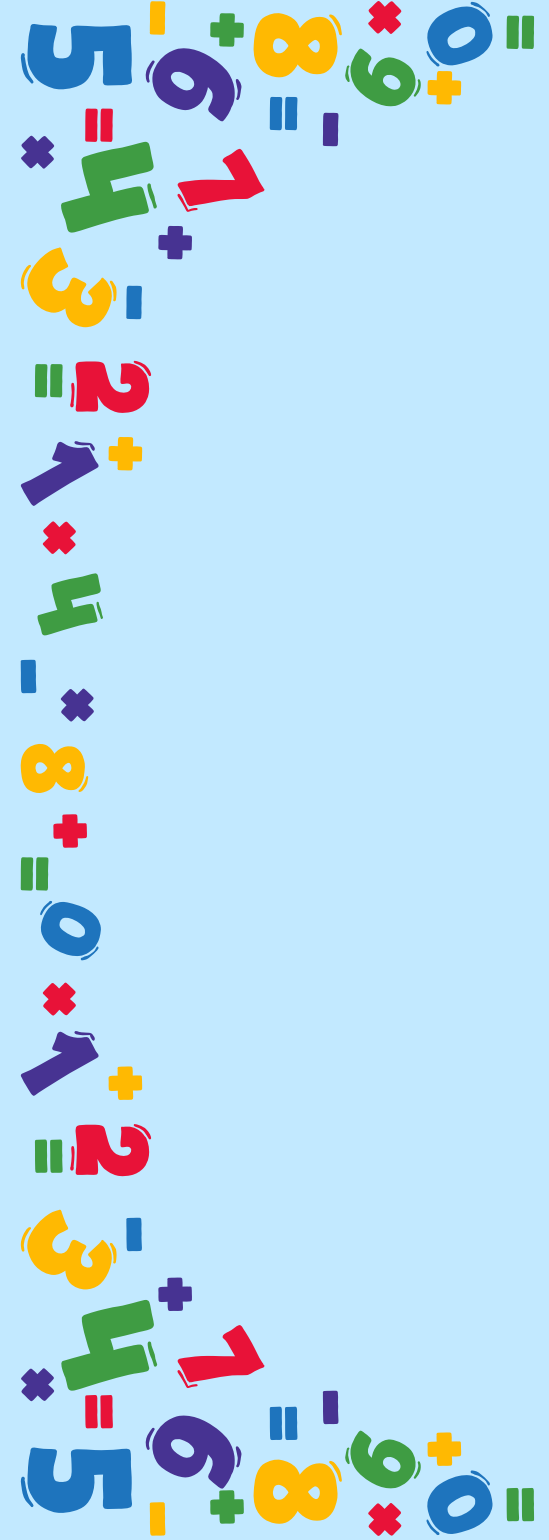
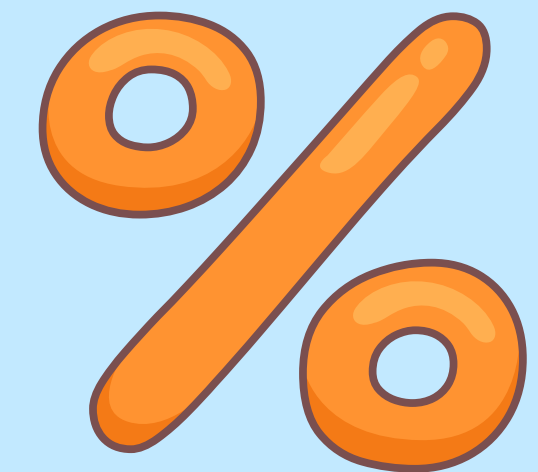
$$\frac{2}{5} = 40\%$$

Recommendations!!!

Often, on the test, we need to approximate percents from fractions or from division.

$$\frac{9}{33} = \frac{27}{99} > \frac{27}{100} = \mathbf{27\%}$$

$$\frac{11}{14} = \frac{77}{98} > \frac{77}{100} = \mathbf{77\%}$$



A vertical arrangement of colorful mathematical symbols and numbers. From top to bottom, the elements are: a red plus sign, a blue number 9, a blue equals sign, a yellow number 3, a blue number 6, a yellow number 5, a red plus sign, a yellow number 1, a red plus sign, a red multiplication sign, an orange number 8, a blue number 1, an orange minus sign, a yellow number 3, a blue plus sign, a red multiplication sign, an orange number 8, an orange minus sign, a yellow number 3, a blue plus sign, and an orange percent sign.

An illustration of three geometric shapes on a light blue background. In the foreground is a red circle with a small black outline and a simple smiley face. Behind it is a green square, also with a black outline and a smiley face. In the background is an orange triangle with a black outline and a smiley face. All shapes have a slight grey shadow beneath them, giving them a 3D appearance.

Example

Find 15% of 80.

$$15\% = 0.15$$
$$0.15 \times 80 = 12$$

10% of 80 = 8

15% = 8 + 4 = 12

Ans: 220

Finding the whole

Example

18 students are 30% of the class.

How many students are in the class?

$$30\% = 0.30$$

$$18 \div 0.30 = 60$$

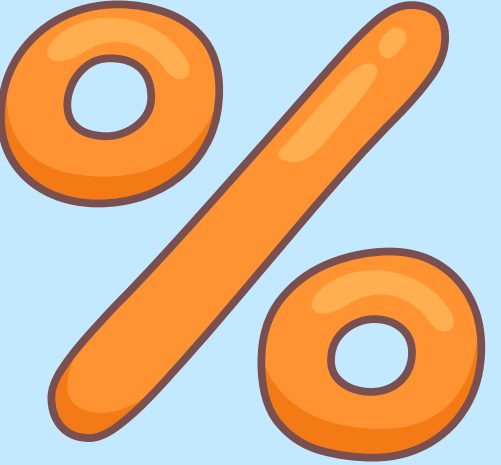
240 is 30% of what number?

30%
means 18 students



30%

100% = ?



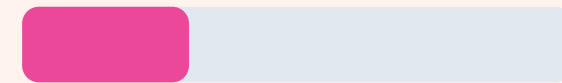
Method One: Per cents as Multipliers

The **decimal form of a percent** is called the **multiplier** for that per cent. This is because we simply can multiply by this form to take a per cent of a number

1. “**is**” means “**equals**”
2. “**of**” means “**multiply**”
3. **Change** any per cent to the **multiplier** form
4. Replace **unknowns** with a **variable**

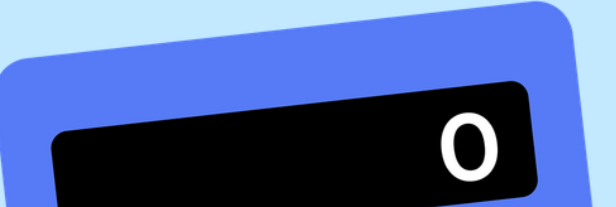


240 is 30% of what number?



30%

100% = ?



% ក្នុងរូបរាងទម្រង់ទុក្ខ = multiplier

The **decimal form** of a **percent** is called the **multiplier** for that per cent. This is because we simply can multiply by this form to take a per cent of a number

=

1. "is" means "equals"
2. "of" means "multiply" x
3. **Change** any per cent to the multiplier form
4. Replace **unknowns** with a variable

ប្រើតួអក្សរតំណាងដោយតួអក្សរ

What is 80% of 200?

Multiplier is 0.80

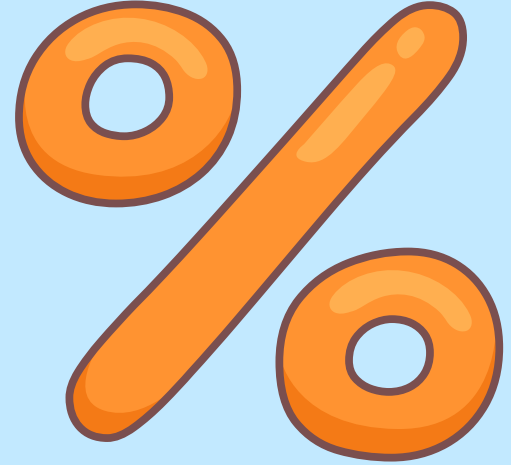
Then $x = (0.80)(200) = 160$



240 is 30% of what number?

$$240 = (0.30)x$$

$$x = \frac{240}{0.30} = \frac{2400}{3} = 800$$



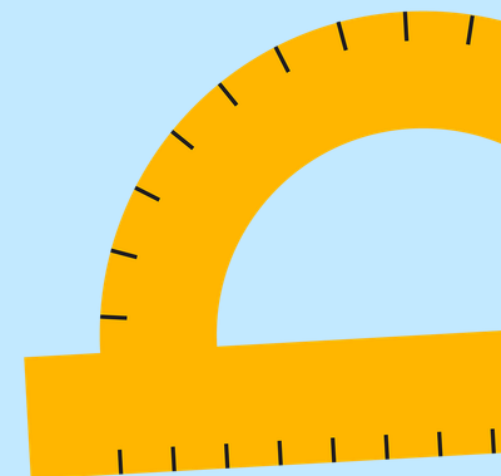
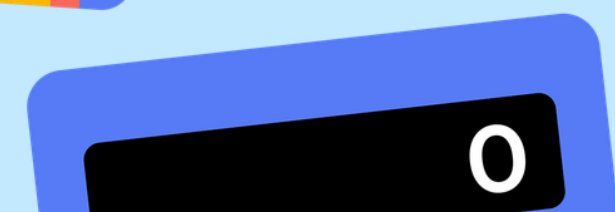
Finding Per cent

Method Two: Finding the Per cent

56 is what per cent of 800

$$56 = x(800) \quad (x \text{ is multiplier})$$

$$x = \frac{56}{800} = \frac{7}{100} = 7\%$$



Per cent and Fraction

Use this approach only
if the % is a very easy
fraction.

for example, $\frac{1}{2}$ or $\frac{1}{4}$

What is 75% of 280 ?

$$75\% \times 280 = \frac{3}{4} \times 280 = 210$$

1. What is 60% of 60 ?

36

2. 52 is 40% of what number ?

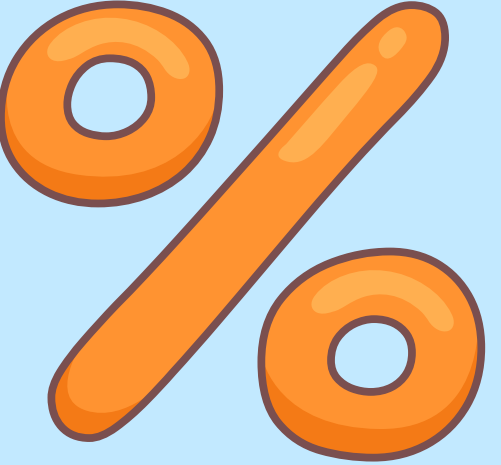
130

3. 18 is what per cent of 45 ?

$$\frac{18}{45} = \frac{6}{15}$$

4. What is 50% of 128 ?

64



3. Number Sense and Per cent

Beginning suggestion :

think about 10% of the whole, and work from there.
Sometimes it also help to find 1%

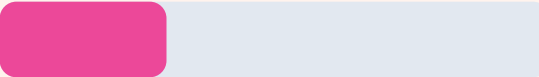
What is 80% Of 200?

10% of 200 is clearly 20

We want eight of these → $8(20) = 160$

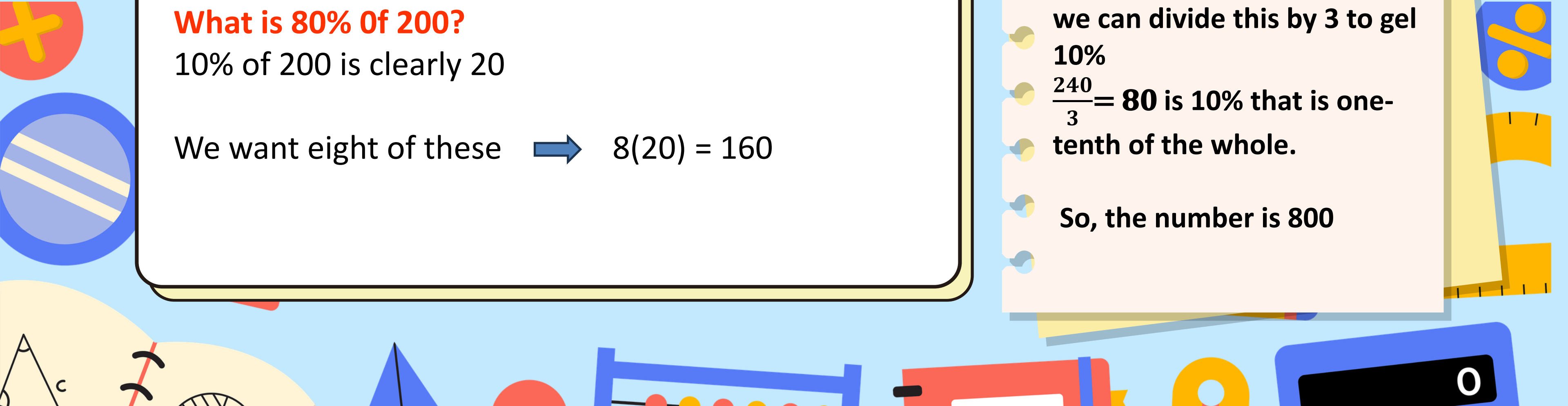


240 is 30% of what number?



If 30% is 240,
we can divide this by 3 to get
10%
 $\frac{240}{3} = 80$ is 10% that is one-tenth of the whole.

So, the number is 800





56 is what per cent of 800?

$$10\% \text{ of } 800 = 80$$

$$1\% \text{ of } 800 = 8$$

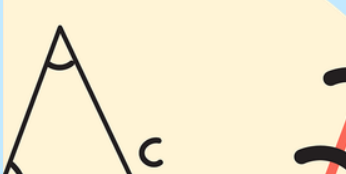
We need 7 times last piece. Thus $7\% \text{ of } 800 = 56 \rightarrow 7\%$



What is 55% of 400?

$$50\% \text{ of } 400 = 200$$

$$5\% \text{ of } 400 = 20$$

$$\text{Thus } 55\% \text{ of } 400 = 200 + 20 = 220$$




What is 37% of 700?

$10\% \text{ of } 700 = 70 \rightarrow 30\% \text{ of } 700 = 70 \times 3 = 210$

$1\% \text{ of } 700 = 7 \rightarrow 7\% \text{ of } 700 = 7 \times 7 = 49$

Thus $37\% \text{ of } 700 = 210 + 49 = 259$

Practice these

1. What is 70% of 50?

35

2. 48 is 30% of what number?

160

3. 72 is what per cent of 300?

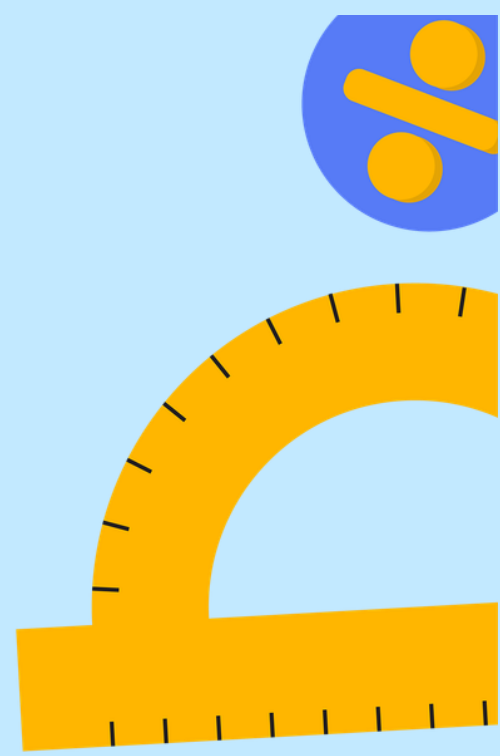
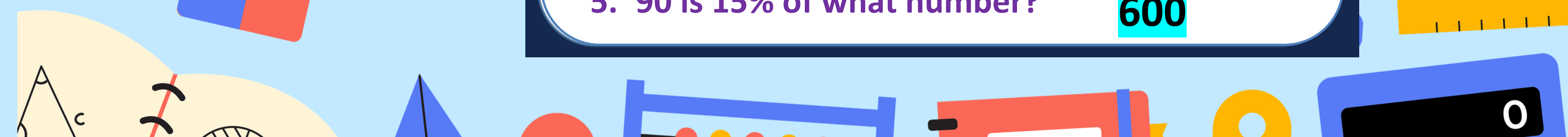
24%

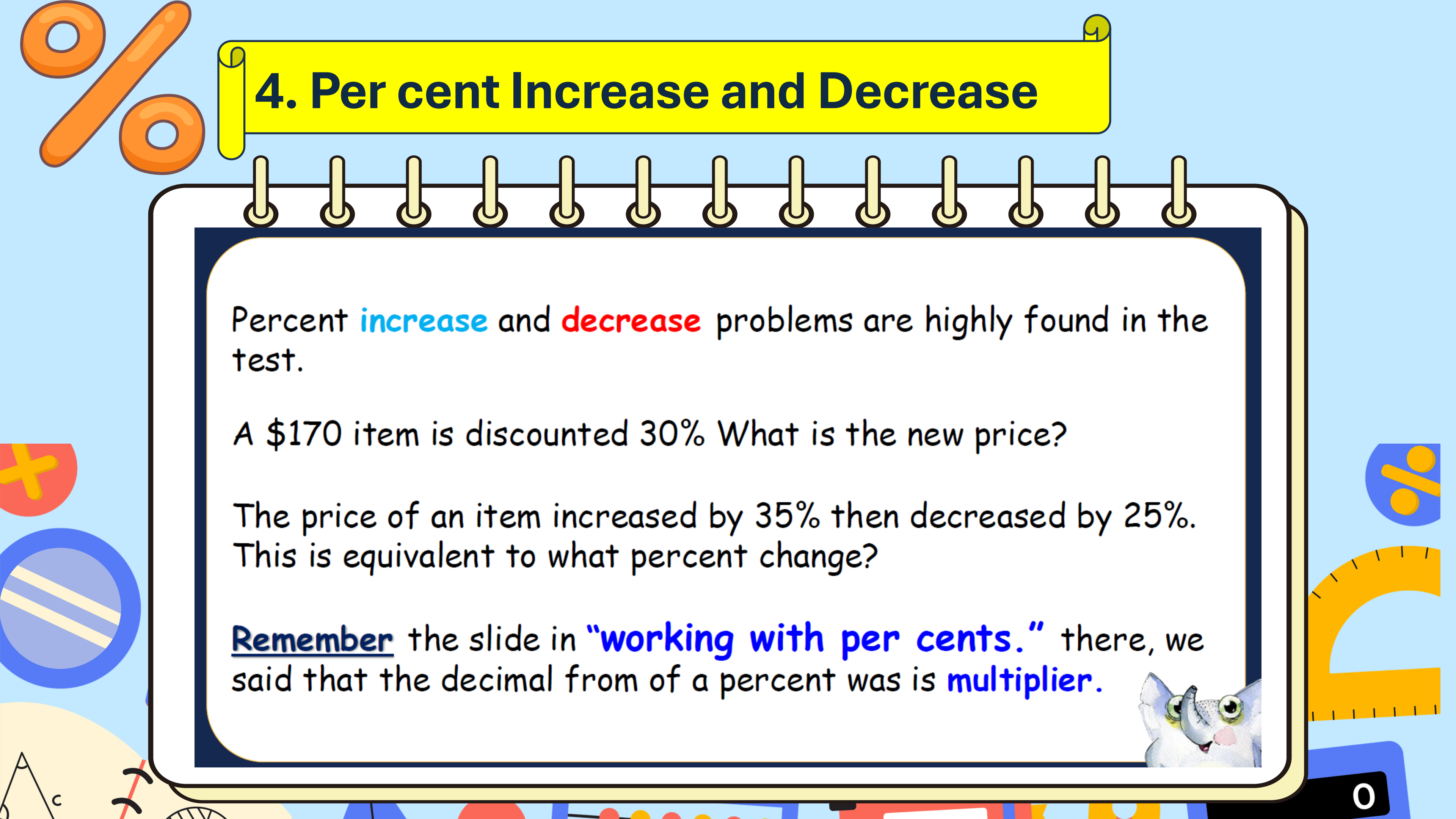
4. What is 35% of 180?

63

5. 90 is 15% of what number?

600





4. Per cent Increase and Decrease

Percent **increase** and **decrease** problems are highly found in the test.

A \$170 item is discounted 30% What is the new price?

The price of an item increased by 35% then decreased by 25%.
This is equivalent to what percent change?

Remember the slide in "**working with per cents.**" there, we said that the decimal form of a percent was is **multiplier**.





4. Per cent Increase and Decrease

The decimal from of P per cent is the multiplier for finding P per cent of something. 0.30 is the multiplier to find 30% of something, but we will use different multiplier for a **30% increase** or a **30% decrease**.

Per cent Increases

This might be phrased as :

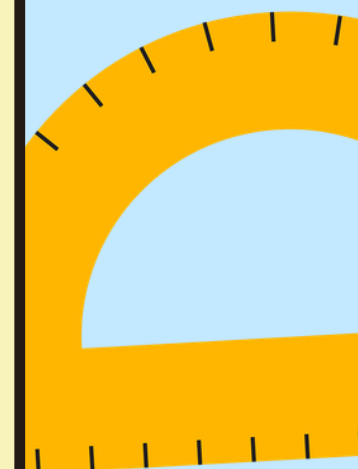
X มากกว่า Y อยู่ 30 %

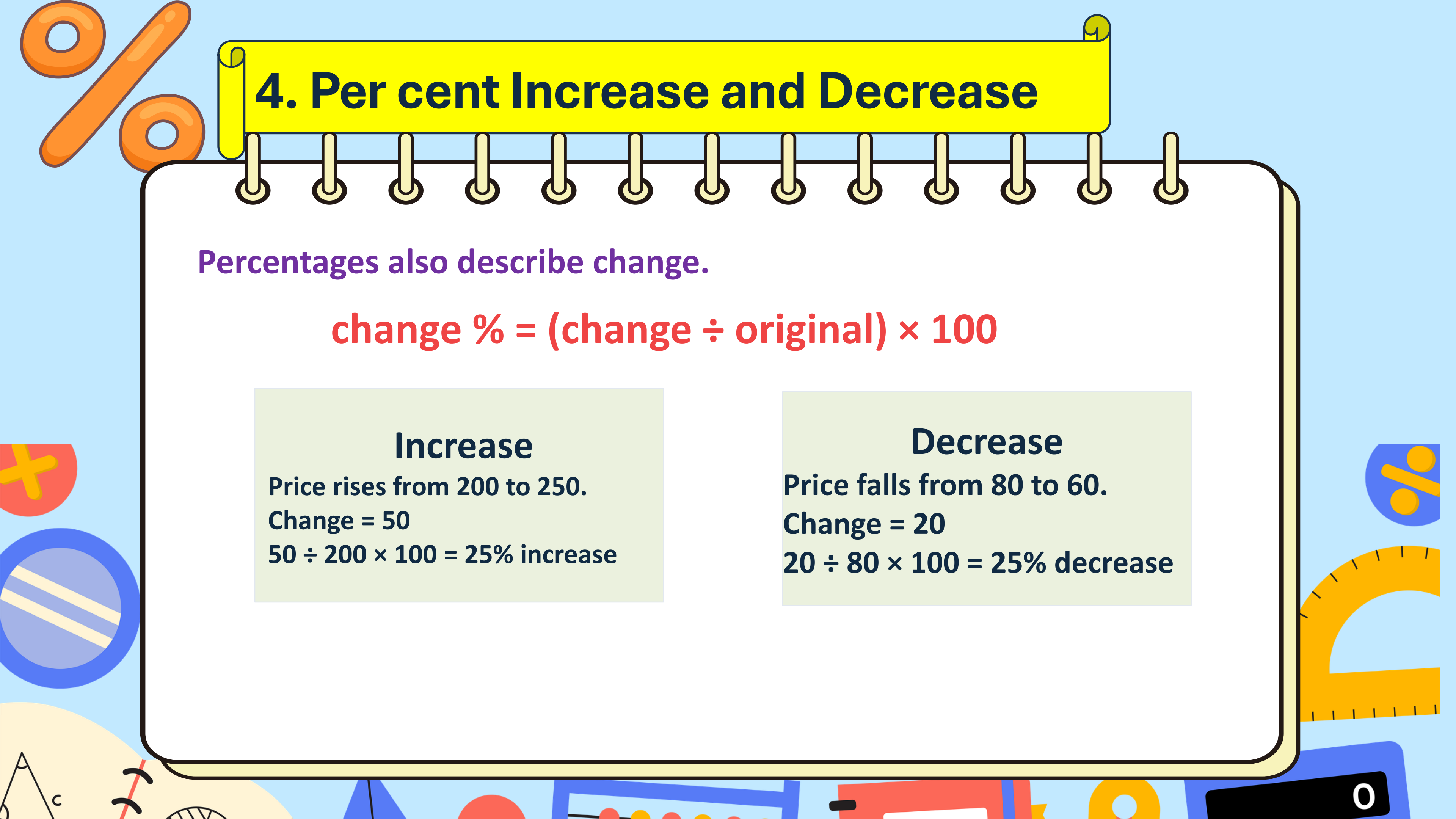
Y increased by 30% Or X is 30% greater than Y.

Either way, this is per cent increase for Y. Let's think about this.

If Y increase by 30%, the whole original part of Y is still there, plus

30% increase is : $1 + 0.30 = 1.3$





4. Per cent Increase and Decrease

Percentages also describe change.

$$\text{change \%} = (\text{change} \div \text{original}) \times 100$$

Increase

Price rises from 200 to 250.

Change = 50

$$50 \div 200 \times 100 = 25\% \text{ increase}$$

Decrease

Price falls from 80 to 60.

Change = 20

$$20 \div 80 \times 100 = 25\% \text{ decrease}$$



4. Per cent Increase and Decrease

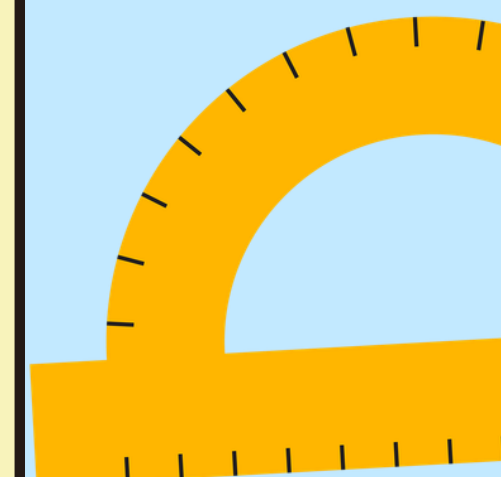
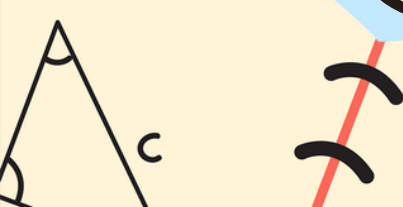
The decimal from of P per cent is the multiplier for finding P per cent of something. 0.30 is the multiplier to find 30% of something, but we will use different multiplier for a **30% increase** or a **30% decrease**.

Per cent Increases

This might be phrased as :

Y increased by 30% Or X is 30% greater than Y.

Either way, this is per cent increase for Y. Let's think about this. If Y increase by 30%, the whole original part of Y is still there, plus 30% increase is : $1 + 0.30 = 1.3$





4. Per cent Increase and Decrease

In general, if the problem talks about a P% increase,

(multiplier for a P% increase) = $1 + (\text{P\% as a decimal})$

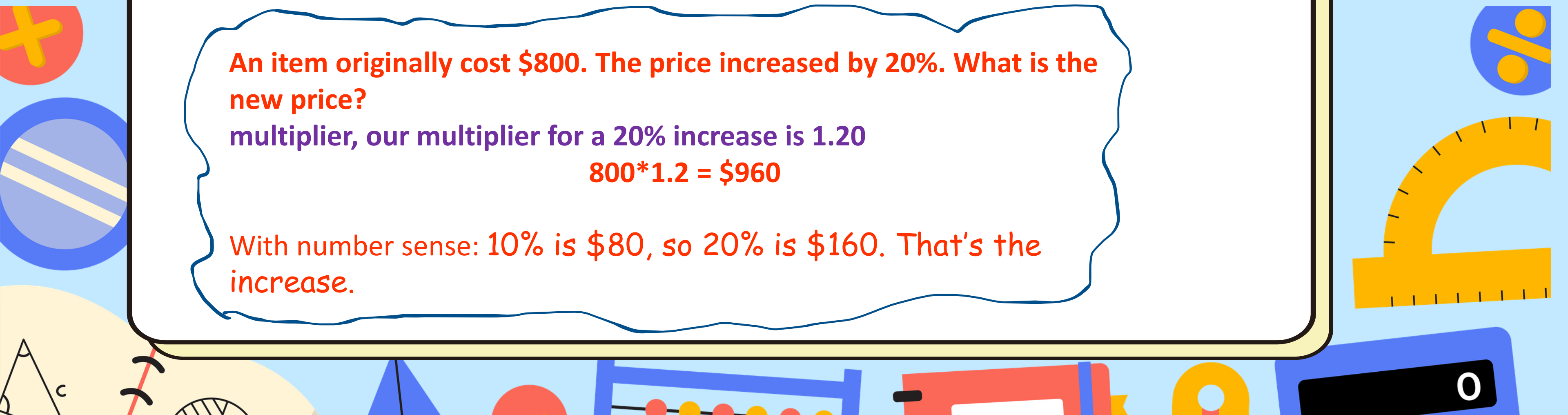
Thus, the multiplier for a 46% increase is $1 + 0.46 = 1.46$

An item originally cost \$800. The price increased by 20%. What is the new price?

multiplier, our multiplier for a 20% increase is 1.20

$$800 * 1.2 = \$960$$

With number sense: 10% is \$80, so 20% is \$160. That's the increase.

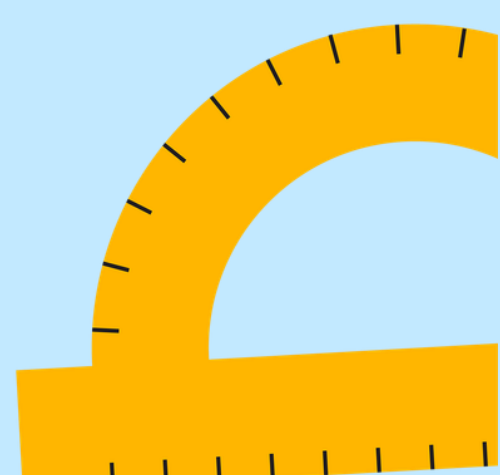
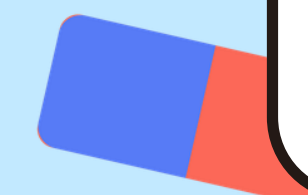
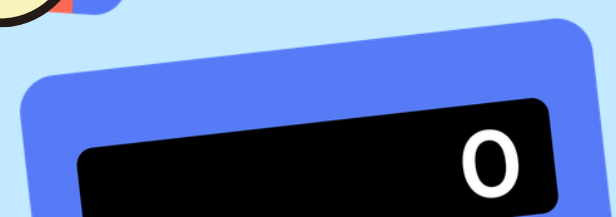




**After a 30% increase, the price of something is \$78.
What was the original price?**

Multiplier = 1.3 The unknown original is multiplied by this

$$x \times 1.3 = 78$$

$$x = \frac{78}{1.3} = \frac{780}{130} = \$60$$


Per cent Decreases

This might be phrased as :

Y decreased by 30% Or X is 30% less than Y

Either way, this is per cent decrease for Y. Let's think about this. If Y decrease by 30% the whole original part of Y is still here, except for a 30% that's now missing. Therefore, the multiplier for a 30% decrease is $1 - 0.30 = 0.7$

In general, if the problem talks about a P% decrease

(multiplier for a P% decrease) = $1 - (P\% \text{ as a decimal})$



As \$ 170 item is discounted 30%. What is the new price?

$$\text{Multiplier} = 1 - 0.30 = 0.70$$

$$170 \times (0.70) = \$119$$

I might notice:

$$10\% = 17, \text{ so } 30\% \text{ is } 3 \times 17 = 51$$

The price goes down by \$ 51 = 50+1

$$170 \text{ down by } 50 = 120$$

$$120 \text{ down by } 1 = 119$$



After an item discounted 80% the new price is \$150. What was the original price?

Notice that:

If 80% is gone , only 20% is left

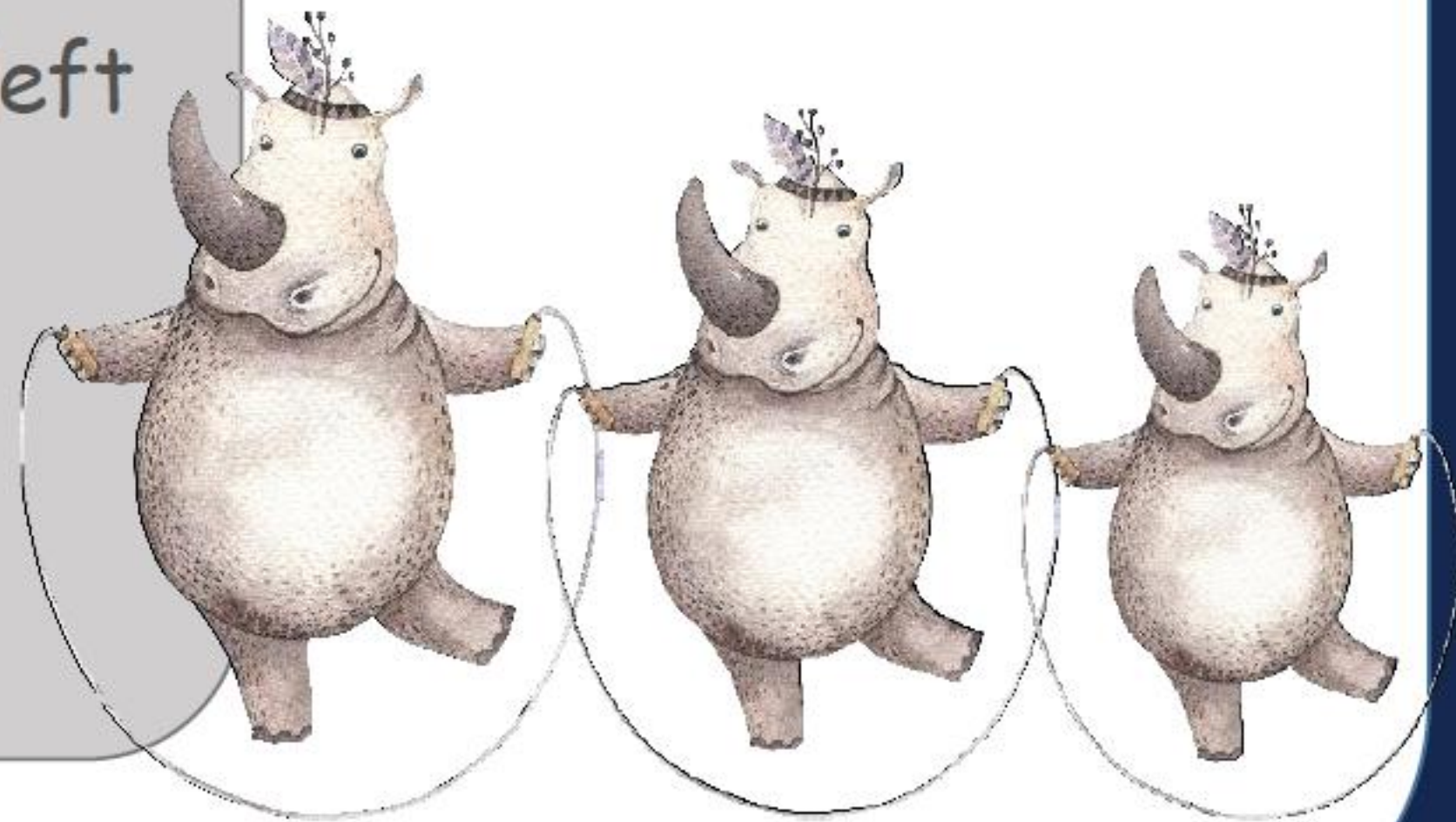
20% is 150

Therefor

10% = 75

Therefor

100% = 750



Finding the per cent

Some problem give us the starting and ending values,
and ask us to find the per cent of increase or decrease

Since $(\text{new}) = (\text{multiplier}) * (\text{old})$

We could say :

$$\text{Multiplier} = \frac{\text{New}}{\text{old}}$$

**** Remember that ratio gives us the multiplier, and we have
to change from that back to a per cent



The price of an item increase from \$60 to \$102.
What was the per cent increase?

$$\text{Multiplier} = \frac{\text{new}}{\text{old}} = \frac{102}{60} = \frac{17}{10} = 1.7$$

That's the multiplier for a 70% increase. 70%

I might notice:

$$102 - 60 = 42$$

$$10\% \text{ of } 60 = 6$$

$$70\% \text{ of } 60 = 42$$

Therefore

102 is 70% greater than 60



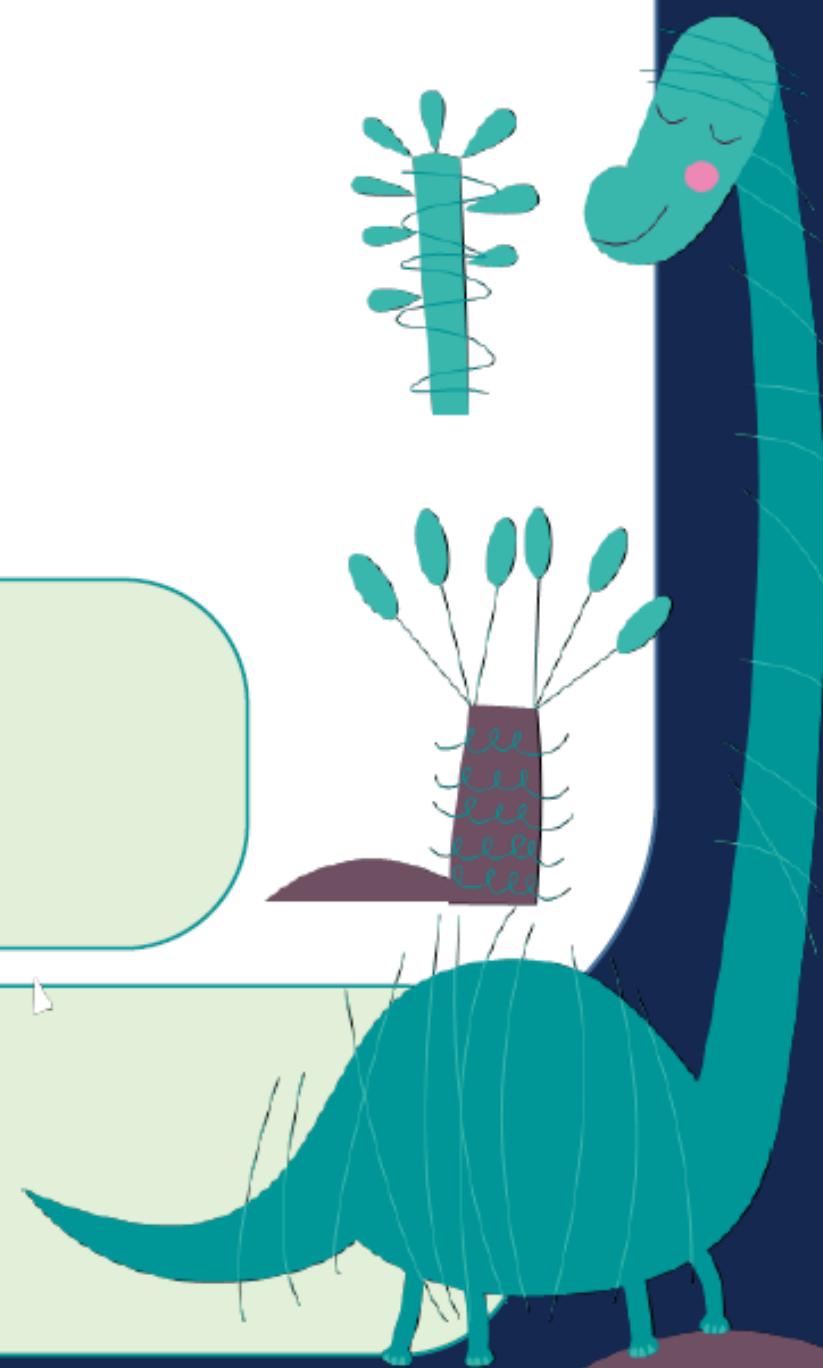
The price of an item decrease from 250\$ to 200\$. What was the per cent decrease?

$$m = \frac{\text{new}}{\text{old}} = \frac{200}{250} = \frac{20}{25} = \frac{80}{100} = 0.8 <$$

That's the multiplier for a 20% decrease

Notice that price drop \$50 is $\frac{1}{5}$ of \$250
That's a drop of $\frac{1}{5}$, or 20%

The price of an item increase from \$200 to \$800.
What was the per cent increase?





5. Sequential Per Cent Changes

Why “+30% then -30%” does not return to the original value \$100



Common test trap

Use multipliers, not simple addition or subtraction of percentages.

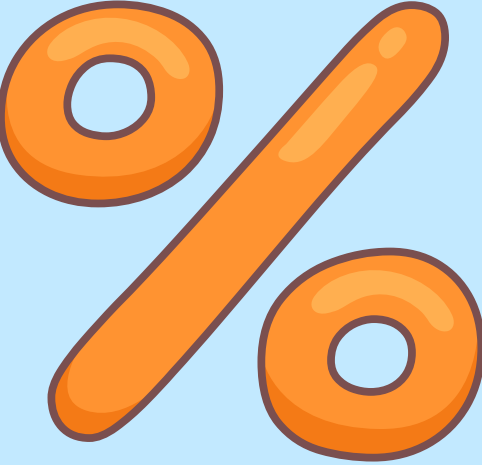
$\times 1.30$

\$130

$\times 0.70$

\$91





Common mistakes

Test loves to exploit these common mistakes



Mistake #1 : an increase by same per cent do NOT get us back to the original starting point. These mistake center on scenarios in which two or more per cent changes follow in sequence (e.g. a% price increase and then a% employee discount)



Common mistakes

Consider this question:

An item initially cost \$100. At the beginning of the year, the price increase 30%. After the increase, an employee purchased it with a 30% discount. What price did the employee pay?

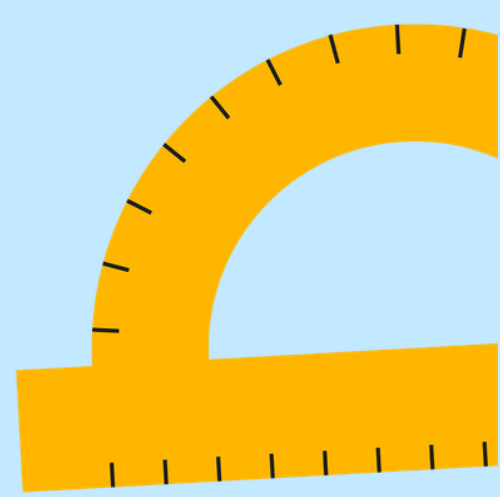
The mistake : $+30\% - 30\% = 0$

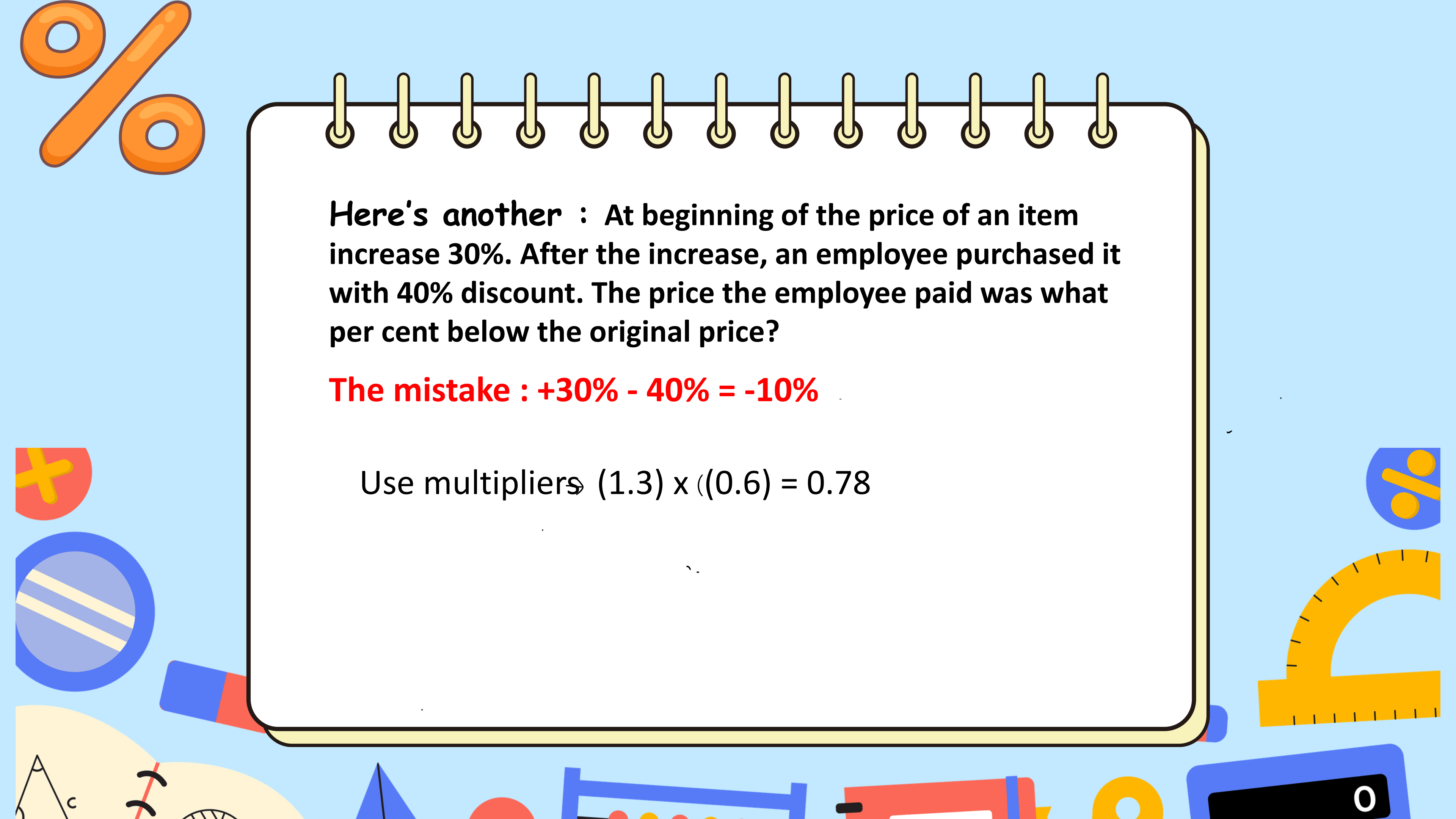
The answer is NOT 100\$

We answer this using a product of multipliers

30% increase = 1.3

30% decrease = 0.7

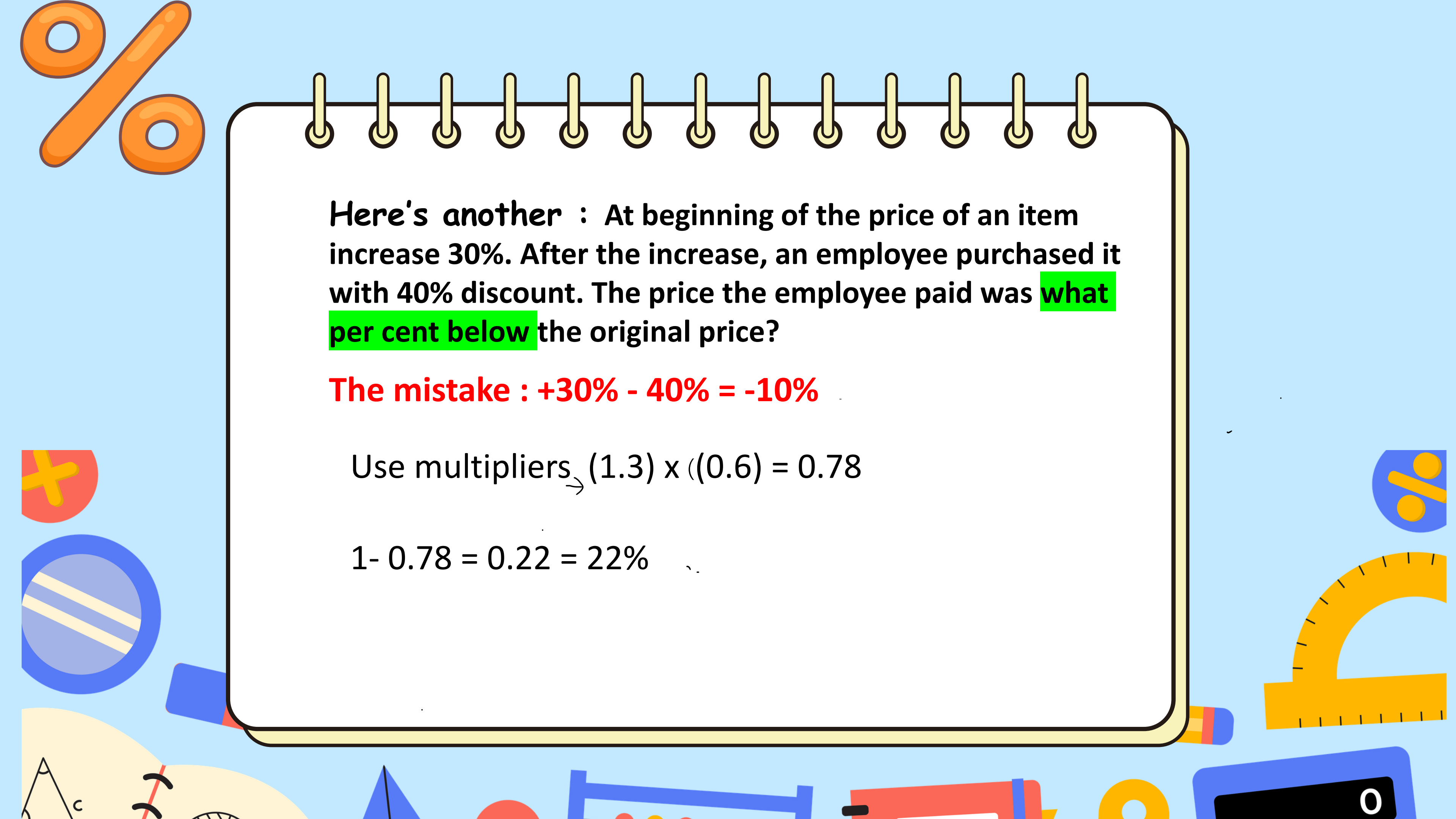
$$100 \times (1.3) \times (0.7) = \$91$$




Here's another : At beginning of the price of an item increase 30%. After the increase, an employee purchased it with 40% discount. The price the employee paid was what per cent below the original price?

The mistake : $+30\% - 40\% = -10\%$

Use multipliers $(1.3) \times (0.6) = 0.78$

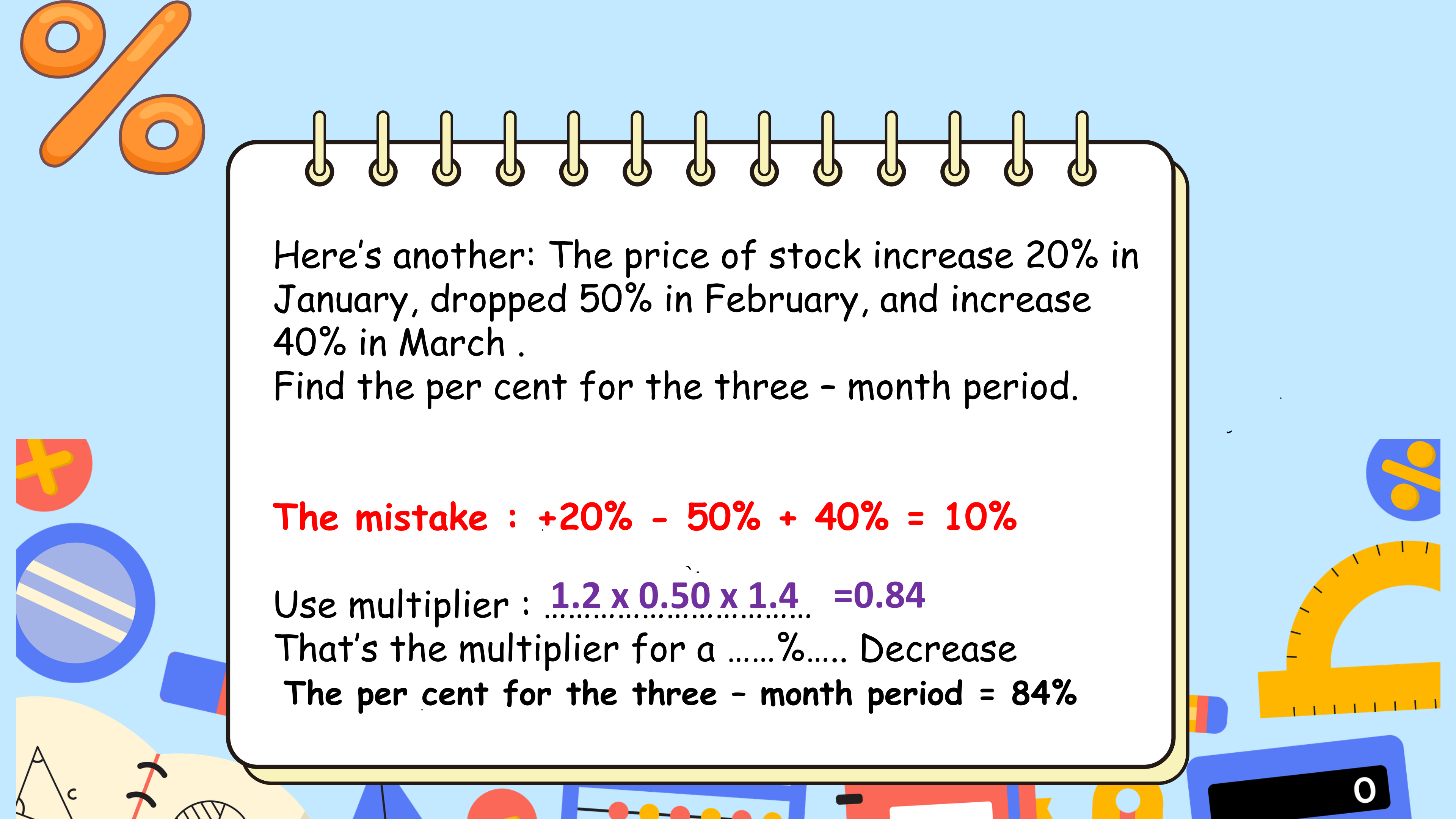


Here's another : At beginning of the price of an item increase 30%. After the increase, an employee purchased it with 40% discount. The price the employee paid was **what per cent below** the original price?

The mistake : $+30\% - 40\% = -10\%$

Use multipliers $(1.3) \times (0.6) = 0.78$

$$1 - 0.78 = 0.22 = 22\%$$



Here's another: The price of stock increase 20% in January, dropped 50% in February, and increase 40% in March .

Find the per cent for the three - month period.

The mistake : $+20\% - 50\% + 40\% = 10\%$

Use multiplier : $1.2 \times 0.50 \times 1.4 = 0.84$

That's the multiplier for a%..... Decrease

The per cent for the three - month period = 84%

Practice Together

Solve using multipliers.

1. A \$200 item increases by 20%, then has a 20% discount. What is the final price?
2. A price decreases by 10%, then increases by 10%. Is it back to the original price?
3. A \$150 item increases by 25%, then has a 40% discount. What is the final price?

Tip: Increase $\rightarrow 1 + p/100$
Decrease $\rightarrow 1 - p/100$

Practice Answers

Check the multiplier, then calculate.

1. $\$200 \times 1.20 \times 0.80 = \192

2. No, the price does **not** return to the original price. It is **\$1 lower**, which is a **1% overall decrease**.
 $1 \times 0.90 \times 1.10 = 0.99 \rightarrow 99\% \text{ of original, so it is } 1\% \text{ below.}$

3. $\$150 \times 1.25 \times 0.60 = \112.50

Remember: equal percentage increase and decrease do not cancel out.

6. Simple and Compound Interest

Let's begin with **simple interest** is a mathematical fiction.
Absolutely no one in the real world use simple interest.

With simple interest payment is exactly the same each time

Bob deposits \$1000 in an account that yield 5% simple interest annually.

5% of \$1000 is \$50

$$1 \text{ year} : \$1000 + \$50 = \$1050$$

$$2 \text{ years} : \$1050 + \$50 = \$1100$$

$$3 \text{ years} : \$1100 + \$50 = \$1150$$

$$4 \text{ years} : \$1150 + \$50 = \$1200$$

$$5 \text{ years} : \$1200 + \$50 = \$1250$$

$$6 \text{ years} : \$1250 + \$50 = \$1300$$

$$7 \text{ years} : \$1300 + \$50 = \$1350$$

$$8 \text{ years} : \$1350 + \$50 = \$1400$$



Compound Interest

The big idea of compound interest is: interest on interest. In compound interest, we get the per cent of interest paid on the total amount that has already increased in the **account, principal + all previous interest payment**. No two successive interest payments are ever the same. The entire amount experiences the same percentage increase in each period in each period



Bob deposits \$1000 in an account that yield 5% compound interest annually

✓
គិតចំណា ប្រើប្រាស់ ចំណា
ប្រើប្រាស់

Start = \$1000

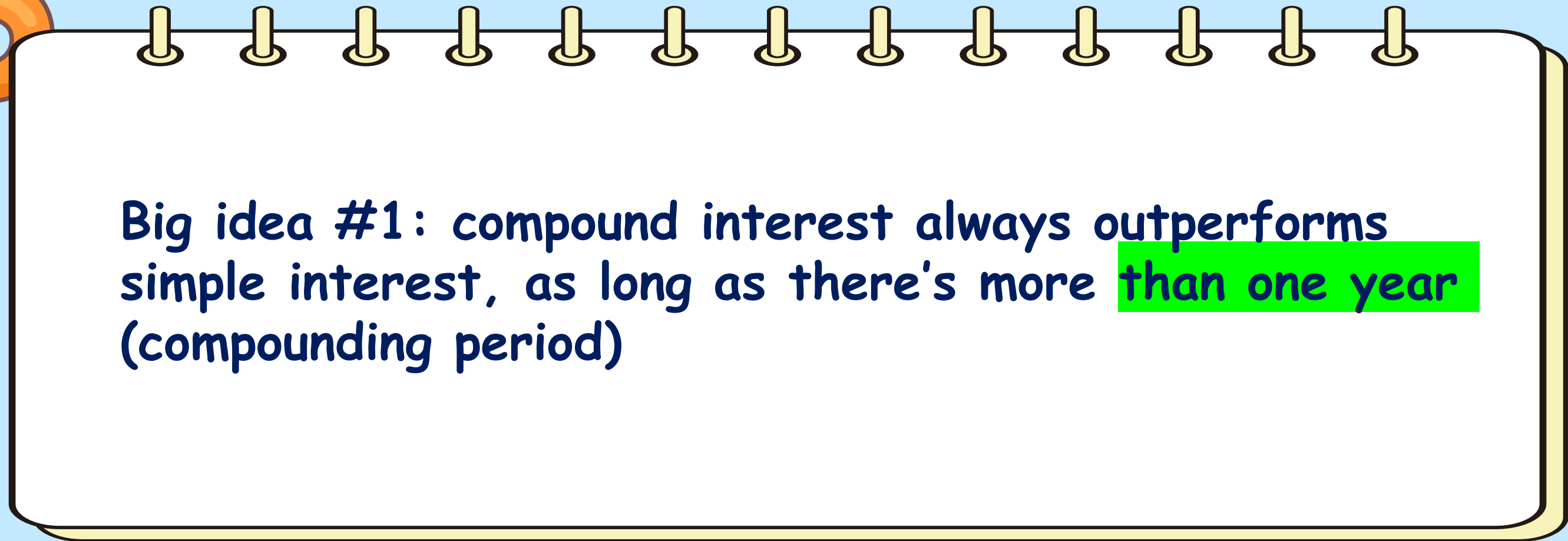
$$1 \text{ year} : \$1000 \times (1.05) = \$1050$$

$$2 \text{ years} : \$1050 \times (1.05) = \$1102.5$$

$$3 \text{ years} : \$1102.5 \times (1.05) = \$1157.625$$

$$4 \text{ years} : \$1157.625 \times (1.05) = \$1215.50625$$

With large amounts of money and/or long periods of time, the difference successive interest payments becomes substantial.



Big idea #1: compound interest always outperforms simple interest, as long as there's more than one year (compounding period)

Using the Compound Interest Formula

Instead of calculating one year at a time, we can use:

$$A = P(1 + r)^t$$

where:

- A = final amount
- P = principal or starting amount
- r = annual interest rate as a decimal
- t = number of years

For this example:

$$A = 1000(1 + 0.05)^4$$

$$A = 1000(1.05)^4$$

$$A = 1000(1.21550625)$$

$$A = \boxed{\$1,215.51}$$

Using the Compound Interest Formula

Notice also: in y years, the principal will be multiplied by the percent increase and multiplier y time.

Let P be the **principal** and r be the **multiplier**.

The total amount in the account after y year is given by:

$$A = P(r^y)$$

If the annual interest rate is I , then the multiplier is simply $r = 1 + \frac{I}{100}$

$$A = P(r^y) = P \left(1 + \frac{I}{100} \right)^y$$

Using the Compound Interest Formula

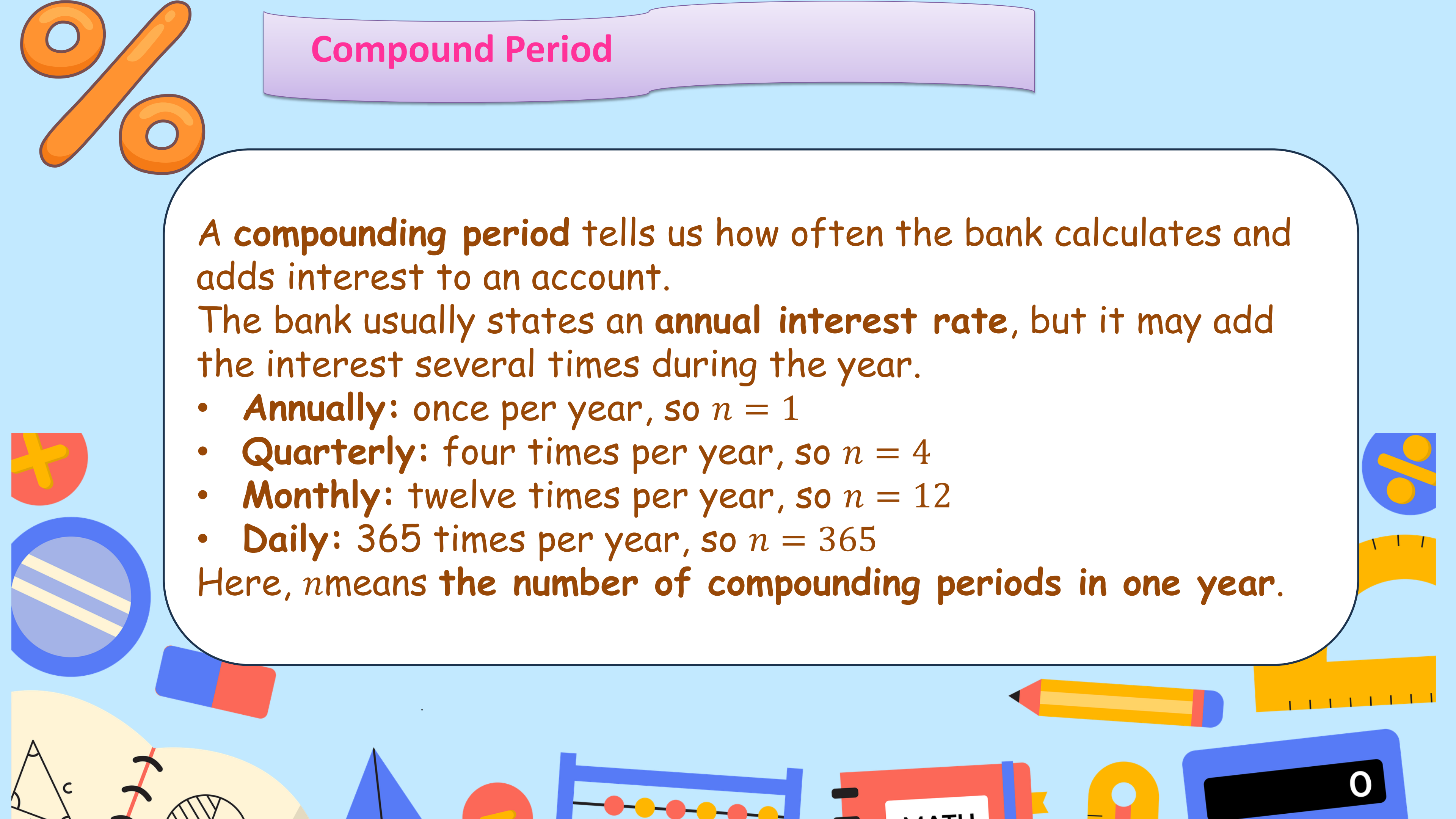
Sheila invests \$4000 in an account that yield 6% compounding annually for 8 years. What is the total amount after 8 years?

Multiplier for a for 6% increase =

The account is multiplied by this multiplier eight times

$$A = 4000(1.06)^8$$

$$= 4000 \times 1.6 = 6400$$



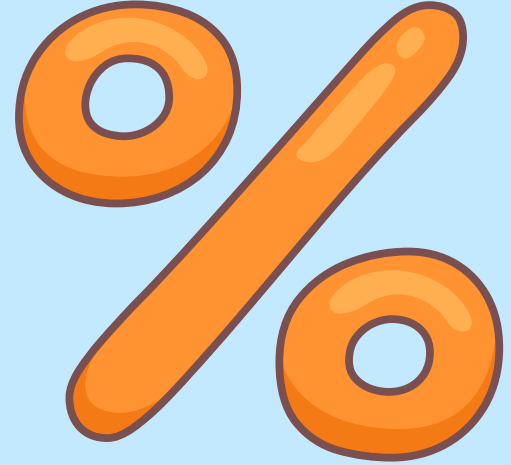
Compound Period

A **compounding period** tells us how often the bank calculates and adds interest to an account.

The bank usually states an **annual interest rate**, but it may add the interest several times during the year.

- **Annually:** once per year, so $n = 1$
- **Quarterly:** four times per year, so $n = 4$
- **Monthly:** twelve times per year, so $n = 12$
- **Daily:** 365 times per year, so $n = 365$

Here, n means the number of compounding periods in one year.



Compound Period

Example: 5% compounded quarterly

The bank does **not** pay 5% every quarter.
The 5% is the rate for the whole year.

Because there are four quarters:

$$\text{Interest per quarter} = \frac{5}{4} = 1.25\%$$

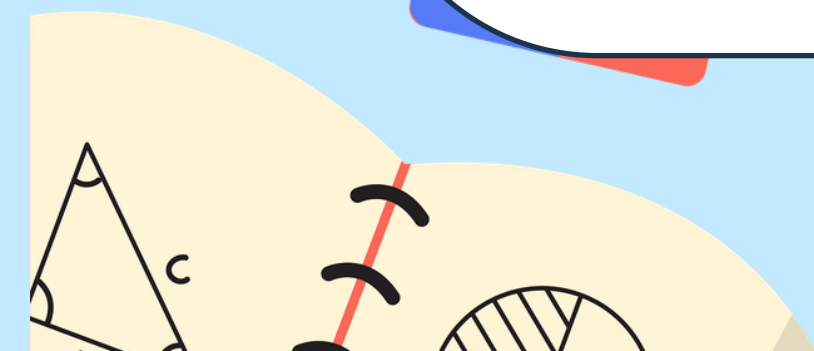
Change 1.25% into a decimal:

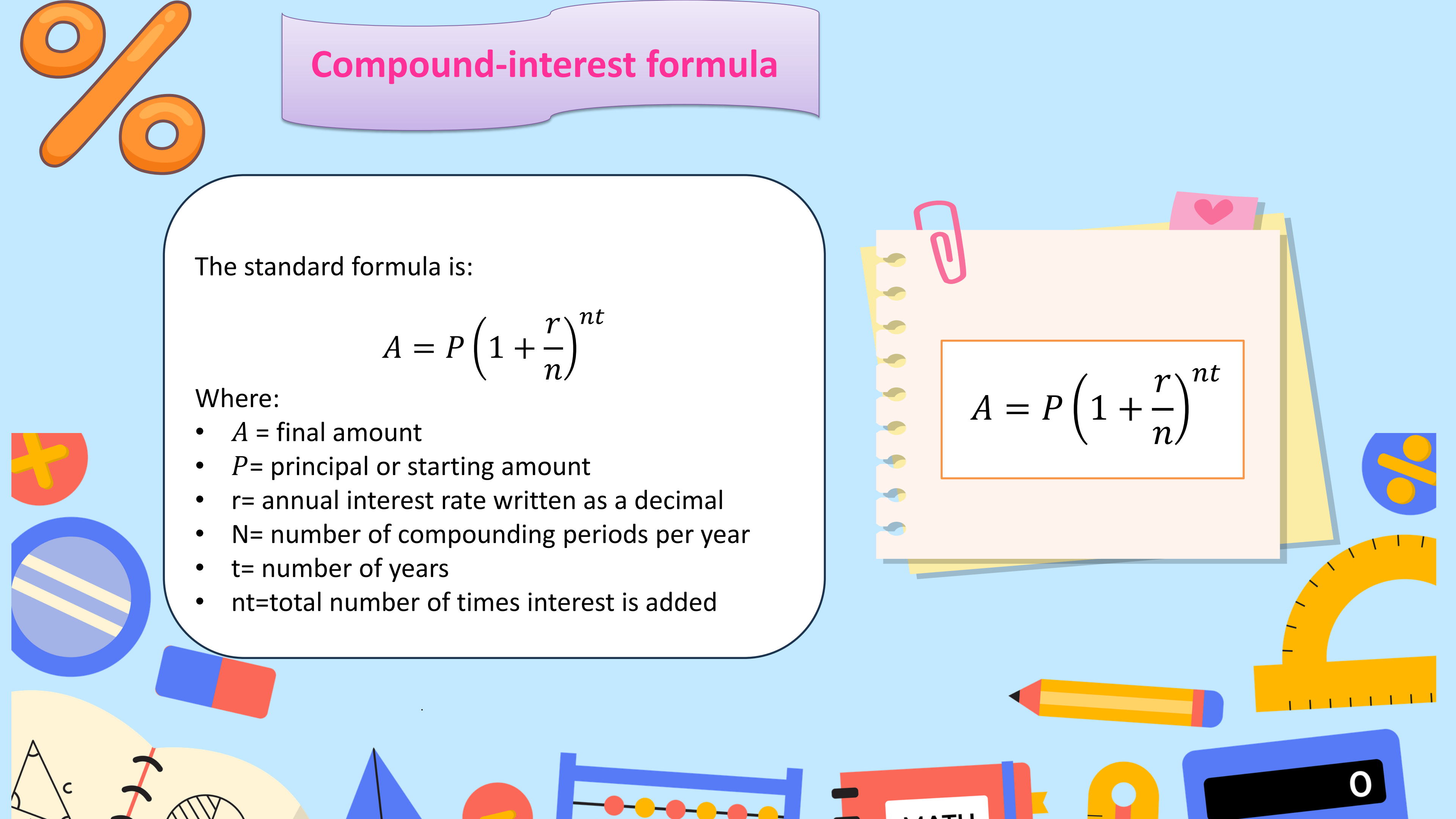
$$1.25\% = 0.0125$$

Therefore, the amount is multiplied by:

$$1 + 0.0125 = 1.0125$$

Simple explanation: Every three months, the account increases by 1.25%.





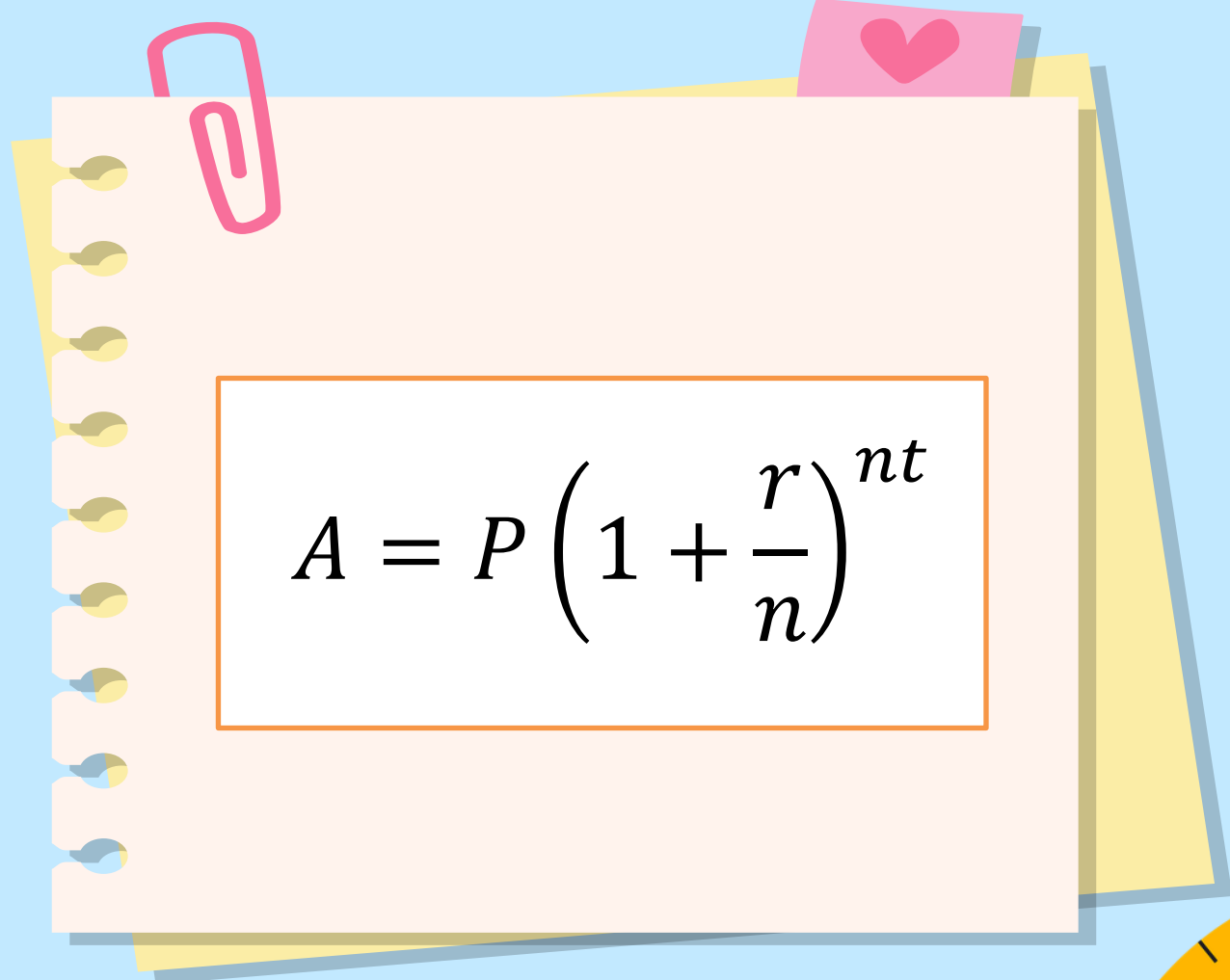
Compound-interest formula

The standard formula is:

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

Where:

- A = final amount
- P = principal or starting amount
- r = annual interest rate written as a decimal
- N = number of compounding periods per year
- t = number of years
- nt = total number of times interest is added


$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

If Susan invests \$1000 in an account that yield 5% annual, compounding quarterly, then how much does she have after 6 years.

$$\text{Quarterly percent} = \frac{5\%}{4} = 1.25\% = 0.0125$$

$$\text{Thus multiplier} = 1.0125$$

The amount in the account experiences that percent increase 4 times each year, or $6 \times 4 = 24$ times in 6 years.

$$A = 1000(1.0125^{24})$$

↑ 24 times

$$\text{Estimate} = \text{simple interest} = 50 \times 6 + 1000 = \$1300$$

The compound interest is slightly more than simple interest



$$A = 1000 \left(1 + \frac{0.05}{4} \right)^{4 \times 6}$$

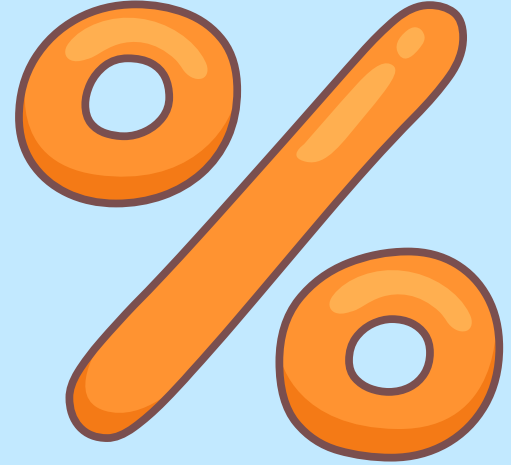
$$A = 1000(1.0125)^{24}$$

$$A \approx \$1,347.35$$

After six years, Susan has approximately **\$1,347.35**.

Her total interest is:

$$\$1,347.35 - \$1,000 = \$347.35$$



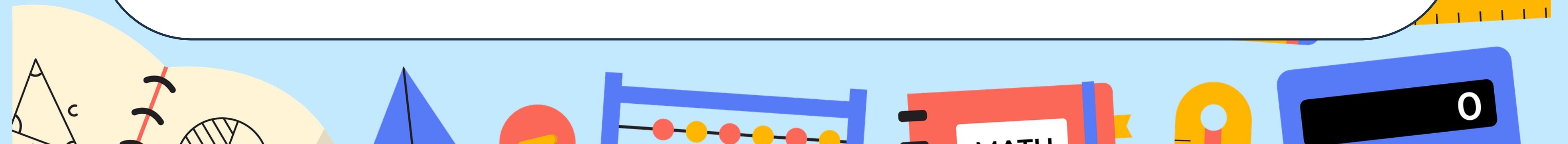
Does more frequent compounding produce more money?

Yes. When interest is compounded more frequently, the final amount becomes slightly larger.

For example:

- Annual compounding: interest is added once per year.
- Quarterly compounding: interest is added four times per year.
- Monthly compounding: interest is added twelve times per year.
- Daily compounding: interest is added 365 times per year.
- Hourly compounding: interest is added every hour.

The annual rate is divided into smaller rates, but interest begins earning additional interest sooner.



For this, we will need a large principal and a long amount of time. Let the annual interest rate is 15%, the principal is \$1,000,000, and the time is 20 years. Simple interest: \$2,000,000

Compounding annually:

\$2,653,297.71

Compounding quarterly:

\$2,701,484.9

Compounding monthly:

\$2,712,640.29

Compounding daily:

\$2,718,095.80

Compounding hourly:

\$2,718,274.07

จำนวน
ที่มากขึ้น
เท่าไร
จากจำนวน

As compounding period decreases, the overall amount of interest earned increases.



7. Intro to Ratios

Ratio and Proportion

Understanding “parts”, scale factor, and total quantity

3 boys : 4 girls

part-to-part

proportion

Goal: read ratio information and solve ratio word problems clearly.

1. What is a ratio?

A ratio is a comparison of two quantities.

part-to-part
Boys : Girls

part-to-whole
Boys : Total class

whole-to-part
Class : Girls

Example: If there are 3 boys and 4 girls, the ratio of boys to girls is 3:4.

Important: The first number matches the first word.

1. What is a ratio?

A ratio is a comparison of two quantities.

part-to-part
Boys : Girls

part-to-whole
Boys : Total class

whole-to-part
Class : Girls

Example: If there are 3 boys and 4 girls, the ratio of boys to girls is 3:4.

Important: The first number matches the first word.
This is one of the tricky things about ratio. The test
will ALWAYS give you ratio in their simplest form.



Same ratio, different real class sizes

15 boys and 20 girls or
300 boys and 400 girls

21 boys and 28 girls
or 3000 boys and 4000 girl

What is the absolute size of the group? $\frac{3}{4}$

All of these classes have the same ratio $\frac{3}{4}$:

3 boys and 4 girls $\rightarrow n = 1$

15 boys and 20 girls $\rightarrow n = 5$

21 boys and 28 girls $\rightarrow n = 7$

300 boys and 400 girls $\rightarrow n = 100$

Formula: boys = $3n$, girls = $4n$

For some positive integer n , we have: $(3n)$ boys and $(4n)$ girls. n is sometimes called the **scale factor**

Scale factor

Scale factor tells us how many times each ratio part is repeated.

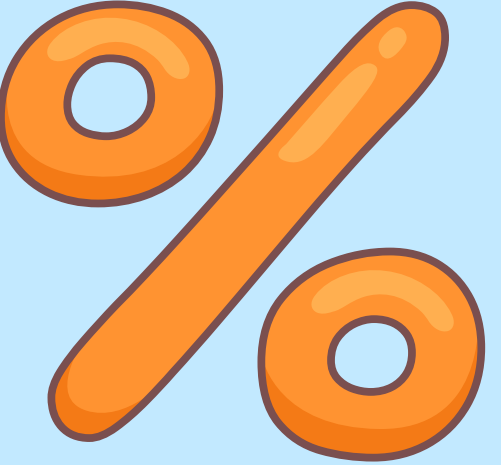
Ratio: $3 : 4$

$\times n$

Real quantities: $3n : 4n$

If $n = 8$, then boys = $3 \times 8 = 24$ and girls = $4 \times 8 = 32$.

Notice, if we are given a ratio, 3 to 4, we have **no idea** about the absolute size of either group. That's a big idea, to which we will return.



Four ways to write a ratio



Words

3 to 4

Fraction

$\frac{3}{4}$

Colon

3:4



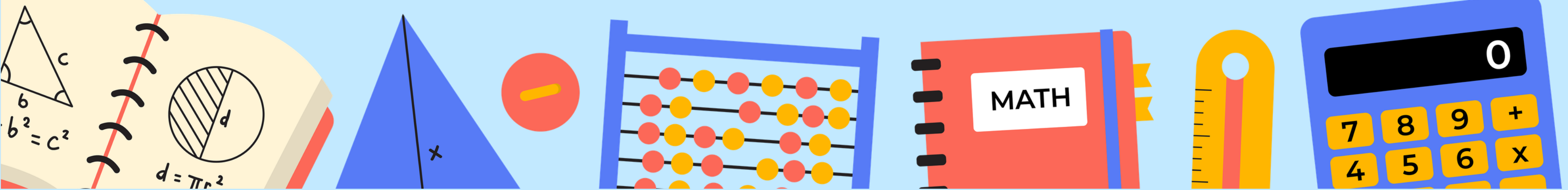
Sentence

For every 3 boys, there are 4 girls.

There are different ways of presenting ratio information.

1. P to q form : "the ratio of boys to girls is 3 to 4."
 2. Fraction form : "the ratio of boys to girls is $\frac{3}{4}$."
 3. Colon form : "the ratio of boys to girls is 3:4."
 4. Idiom form : "for every 3 boys, there are 4 girls."
- 

All forms mean the same thing, but the order must stay the same.



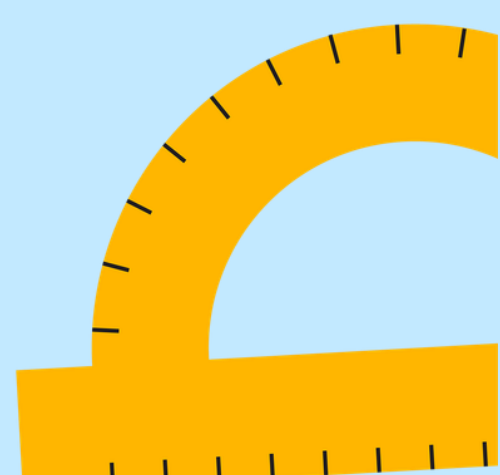
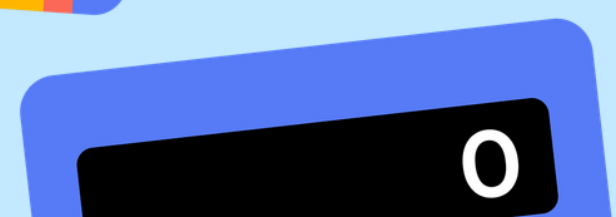


Boys to girls = 3:4 $\leftrightarrow$ Girls to boys = 4:3

When the words switch, the numbers must switch too.

Check the question carefully: What is compared first?

Use **fraction = fraction** to solve the major of ratio problems An equation of this form is known as a **proportion**.



In a class, the ratio of boys to girls is 5:8. If there are 40 girls, how many boys are there?

Let x is a numbers of boys

$$\therefore \frac{x}{40} = \frac{5}{8} \rightarrow x = 25$$

There are 25 boys in a class.





Solving with scale factor

**Problem: Boys to girls is 3:7. There are 32 more girls than boys.
How many boys?**

Ratio parts: boys = 3 parts, girls = 7 parts

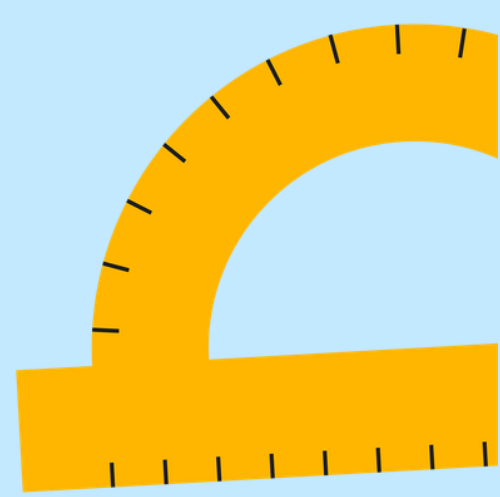
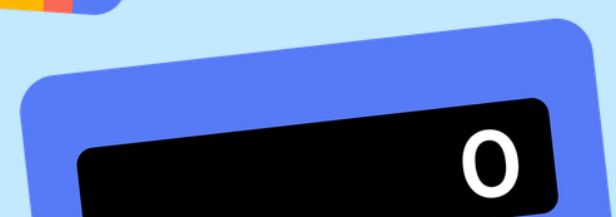
Difference = 7 parts – 3 parts = 4 parts

4 parts = 32 → 1 part = 8

Boys = 3 parts = $3 \times 8 = 24$



Answer: 24 boys





Solving with scale factor

Ratio to the whole

If boys to girls is 3:5, what fraction of the whole class are boys?

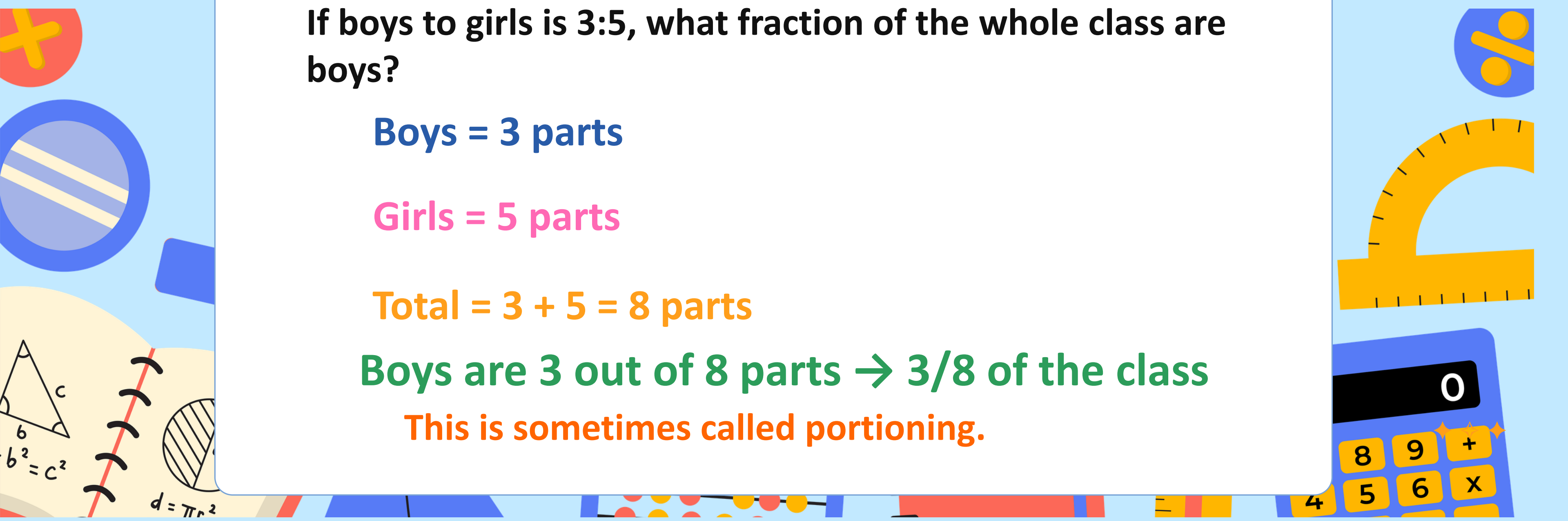
Boys = 3 parts

Girls = 5 parts

Total = $3 + 5 = 8$ parts

Boys are 3 out of 8 parts $\rightarrow 3/8$ of the class

This is sometimes called portioning.

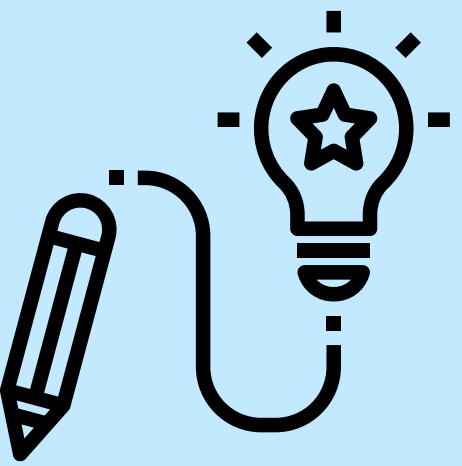




Question

General propose concrete using a 1:2:3 ratio of cement to sand to gravel. If we have 150 kilograms of sand available, how many kilograms of concrete can we make. (assume we have more than enough cement and gravel)





Real-life ratio problem

Concrete mix ratio: cement : sand : gravel = 1 : 2 : 3

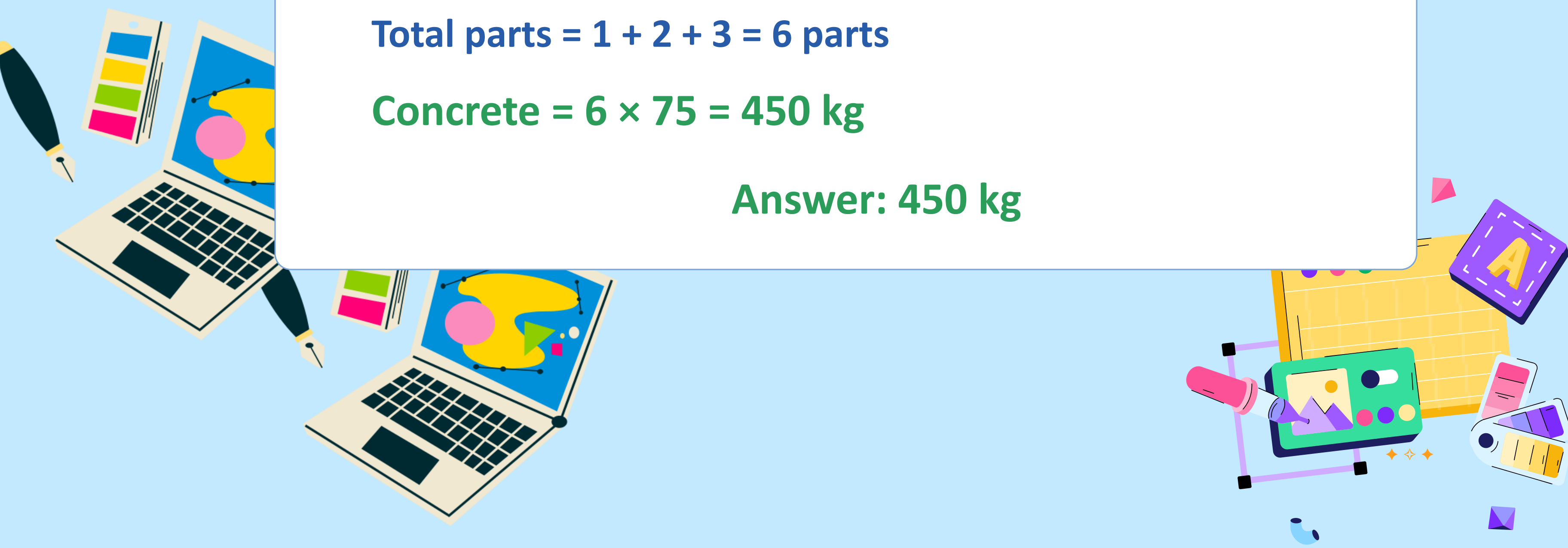
If sand = 150 kg, how many kg of concrete can we make?

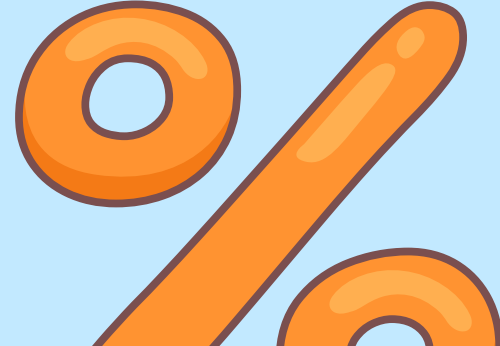
Sand is 2 parts = 150 kg → 1 part = 75 kg

Total parts = 1 + 2 + 3 = 6 parts

Concrete = 6 × 75 = 450 kg

Answer: 450 kg





8. Combining Ratios

What does “combining ratios” mean?

Sometimes one problem gives two separate ratios.

On a certain high school team, the ratio of sophomores to juniors is 2:3, and ratio of juniors to seniors is 5:6, Sophomores are what fraction of the whole team?

Strategy: identify the common element. Find equivalents of both ratios so that the common element is equal.

Main idea

When two ratios share one group, make that common group equal.

Then write one combined ratio for all groups.

Example pattern

$$S : J = 2 : 3$$

$$J : Sr = 5 : 6$$

J is the common group

We must make J equal before combining.

Example 1: Find a fraction of the whole team

Sophomores to juniors = 2:3 and juniors to seniors = 5:6

Question: Sophomores are what fraction of the whole team?

Step 1: Identify the common group

$$\mathbf{S : J = 2 : 3}$$

$$\mathbf{J : Sr = 5 : 6}$$

Common group = J

Step 2: Make J equal

$$\text{LCM of 3 and 5} = 15$$

$$\mathbf{S : J = 10 : 15}$$

$$\mathbf{J : Sr = 15 : 18}$$

Step 3: Combine and answer

$$\mathbf{S : J : Sr = 10 : 15 : 18}$$

$$\text{Whole team} = 10 + 15 + 18 = 43 \text{ parts}$$

Sophomores = 10/43 of the whole team

Example 2: Company staff

In a company, directors to marketers is 3:8, and customer service reps to marketers is 2:3. If there are 27 directors, how many CSRs are there?

$$D : M = 3 : 8$$

$$CSR : M = 2 : 3$$

Common group = Marketers
(M)

Make M equal

$$\text{LCM of 8 and 3} = 24$$

$$D : M = 9 : 24$$
$$CSR : M = 16 : 24$$

Combine

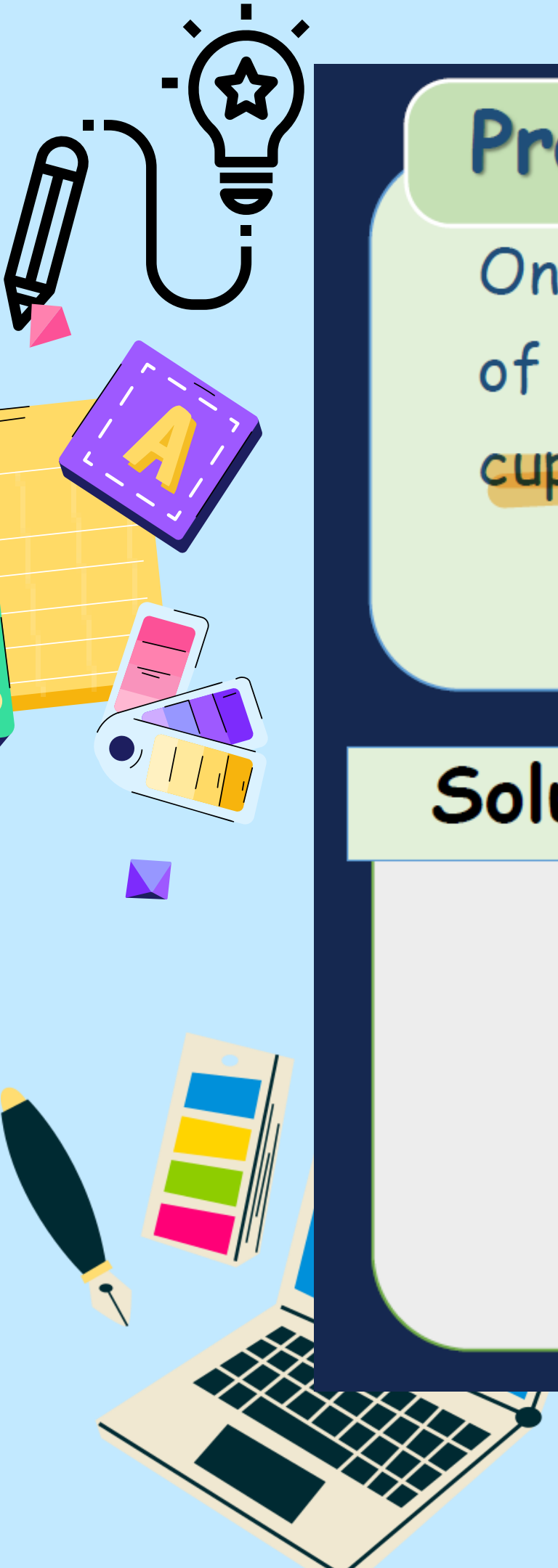
$$D : M : CSR = 9 : 24 : 16$$

$$9 \text{ parts} = 27 \text{ directors}$$

$$1 \text{ part} = 3$$

$$CSR = 16 \text{ parts} = 48$$

**Answer: 48
CSRs**



Problem

One cup of butter is enough for 12 of Kathy's cookies, and cup of sugar is enough for 8 of her cookies. If she used five more cups of sugar than of butter, how many cookies did she make?

Solution

Let B = cups of butter and S = cups of sugar.

$$B = 12, S = 8$$

$$B : S = 12 : 8$$

$$B : S = 3 : 2$$

Answer: 120 cookies

Use the difference

$$S - B = 5 \text{ cups}$$

$$3n - 2n = 5$$

$$n = 5$$

Butter = 10 cups

$$\text{Cookies} = 10 \times 12 = 120$$

Practice problems

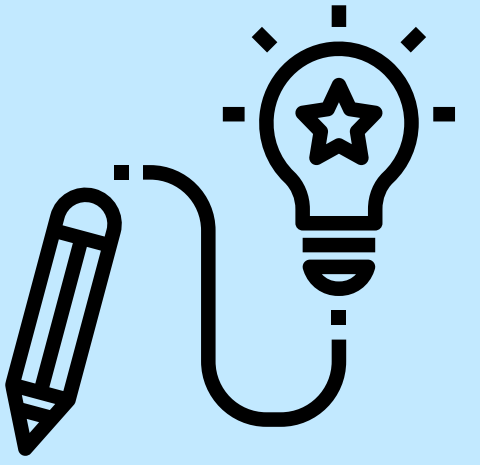
Students solve first, then check with a partner.

1 $A : B = 4 : 5$ and $B : C = 3 : 2$. What is $A : B : C$?

2 Girls : Boys = 7 : 5. Boys : Teachers = 10 : 1. If there are 70 girls, how many teachers are there?

3 Red : Blue = 2 : 9 and Green : Blue = 4 : 3. What fraction of the whole is Green?

Answer key in speaker notes



9. Ratios and Rates

Use proportions to solve rate problems

What is a Rate?

A rate is a ratio with different units.

same idea as a ratio

different units

means “per”

Examples

:

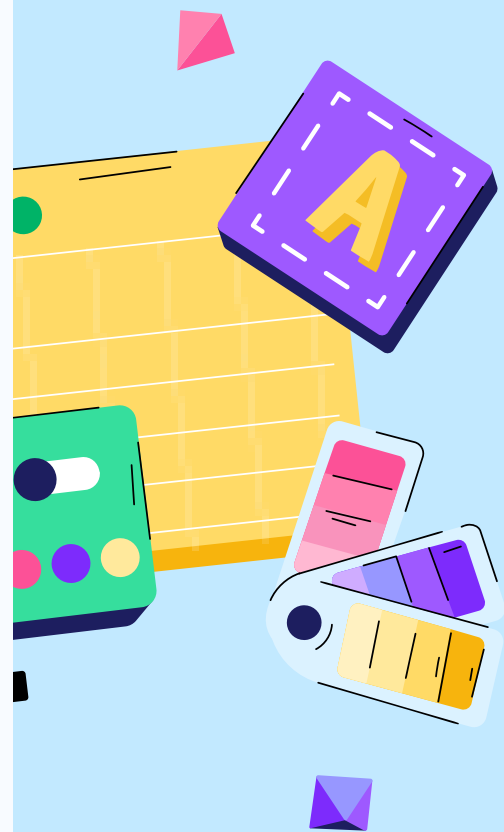
60 minutes / 1 hour

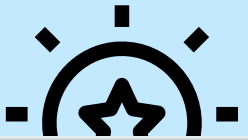
360° / 1 revolution

8 grams / 1 hour

1440 flaps / 8 seconds

Key idea: when solving, the same units must be in the same positions.





Problem

A block of ice is melting at a rate of 8 grams per hour.
At noon there are 30 grams of ice. When will it be entirely melted?

Solution

$$\frac{8 \text{ grams}}{1 \text{ hour}}$$

=

$$\frac{30 \text{ grams}}{x \text{ hours}}$$

$$\begin{aligned} 8x &= 30 \\ x &= 30 \div 8 \\ x &= 3.75 \\ &\text{hours} \end{aligned}$$

3.75 hours = 3 hours
45 minutes
Noon + 3 h 45 min =
3:45 p.m.

Answer: The block will be entirely melted at 3:45 p.m.

ed?
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$x = \frac{15}{4} = 3.75$

The block first will be entirely
melted at 17.15



Problem

A bumblebee's wings flap 1440 times in 8 seconds.
How many times does it flap in 1 minute?

Solution

1 minute = 60 seconds

$$\frac{1440 \text{ times}}{8 \text{ seconds}} = \frac{x \text{ times}}{60 \text{ seconds}} \quad \begin{aligned} x &= 1440 \times 60 \div 8 \\ x &= 10,800 \end{aligned}$$

Answer: The wings flap 10,800 times in one minute.

Wrap-Up & Reflection

Wrap-Up

- A ratio compares two quantities.
- A rate is a ratio with different units.
- We solve rate problems by setting up proportions.
- Always match the units carefully.

Reflection

- What is the difference between a ratio and a rate?
- What step helped you most today?
- Which problem was easiest for you?
- What do you still want to practice?



Exit Ticket: Write one example of a ratio and one example of a rate.

