History of Mathematics

Chapter4

The period from 300 B.C to 500 A.D.

Topics

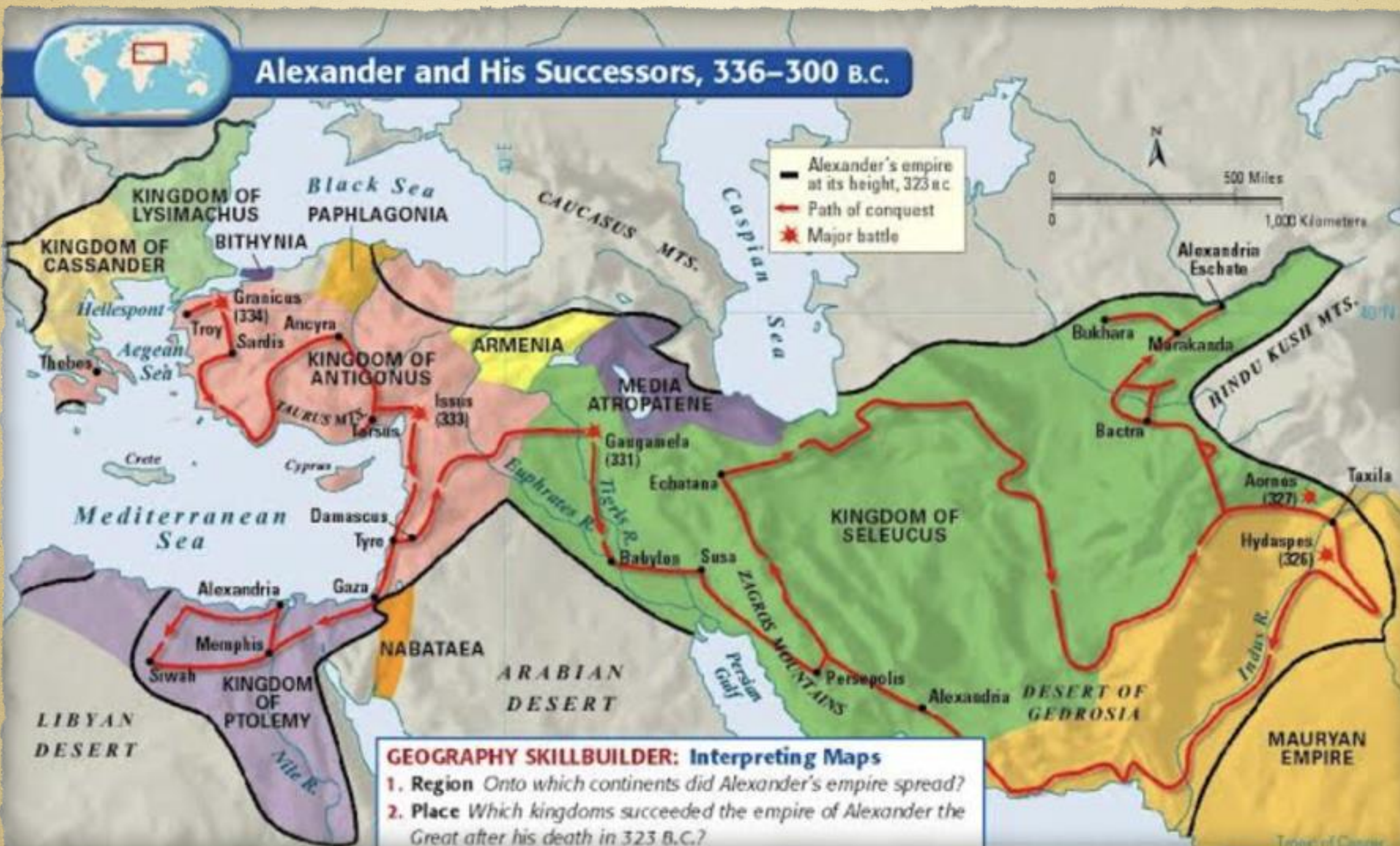
- Euclid
- Eratosthenes and Archimedes
- Apollonius
- Period of Menelaus
- Ptolemy
- Diophantus
- The Orient

1. Euclid

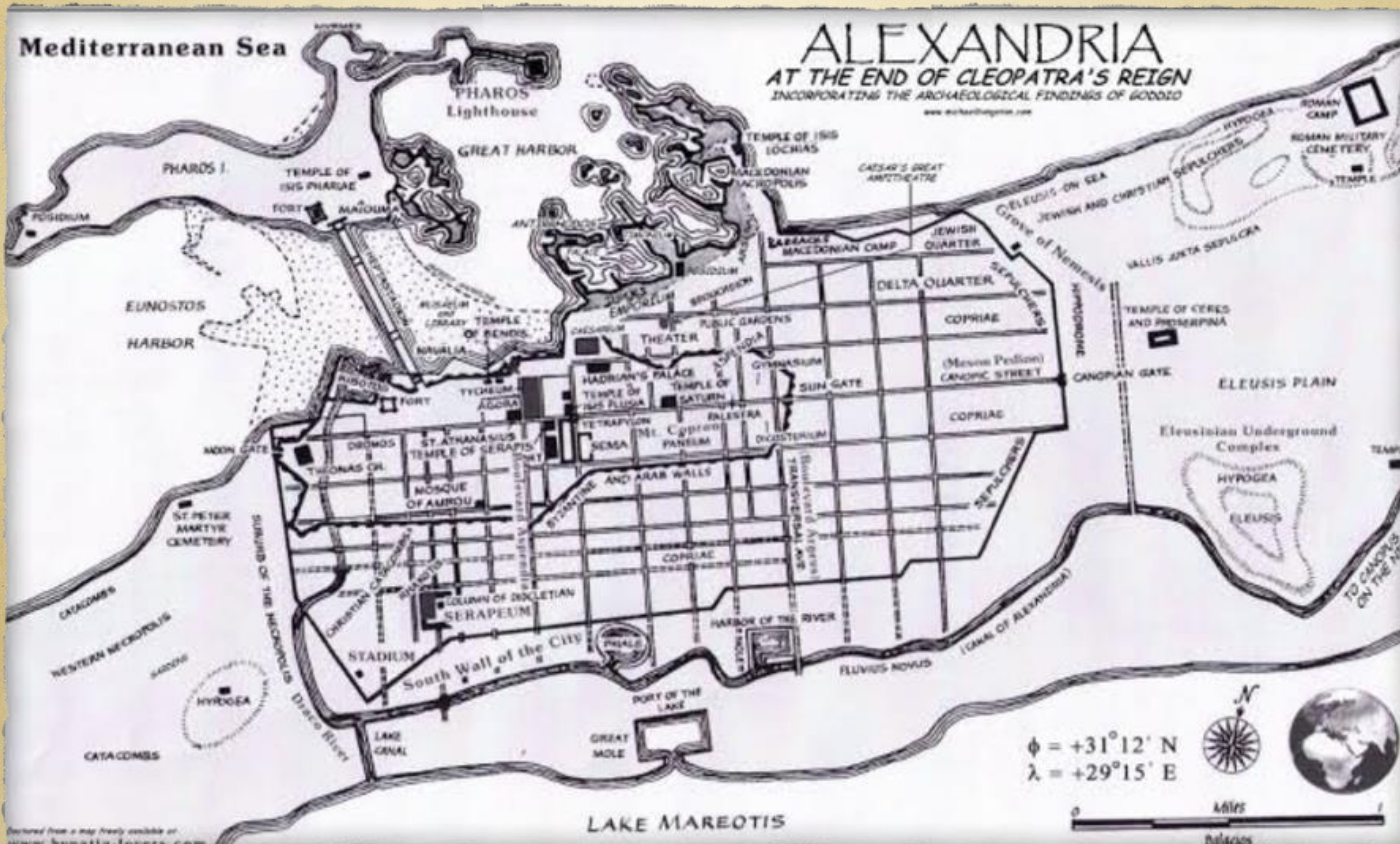
School of Alexandria

The greatest mathematical center of ancient times

Alexandria's Empire

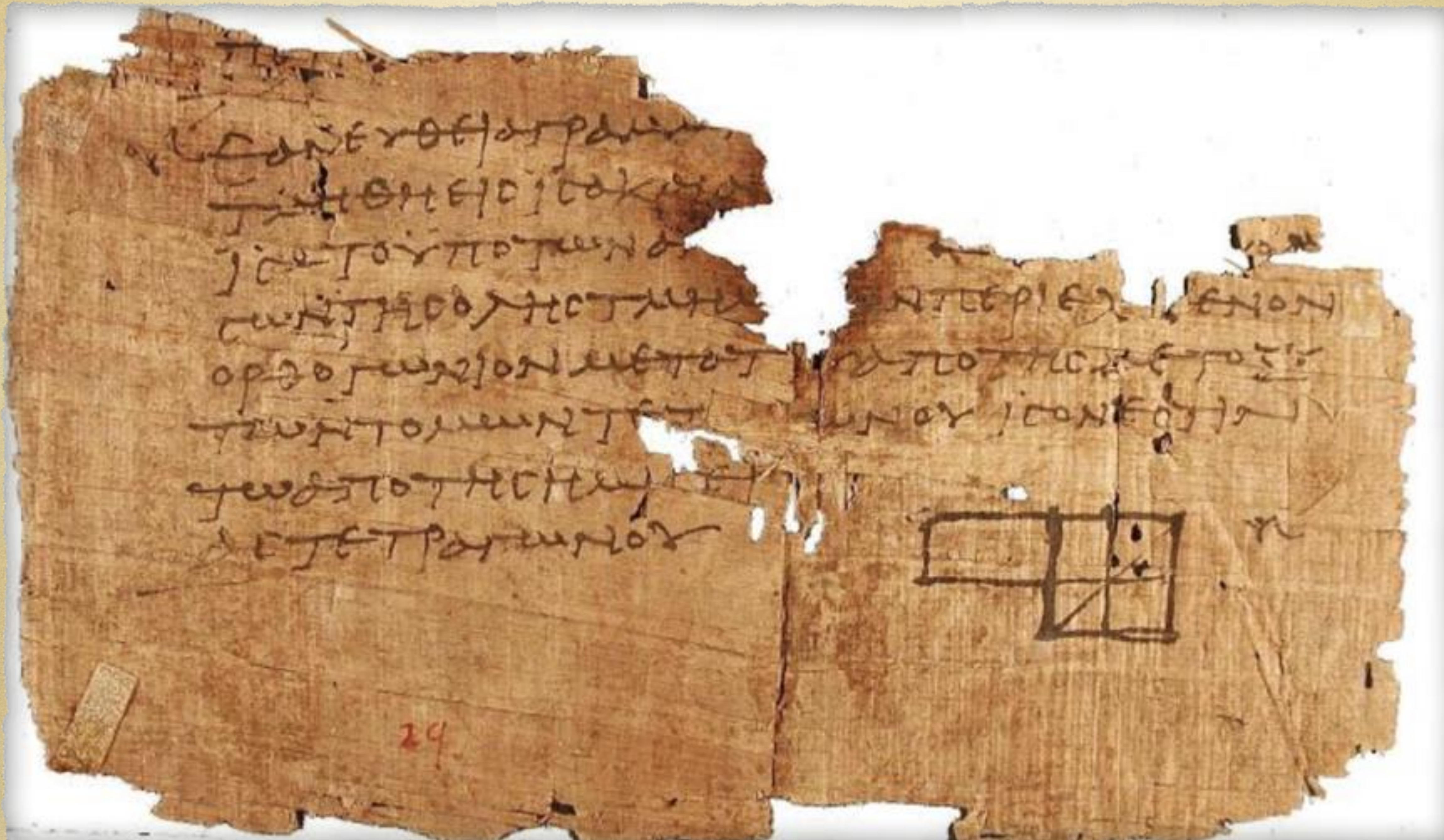


Alexandria


ALEXANDRIA
AT THE END OF CLEOPATRA'S REIGN
INCORPORATING THE ARCHAEOLOGICAL FINDINGS OF GODDIO
Mediterranean Sea
PHAROS
Lighthouse
GREAT HARBOR
PHAROS I.
TEMPLE OF ISIS PHARIAE
FORT
TEMPLE OF ISIS LOCHIAS
MACEDONIAN ACROPOLIS
CAESAR'S GREAT AMPHITHEATRE
ROMAN CAMP
ROMAN MILITARY CEMETERY
TEMPLE
HYPOGEA
ELEUSIS ON SEA
JEWISH AND CHRISTIAN SEPULCHERS
JEWISH QUARTER
MACEDONIAN CAMP
Grove of Nemesis
VALLIS JUXTA SEPULCRA
DELTA QUARTER
SEPULCHERS
EUNOSTOS
HARBOR
TEMPLE OF BENDIS
EMPORIUM
CAESARIUM
THEATER
PUBLIC GARDENS
COPRIAE
HIPPODROME
TEMPLE OF CERES AND PROSERPINA
CANOPIC STREET
CANOPIAN GATE
GYMNASIUM
SUN GATE
HADRIAN'S PALACE
TEMPLE OF SATURN
AGORA
FORT
ELEUSIS PLAIN
PALESTRA
TETRAPYLON
MOON GATE
DROMOS
ST ATHANASIUS
TEMPLE OF SERAPIS
SEMA
PANEUM
Mt. Copron
DICASTERIUM
Eleusinian Underground Complex
HYPOGEA
ELEUSIS
BYZANTINE AND ARAB WALLS
MOSQUE OF AMROU
SEPULCHERS
ST. PETER MARTYR CEMETERY
SUBURB OF THE NECROPOLIS
TRANSVERSAL AVE
COPRIAE
COLUMN OF DIOCLETIAN
SERAPEUM
CATACOMBS
WESTERN NECROPOLIS
HARBOR OF THE RIVER
TO CANOPUS ON THE NILE
CANAL OF ALEXANDRIA
STADIUM
South Wall of the City
FLUVIUS NOVUS
HYPOGEA
PORT OF THE LAKE
Draco River
LAKE CANAL
GREAT MOLE
CATACOMBS
N
ϕ = +31°12' N
λ = +29°15' E
LAKE MAREOTIS
Miles
www.hypatia-lovers.com

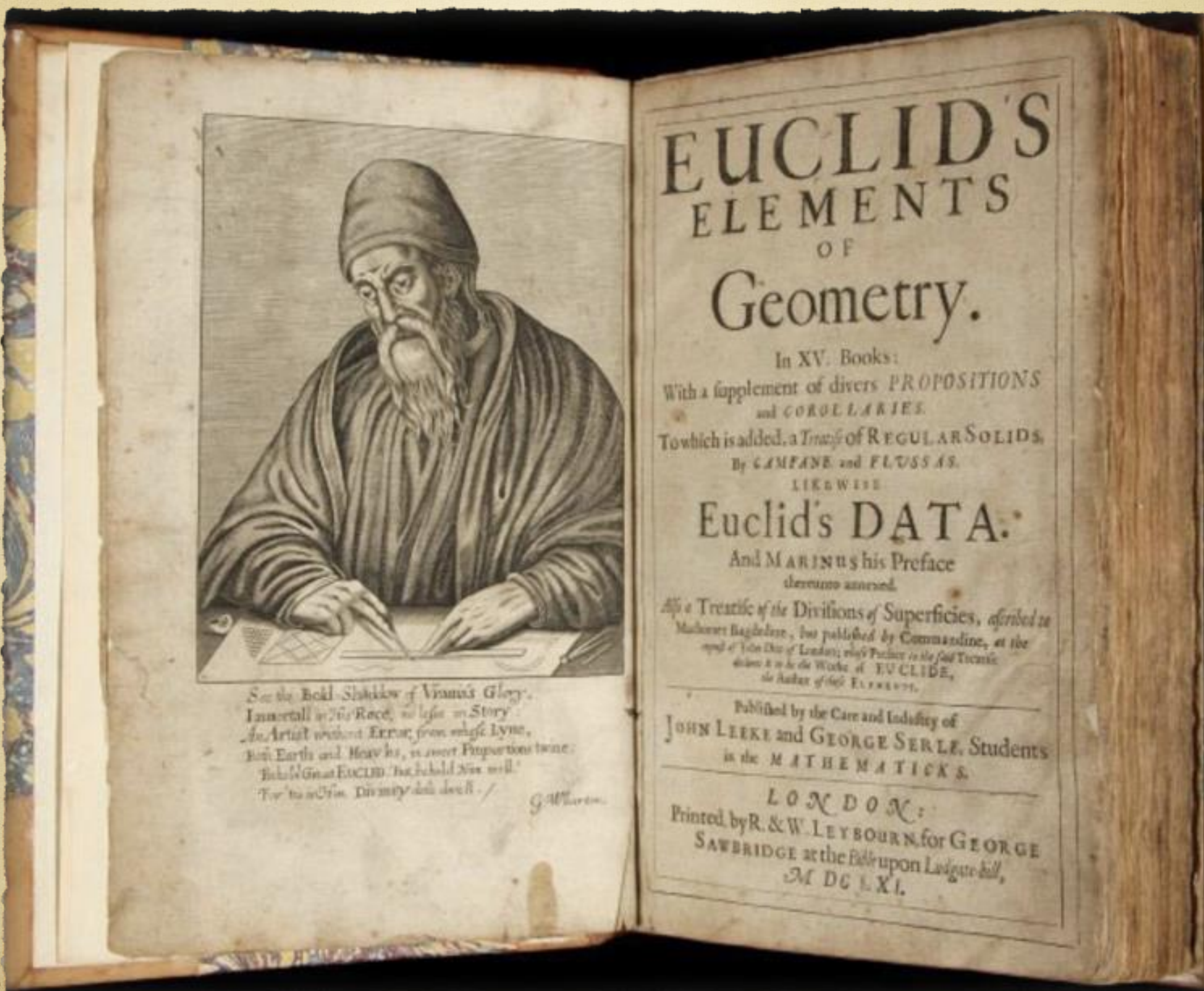
Alexander

The Element



The first English transition of Euclid

Euclid's Elements



Contents of Elements

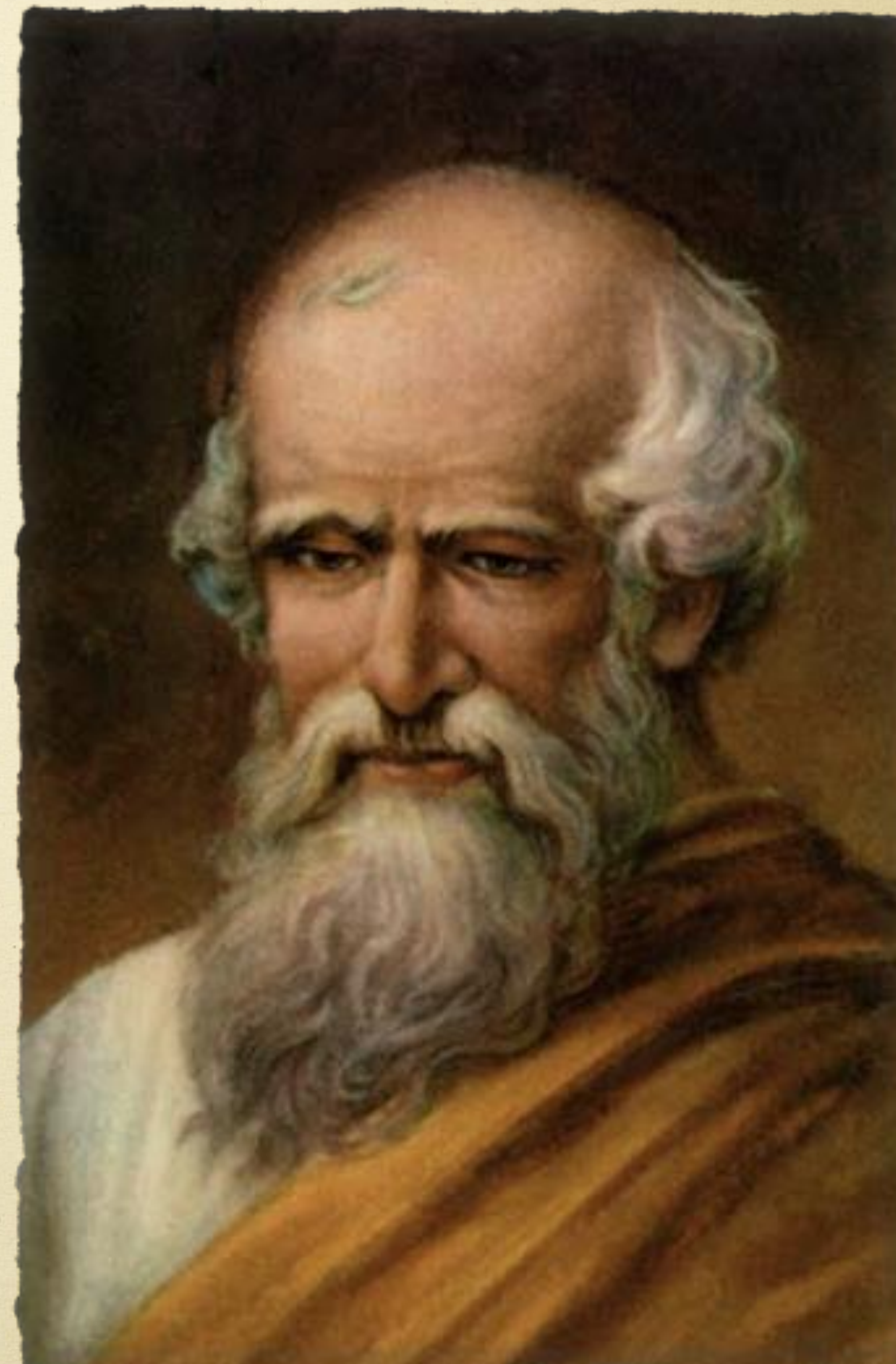
- I Congruence, parallel, the Pythagoras Theorem
- II Identities of algebra
- III Circles
- IV Inscribed and circumscribed polygons
- V Proportion treated geometrically; in part, a geometric way of solving fractional algebraic equation
- VI Similarity of polygons
- VII-IX Arithmetic (the ancient theory of numbers) treated geometrically
- X Incommensurables magnitudes
- XI-XIII Solid geometry

Minor Writers

Minor Writers. There were, for example, at first such minor writers as Co'non[4] of Samos, who, influenced by his observations of the coiled basketry work of the Egyptians, may have invented the spiral of which Archimedes developed the properties. He is also mentioned by Apollonius (*c.* 225 B.C.) as having studied the number of points of intersection of two conics. There was also Nicot'eles[5] of Cyrene, possibly a student in Alexandria, of whom Apollonius speaks as his predecessor in the study of conics. Still another writer of influence appeared in the person of the astronomer Aristar'chus,[6] a native of Samos but a teacher at Alexandria. It was he who first showed how to find, by means of the Pythagorean triangle, the relative distances of the sun and the moon from the earth, and for nearly two thousand years no better plan was known. His instruments of observation were such as to make his result far from being even approximately correct.[7] His greatest glory, however, lies in the fact that he was the first to place the sun in the center of the universe, asserting that the earth and the other planets revolved about it, thus anticipating Copernicus by seventeen centuries. In the field of arithmetic he found $\sqrt{2}$, possibly by a method analogous to that of continued fractions.[1]

There are also extant various papyri of the Ptolemaic period containing information about the financial problems of Egypt. These problems relate chiefly to taxes and the cost of various commodities, but they add nothing to our information as to methods of calculation in ancient times.[2]

2. Eratosthenes and Archimedes



Eratosthenes

Lived some year after Euclid

The second Plato

Beta-nickname

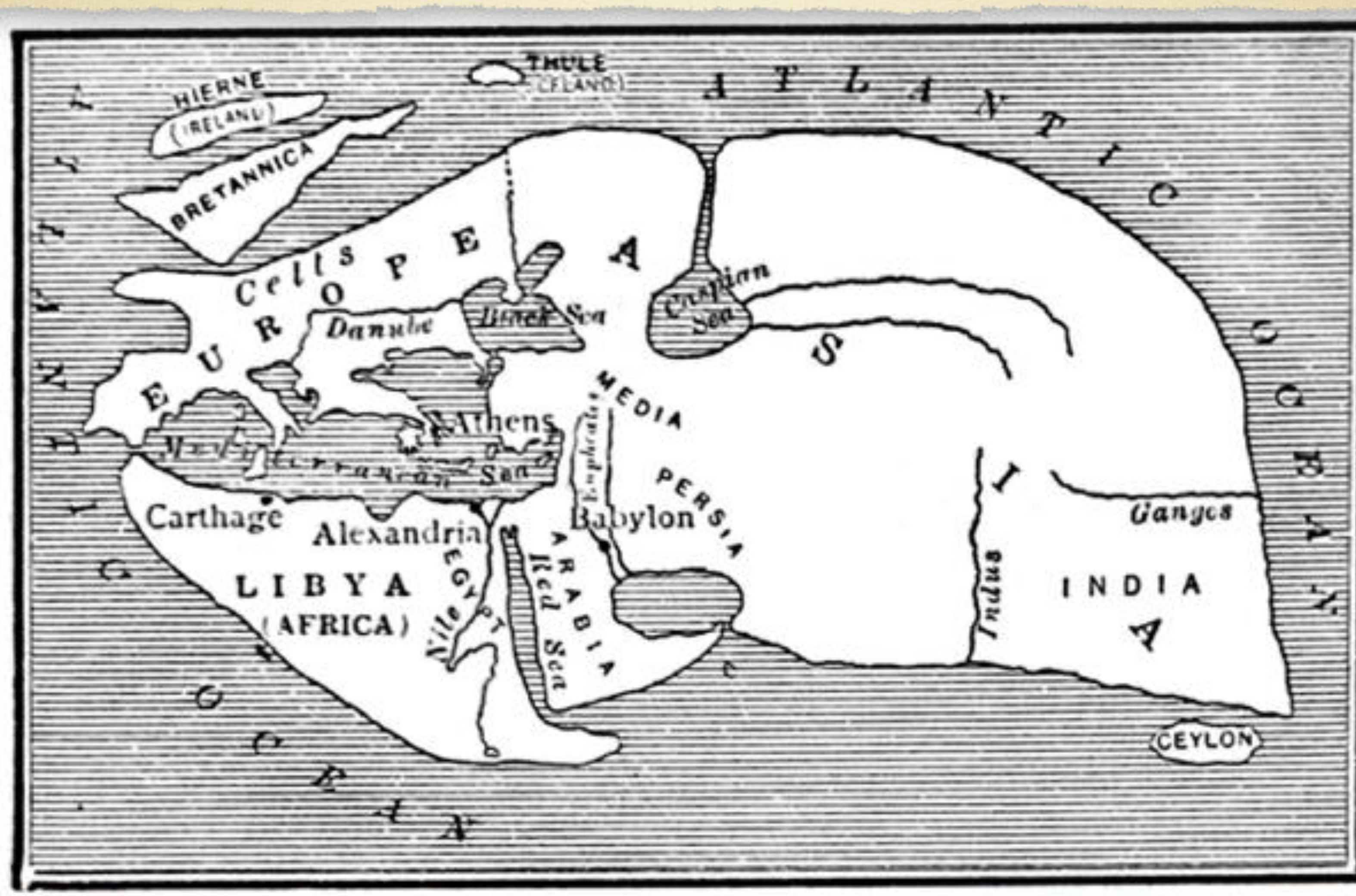
The first prominent geographer of antiquity

Prime numbers

and is known to have taught at Alexandria after *c.* 240 B.C., to have been librarian of the university, and to have been a poet of some merit. His contribution to arithmetic was his sieve,[1] a method of sifting out the composite numbers in the natural series, leaving only primes. This he did by writing all the odd numbers and then canceling the successive multiples of each, one after the other, thus: 3, 5, 7, ~~9~~, 11, 13, ~~15~~, 17, 19, ~~21~~, 23, ~~25~~, ~~27~~, 29, 31, ~~33~~, ~~35~~, 37, ~~39~~, . . .[2] Prime numbers

have been studied from that time until the present, but no general formula is yet known for detecting all of them. For example, we do not yet know whether there are an infinite number of primes of the form $x^2 + 1$, whether $2 = x - y$ has an infinite number of prime solutions, or whether a prime number can always be found between n^2 and $(n + 1)^2$.

Map of the world

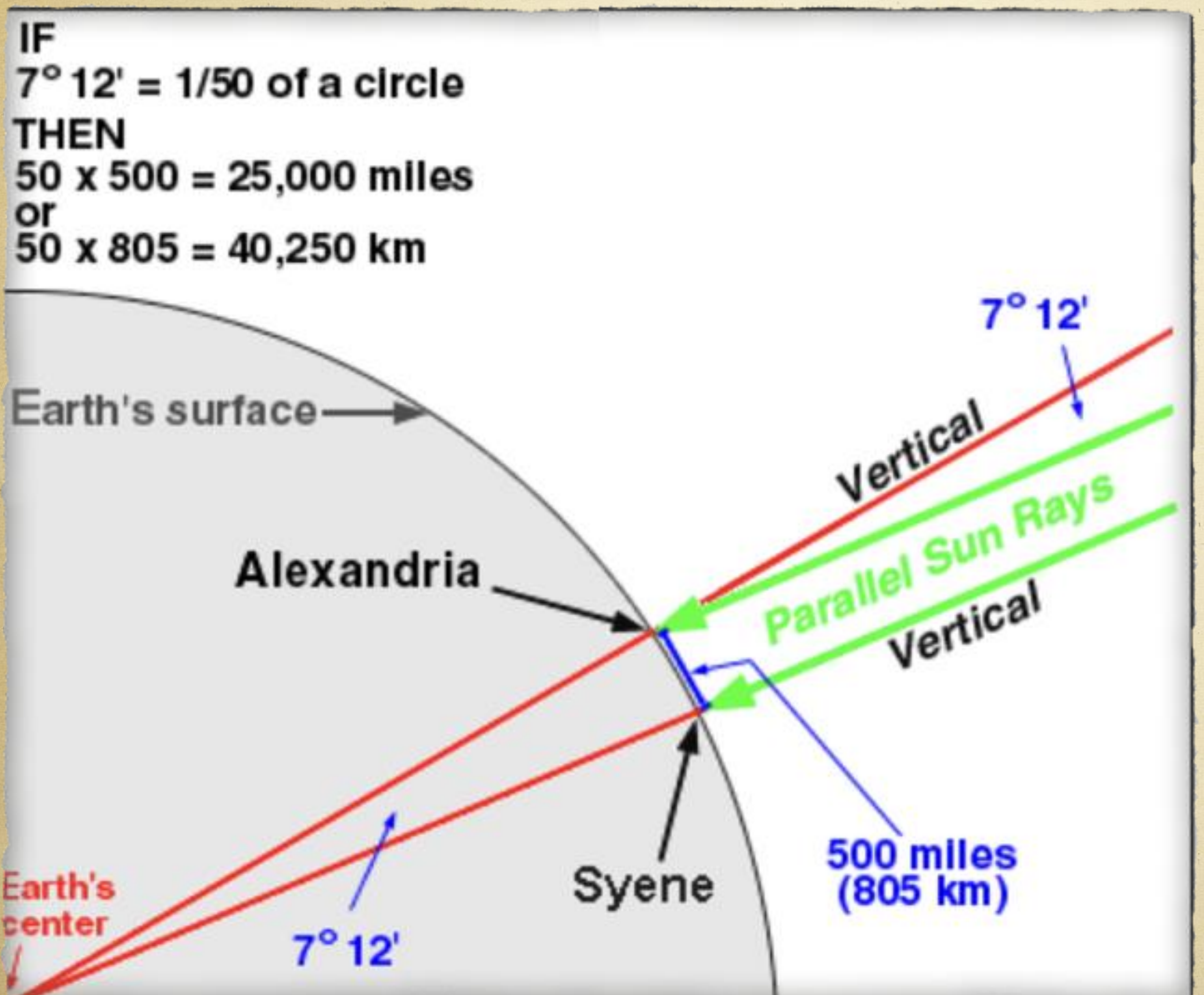


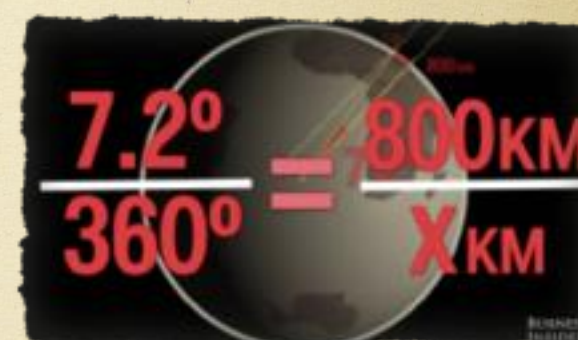
MAP OF THE WORLD ACCORDING TO ERATOSTHENES

This shows the knowledge of the geography of the world in the 3d century B.C. and should be compared with the map of Hecatæus (517 B.C.) shown on page 77. From Breasted's *Ancient Times*

Earth Measure

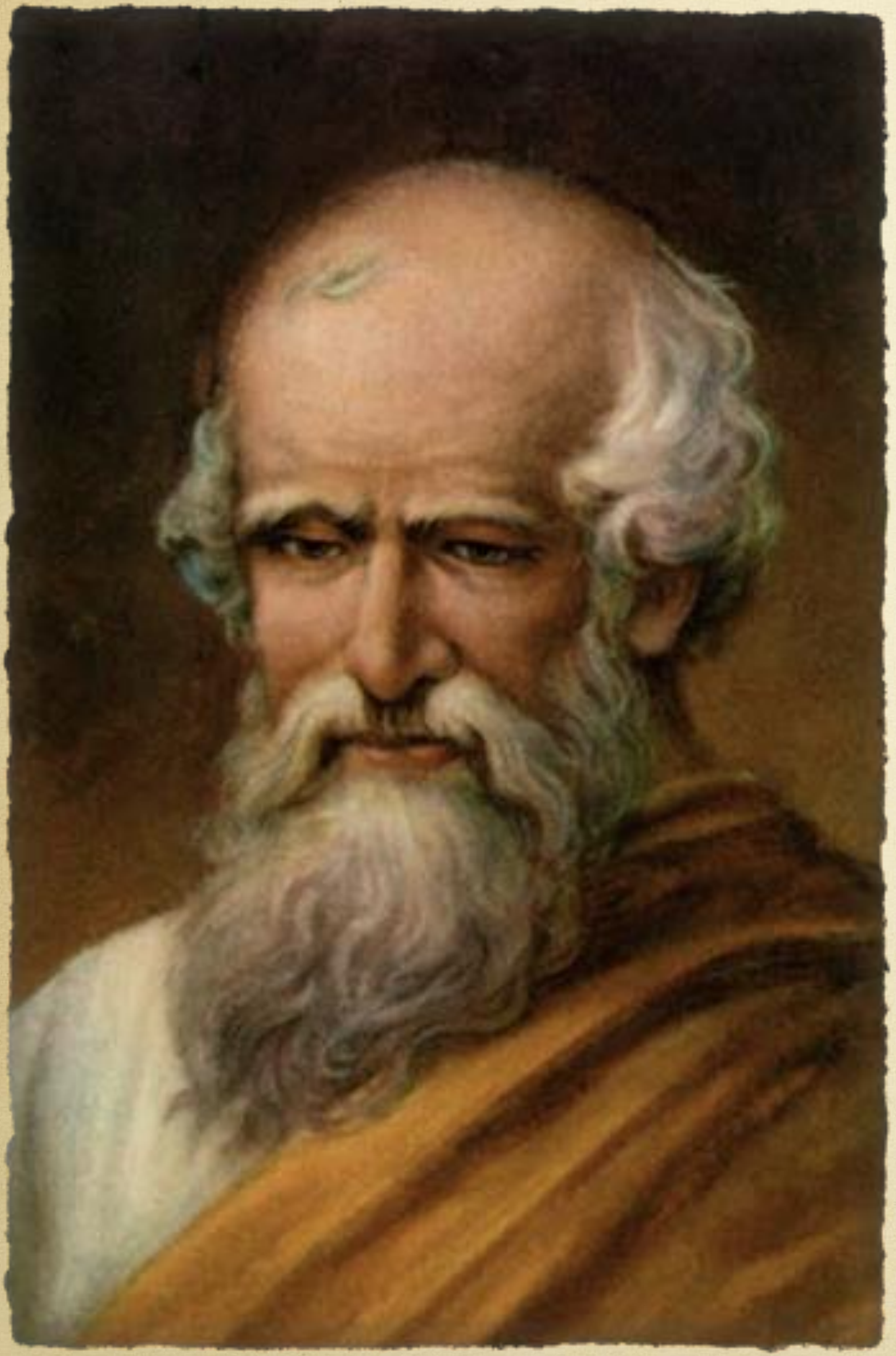
Circumference of Earth





Archimedes

Life of Marcellus

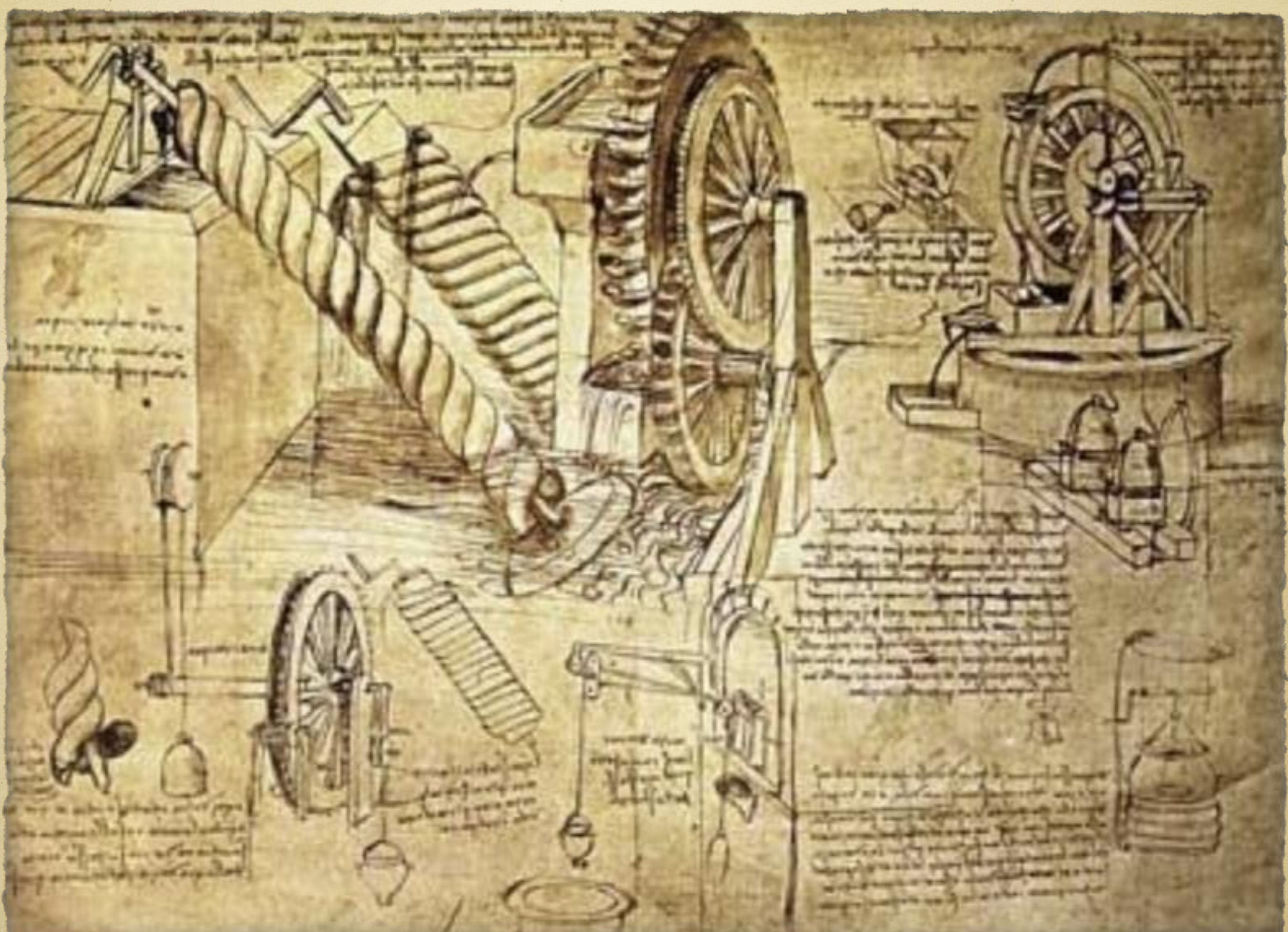


A friend of Eratosthenes

A genius more divine than human

The god of mathematics-Pliny

Archimedes and Mechanic



recognized, in substance, that $a^m a^n = a^{m+n}$, a law that is the basis of our present operations by logarithms.

Other Mathematical Activities. Among his many activities was the summation $\sum_1^n n^2$, the first example of the systematic treatment of higher series of any kind. By the intersection of

[1] Ψαμμίτης (*psammites*); Latin, *arenarius* or *harenarius*.

conics he was able to solve cubic equations which we should now write in the form $x^3 \mp ax^2 \pm b^2c = 0$. He also succeeded in squaring a parabola,[1] that is, in finding the area of a segment, showing that it is two thirds of a circumscribed parallelogram. In the measure of a circle he showed that $3\frac{1}{7} > \pi > 3\frac{10}{71}$. In his work in mensuration Archimedes included the sphere, cylinder, and cone, the rules concerning the two latter having already been known to Menæchmus. He also studied ellipsoids and paraboloids of rotation. In his treatise on the mensuration of circles and round bodies he was aided by the method of exhaustion which had been developed by Menæchmus and others. In the study of specific gravity and the center of

3. Apollonius

The great geometer

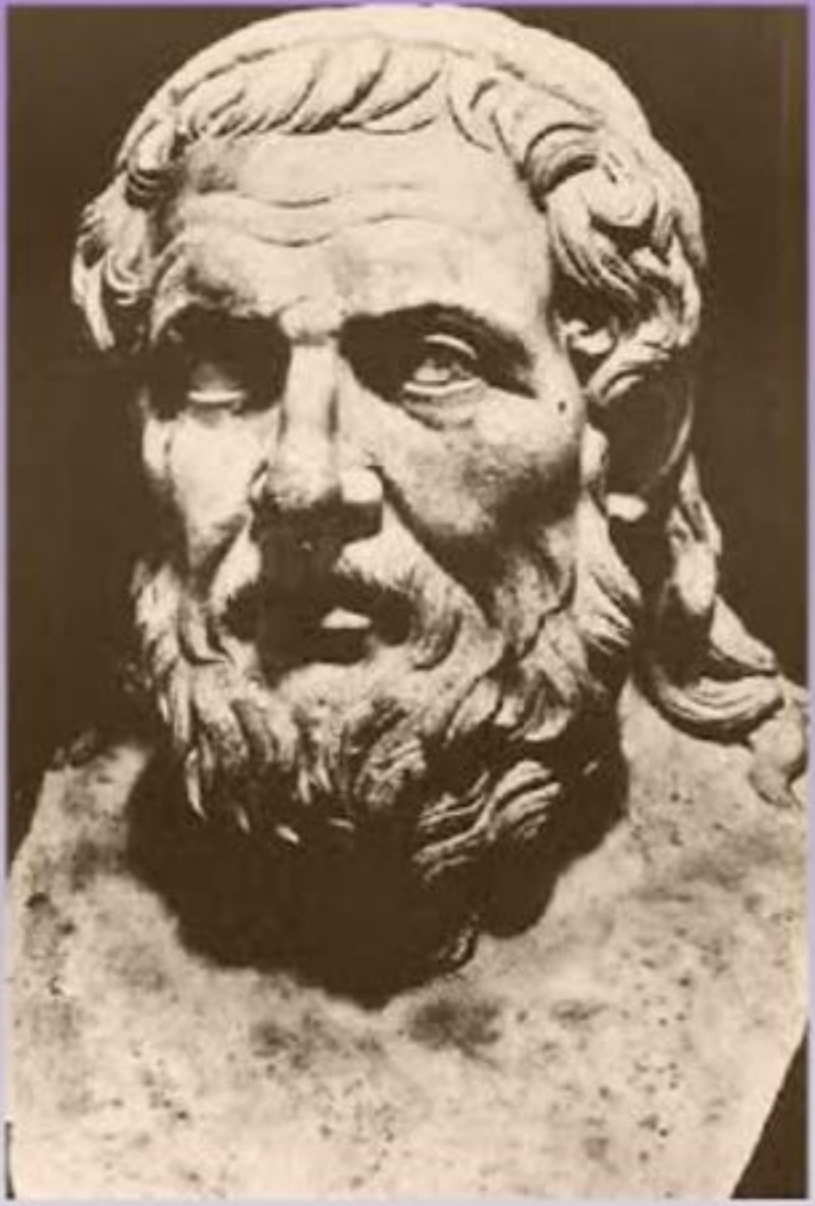
Apollonius of Perga (262-190 BC)

The Great Geometer

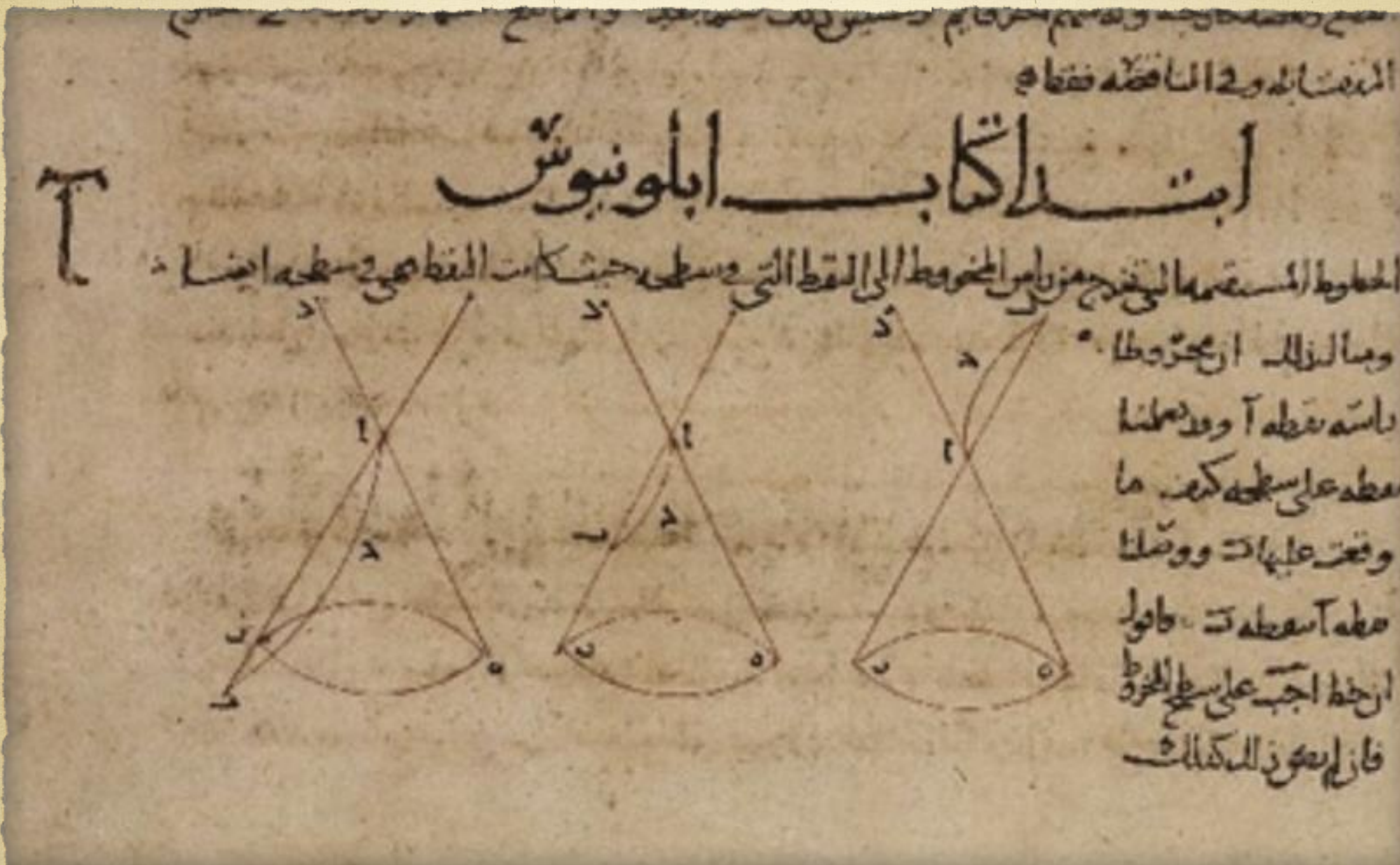
Famous work was *Conics* consisting of 8 Books

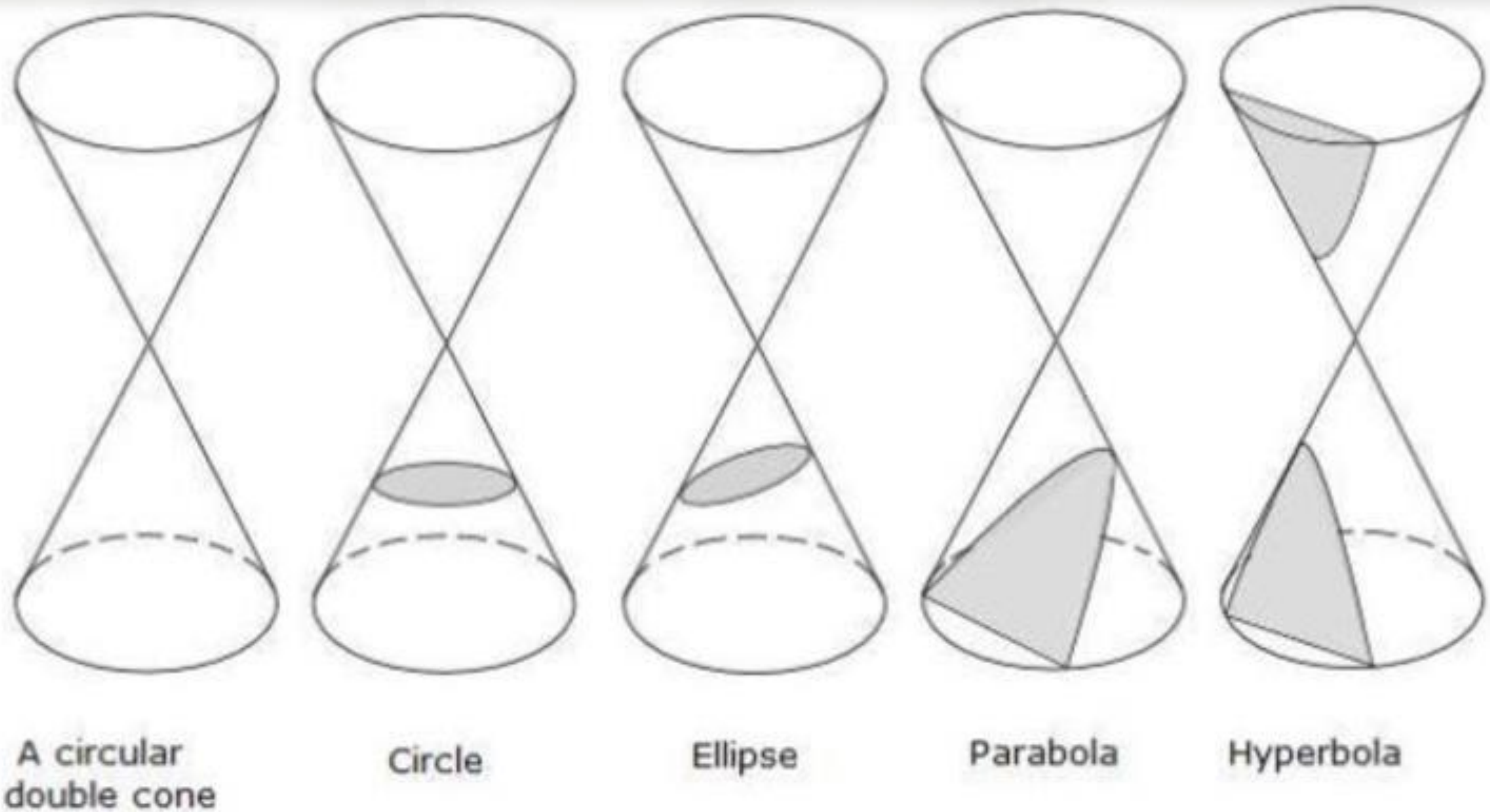
Tangencies - showed how to construct the circle which is tangent to three objects

Computed an approximation for π better than Archimedes.



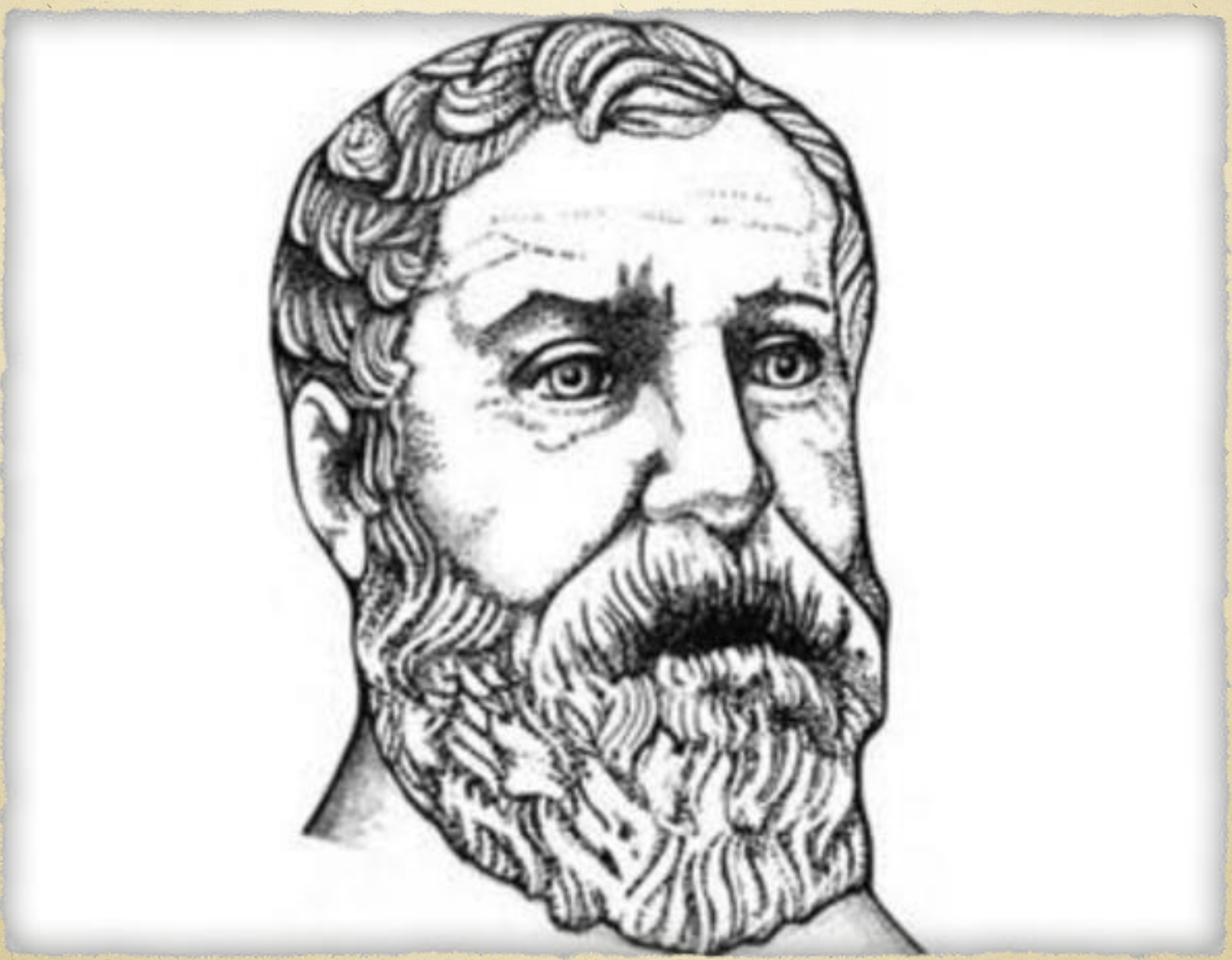
Conic section




A circular double cone
Circle
Ellipse
Parabola
Hyperbola

4. Period of Menelaus

Heron of Alexandria



methods in use by the Egyptians. As is the case with many of the Greek scholars, some of his works are lost. His formula for the area of a triangle, $A = \sqrt{s(s-a)(s-b)(s-c)}$, is well known. It appears in the geodesy,[1] which is contained in his metrics,[2] but the proof is given (possibly an interpolation) in his dioptrics.[3] In his geometry may be found the first definite use of the trigonometric rule which we express by the formula $c = \frac{n}{4} \cdot \cot \frac{180°}{n}$, where n is the number of sides of a regular polygon of area A and side s, and where $c = A/s^2$. He computed c for $n = 3, 4, \cdots, 12$, but his method is unknown. He was able to solve the equation which we write in the form $ax^2 + bx = c$, so that the general quadratic as we know it today was thus fully mastered by the Greek mathematicians.

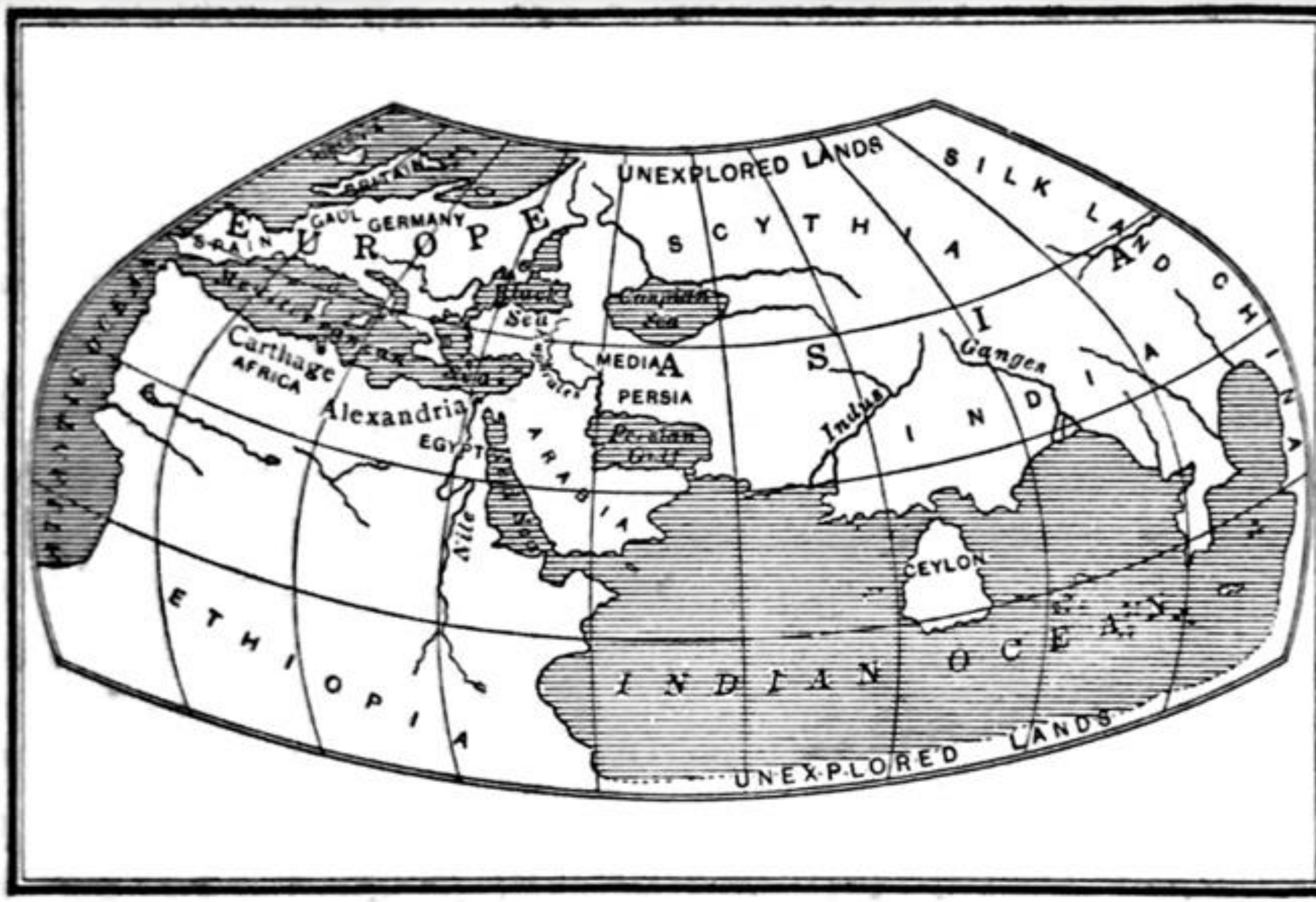
About this time there lived Sere′nus of Antinoop′olis.[4] He was the author of a treatise on the *Section of the Cylinder*, containing thirty-three propositions, and of one on the *Section of the Cone*, with sixty-nine propositions. The latter has considerable work on maxima and minima. He also employed the principle of a harmonic pencil of rays.

5. Ptolemy

Ptolemy

Astronomy what Euclid did for plane geometry, Apollonius for conic, and Nicomachus for arithmetic

Ptolemy's map of the world



PTOLEMY'S MAP OF THE WORLD

This shows the great growth in the knowledge of geography from the time of Eratosthenes. See page 109. From Breasted's *Ancient Times*

6. Diophantus

Works of Diophantus

Works of Diophantus. Diophantus wrote three works: (1) *Arithmetica*, originally in thirteen books, of which six are extant[1]; (2) a tract *De polygonis numeris*[2] of which a portion is extant; (3) a number of propositions under the title of *porisms*. Of these, the work of greatest importance is the *Arithmetica*. This work relates, as the title indicates, to the theory of numbers as distinct from computation, and covers much that is now included in algebra. The equations of the first degree are determinate and are so framed as to give positive values for the unknowns. In solving determinate quadratic equations Diophantus used only one root, even where both are positive. He solved a single special case of a cubic equation, but it is thought that further work on such equations may have been given in the lost books. His indeterminate quadratic equations are generally of the types $Ax^2 + C = y^2$ and $Bx + C = y^2$. His simultaneous quadratics relate only to special cases.[3]

6. The Orient

China

China. The period from 300 B.C. to 500 A.D. was one of mathematical activity in China, and some slight but noteworthy trace remains of an interest in numbers in Japan.[5] At the beginning of this period the event of greatest concern in the history of Chinese mathematics was the burning of all books[6] (213 B.C.), as already mentioned in Chapter II, by order of the emperor Shï Huang-ti,[7] founder of the Ch'in (Ts'in)

Among the mathematicians of this period whose names have come down to us is P'i Yen-tsung (*c.* 400–*c.* 450), who is said to have computed a noteworthy value of π which has since been lost. There is also Tsu Ch'ung-chih (430–501), an expert in mechanics, who revived the knowledge of the "south-pointing vehicle" and constructed a motor boat, all details of which are lost. He gave $\frac{22}{7}$ as an "inaccurate value" of π, and $\frac{355}{113}$ as the "accurate value," and he also showed that π lies between our present decimal forms 3.1415926 and 3.1415927. About the year 450 a new calendar was devised by Ho' Ch'êng-t'ien,

Japan

Japan in Earliest Times. Prior to the year 500 Japan seems to have made no progress either in literature or in science. There is a tradition that Chinese ideograms made their way through Korea and into Japan in the year 284. There is also reference to the *Jindai monji*, or "letters of the era of the gods," in early times, possibly a kind of system of cabala with numerical values assigned to the letters, but nothing is definitely known upon the subject. A tradition also exists that in 660 B.C. the Japanese had a system of numeration extending to very high powers of ten. In this system the special name *yorozu* was used for 10,000, corresponding to the Greek myriad already mentioned, and this may possibly be some slight evidence of the early interrelations between the East and the West.[1]

Of the rest of Japanese mathematics in the early periods we know only that there was a system of measures and that, as among all other ancient peoples of any intellectual standing, a calendar existed.

India

India. The noteworthy contribution of India in this period was probably the Hindu numeral system, which will be discussed later.[2] A second event of importance in the history of mathematics in India, and one which chronologically precedes the writing of the numerals, was the invasion of this country by the army of Alexander the Great (327 B.C.) and the sending of Greek ambassadors to reside in Indian courts. How much

The first important work on astronomy produced in India, so far as now known, was the *Sūrya Siddhānta*,[1] probably written about the beginning of the 5th century, although known to us only in later manuscripts. The ritualistic mathematical formulas of the *Śulvasūtras* now gave place to the mathematics of the stars. This change was possibly due to the influence of Greek scholars whose works might still have been appreciated by the descendants of the ancient Greeks who settled in India after Alexander's time. Varāhamihira, who will be mentioned later, speaks of five *Siddhāntas*, but places the *Sūrya Siddhānta* at the head. Among the five is the *Paulisa Siddhānta*, probably of about the same period. This contains an excellent summary of early Hindu trigonometry, the rules, expressed in modern symbolism, being as follows:

$$\sin 30° = \tfrac{1}{2}, \qquad \pi = \sqrt{10},$$

$$\sin 60° = \sqrt{1 - \tfrac{1}{4}}, \qquad \sin^2\phi = \left(\frac{\sin 2\phi}{2}\right)^2 + \left(\frac{1 - \sin(90° - 2\phi)}{2}\right)^2.$$

Discussion 4

1. The School of Alexandria, its rise, its influence, the great scholars connected with it, and its decay.

2. Euclid, his life, his works, and his influence.

3. The work of Eratosthenes, particularly with respect to geodesy.

4. The life, the works, and the influence of Archimedes.

5. Apollonius and his contribution to the study of conics.

6. The mathematical contributions of the Greek astronomers.

7. Mathematics in the Roman civilization. Causes of the disregard for the science.

8. The life and works of Heron. His influence upon the development of applied mathematics as compared with that of Archimedes.

9. The work of Nicomachus compared with the works of Euclid and Apollonius.

10. The work of Claudius Ptolemæus, or Ptolemy.

11. The life and works of Diophantus.

12. The decay of Greek geometry, with a special consideration of the work of Menelaus, Hypatia, Proclus, and Pappus.

13. Causes and probable effects of the burning of the books in China in 213 B.C.

14. The period in which the *Nine Sections* was written and the general nature of this work.

15. The knotted cords of China and the general subject of knotted cords in the keeping of records and in religious ceremonial.

16. Efforts at opening communications between the East and West at this period, and the probable effect of these efforts on science in general and mathematics in particular.

17. The periods and nature of the *Arithmetic Classic in Five Books* and the *Sea Island Classic*.

18. Influx of Hindu learning into China in this period and the probable effect of this intercourse on the mathematics of both China and India.

19. The invasion of India by Alexander the Great and its effect upon the mathematics of the East.

20. The nature of the *Sūrya Siddhānta* and the bearing of this work upon the mathematics of India.

21. Causes of the decay of mathematics in Mesopotamia in the five centuries after the time of Alexander.