Chapter 5
Word Problems

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Outlines

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Outlines

1

Word Problems

Word Problems

- age questions
- motion questions
- work questions
- growth & decay questions

Typically, questions state the information in words and do not provide variables.

We have to 1) assign variables and 2) write equations

We often have to consider whether to use just one variable or more than one.

2

Assigning Variables

To solve a word problem algebraically, we need to **introduce variables**.

The very first thing I will say is this: be thoughtful in the choice of letter for your variable.

Don't pick plain old x.

Beginning of a problem:

Each week, Saran makes $500 more than Masuk, and Heyun makes $700 more than Masuk. If the sum of their weekly salaries is ...

If you pick x for the salary of one of the women, then you may correctly solve for it, but when you get that number for x, how will you know whether it answers the question?

Do

S = Saran
M = Masuk
H = Heyun

Don't

A = Saran
B = Masuk
C = Heyun

Other considerations in choosing variables:

1) It may be helpful to assign a variable for the smallest value, and then express everything in terms of that.

2) It may be helpful to assign a variable for the target value, what the question is asking.

3) If all the quantities in the problem relate to a single quantity, make this conceptually central quantity the variable.

Best

Practice problem

Sam's car was fined for parking when he gave Joe and Peter a ride, so they decided to help Sam pay the fine. Joe paid $3 more than 1/4 of the fine and Peter paid $3 less than 1/3 of the fine, leaving Sam $4 less than 1/2 the fine to complete the payment. How much did Sam pay?

3

Writing Equations

Translating from Words to Math

1) The words "is/are" means equal sign.
2) The word "of" means multiply.
3) The words "more than" or "greater than" mean addition.
4) The word "less than" means subtraction.

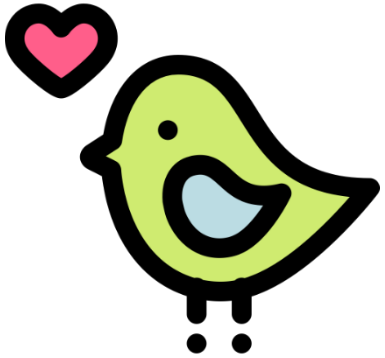
The words "is" or "are" in the text correspond to the equals sign.

If we have "A is 50 more than B":

$$A = B + 50$$

If we have "A is 50 less than B":

$$A = B - 50$$

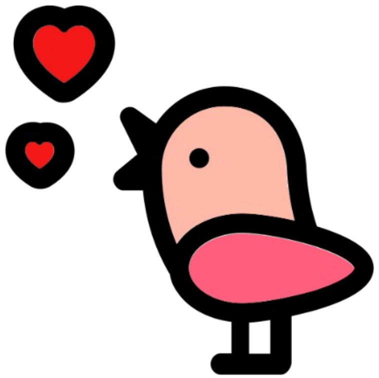


Suppose we have "Twice A is 100 less than three times B":

$$2A = 3B - 100$$

Suppose we have "A is 50% of B":

$$A = 0.5B \quad \rightarrow \quad 2A = B$$



Suppose we have "A is 50% greater than B":

Alice and Bruce each bought a refrigerator, and the sum of their purchases was $900. If twice of what Alice paid was $75 more than what Bruce paid, what did Alice pay for her refrigerator?

In a certain company, 50% of Ed's salary is 20% of Ruth's salary. If the difference between their salaries is $90,000, what is Ruth's salary?

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Number of Variables

A very simple word problem:

Frank has $13 more dollars than Glenda does, and together they have $81. How much does Frank have?

Approach #1: two variables

A very simple word problem:

Frank has $13 more dollars than Glenda does, and together they have $81. How much does Frank have?

Approach #2: one variable

A relatively simple word problem:

Mark is 8 years older than Lisa, and Peter's age is 3 years less than three times Lisa's age. If the sum of their ages is 80, what is Peter's age?

Practice problem:

Wendy left the house with D dollars in cash. She made the following cash purchases: gas, for $3 less than 1/3 of D; a book, for 1/6 of D; stationery, for six dollars more than 1/6 of D; and groceries, for 1/4 of D. After these four purchases, she had $4 left. What is the value of D?

5

Age Questions

One category of word problems concerns people of <u>different ages</u>.

What's tricky about these questions is that mathematical relationships can be specified at two different times.

Practice problem:

Right now, Steve's age is half of Tom's age. In eight years, twice Tom's age will be five more than three times Steve's age. How old is Tom right now?

The big mistake associated with this problem is thinking of the ages as fixed numbers.

The first equation is $S = (1/2)*T$, but the second equation is NOT $2T = 3s + 5$.

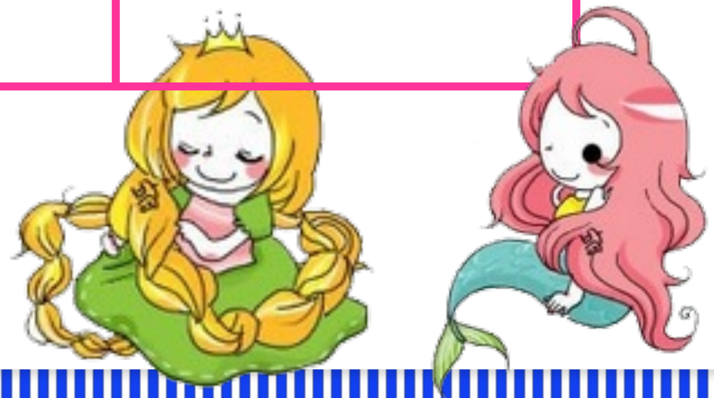
We will use these expressions to create equations that correspond to statements about age relationships in the future or past.

... and in seven years, Edgar will be twice as old as Frieda ...

$$(E + 7) = 2*(F + 7)$$

It is usually to pick the variable to represent the age right now.

	Age five years ago	Age right now	Age seven years later
Frieda	F - 5	F	F + 7
Jannie (Jannie is twice as old as Frieda)	2(F - 5)	2F	2(F + 7)



Practice problem:

Right now, Steve's age is half of Tom's age. In eight years, twice Tom's age will be five more than three times Steve's age. How old is Tom right now?

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Intro to Motion Questions

Perhaps the largest category of word problems concerns moving things and the related quantities **distance**, **rate** (speed), and **time**.

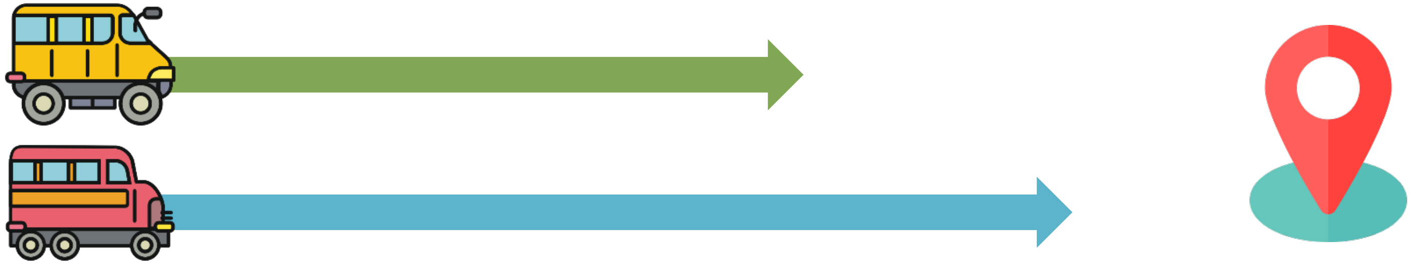
Distance = Rate × Time → **D = RT**

$R = \frac{D}{T}$ or $T = \frac{D}{R}$

Types of motion word problems

1. Two objects going in opposite directions.

2. Both objects going in the same direction, but one goes further.



3. One object going and returning at different rates.

Notice that **D = RT** is already solved for D, and we can easily solve for R or T.

Divide both sides by T to solve for R

$$RT = D \rightarrow R = \frac{D}{T}$$

Divide both sides by R to solve for T

$$RT = D \rightarrow T = \frac{D}{R}$$

It's important to be comfortable moving back and forth between any of the three forms.

Let's do a couple simple calculations for practice.

1) What is the speed of someone who covers 240 miles in 6 hours at a constant speed?

2) How far does someone moving at 8 m/s move in 40 seconds?

3) How much time does it take to move 300 feet at a speed of 20 ft/s?

What if the units given in the prompt and/or the units for which the prompt asks are inconsistent?

Changing the units, using a unit conversion, such as (1 hour = 60 minutes) or (1 foot = 12 inches). Often, the test will give you the conversions for any that aren't very common.

Practice question:

A lichen advances 4 cm each year across a rock slab. If this rate remains constant over time, how many years will it take to cross 30 meters? (1 m = 100 cm)

Practice question:

A car moving at 72 km/hr moves how many meters in one second? (1 km = 1000 m)

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Average Speed

70 mph

50 mph

$$\text{Average Speed} = \frac{70 + 50}{2}$$

$$= \frac{120}{2}$$

$$= 60 \text{ mph.}$$

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

You always need to apply $D = RT$

Be careful!!! You will get the wrong answer if you add the two speeds and divide the answer by two.

Sometimes, the problem won't give you all the numbers.

For example:

Cassandra drove from A to B at a constant 60 mph speed. She then returned, on the same route, from B to A, at a constant speed of 20 mph. What was her average speed?

Trap answer = 40 mph

Casdra drove from A to B at a constant 60 mph speed. She then returned, on the same route, from B to A, at a constant speed of 20 mph. What was her average speed?

Alternate solution: pick a value for the distance.

Let distance = 60 miles

A-to-B takes 1 hour; B-to-A takes 3 hours

Practice question:

An airplane has a 3600 mile trip. It covers the first 1800 miles of a trip at 400 mph. Which of the following is the closest to the constant speed the plane would have to follow in the last 1800 miles so that the average speed of the whole trip is 450 mph?

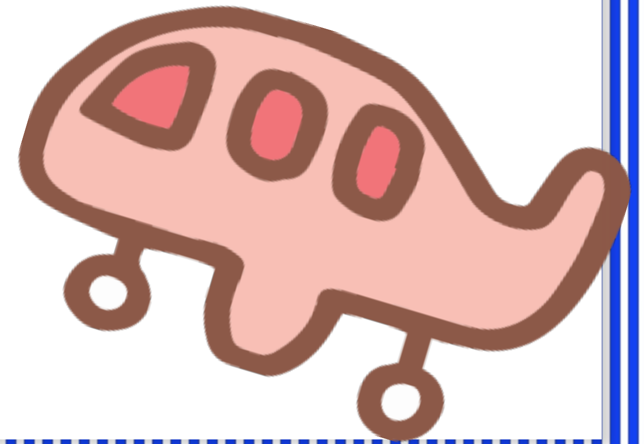
(A) 450 mph

(B) 455 mph

(C) 500 mph

(D) 514 mph

(E) 600 mph



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Multiple Traveler Questions

Some motion-based word problems involve more than one traveler and/or trip of more than one segment.

We already saw a little of this in the previous video on average speed. The basic strategy is that each traveler and each trip gets its own $D = RT$ equation.

Practice problem

Martha and Paul started traveling from A to B at the same time. Martha traveled at a constant speed of 60 mph, and Paul at a constant speed of 40 mph. When Martha arrived at B, Paul was still 50 miles away. What is the distance between A and B?

Practice problem

Frank and Georgia started traveling from A to B at the same time. Georgia's constant speed was 1.5 times Frank's constant speed. When Georgia arrived at B, she turned around immediately and returned by the same route. She crossed paths with Frank, who was coming toward B, when they were 60 miles away from B. How far away are A and B?

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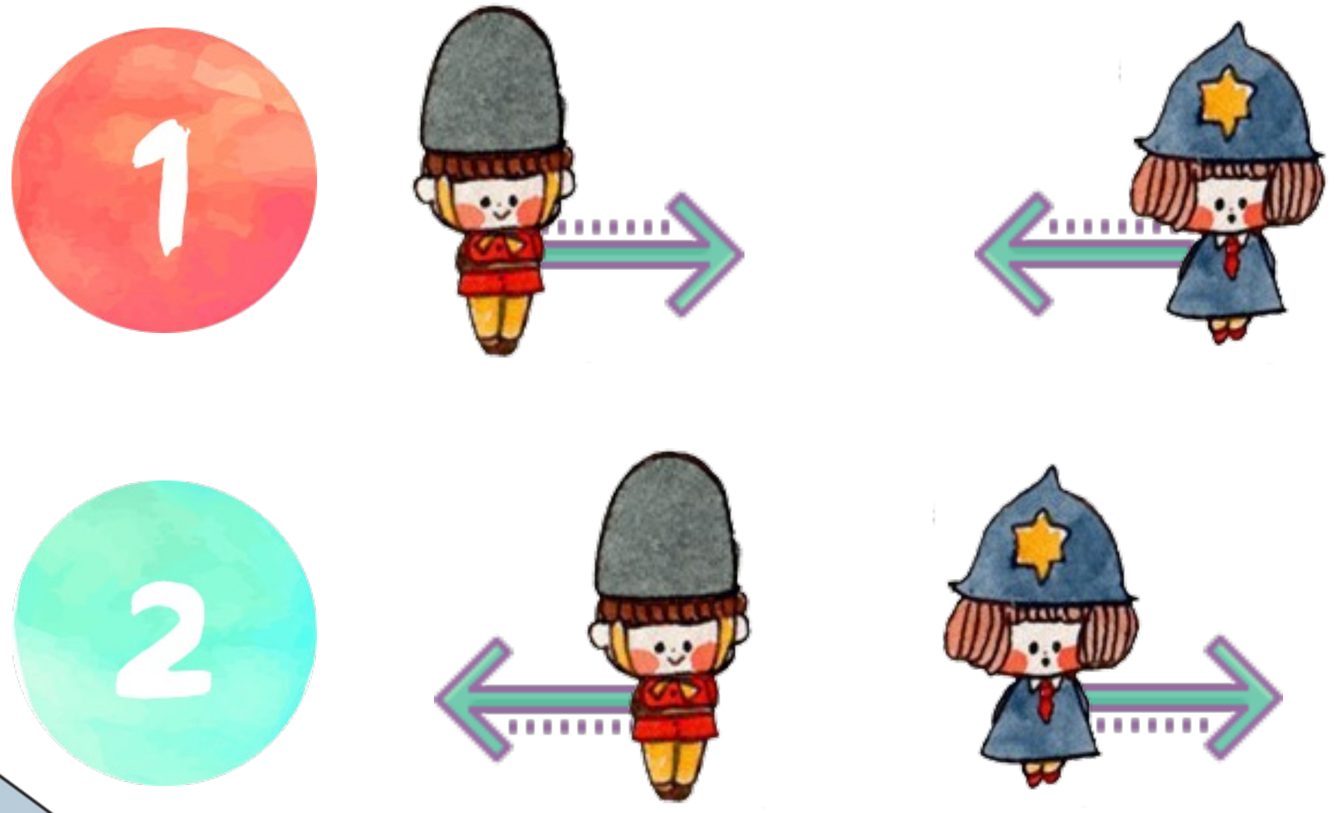
Shrinking and Expanding Gaps

Some motion-based word problems involve two travelers moving toward or away from each other.

Some very basic rules make these problems much easier.

Two travelers in opposite directions

We always add the speeds.



Approaching

the sum of the speeds is the speed at which the gap is shrinking

Receding

the sum of the speeds is the speed at which the gap is expanding.

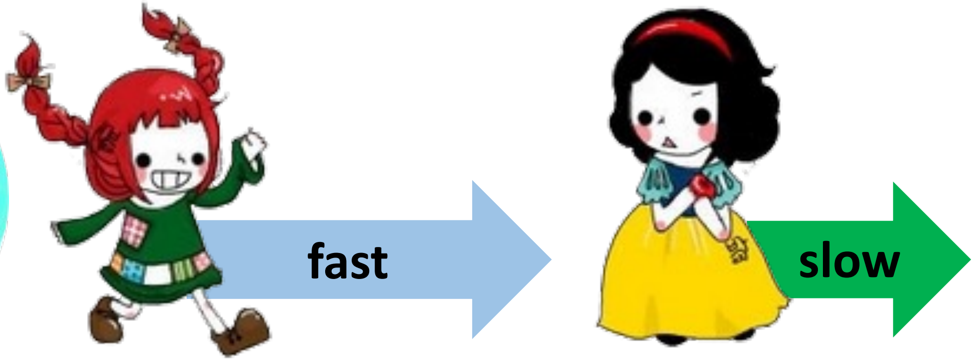
Two travelers in the same direction

We always subtract the speeds (bigger minus smaller).

1

faster traveler is in front the difference in speeds is the speed at which the gap is expanding.

2



slower traveler is in front the difference in speeds is the speed at which the gap is shrinking.

In a problem in which a gap is obviously shrinking of expanding, we could set a D = RT for each traveler, but sometimes, it saves time to set up a D = RT for the gap itself.

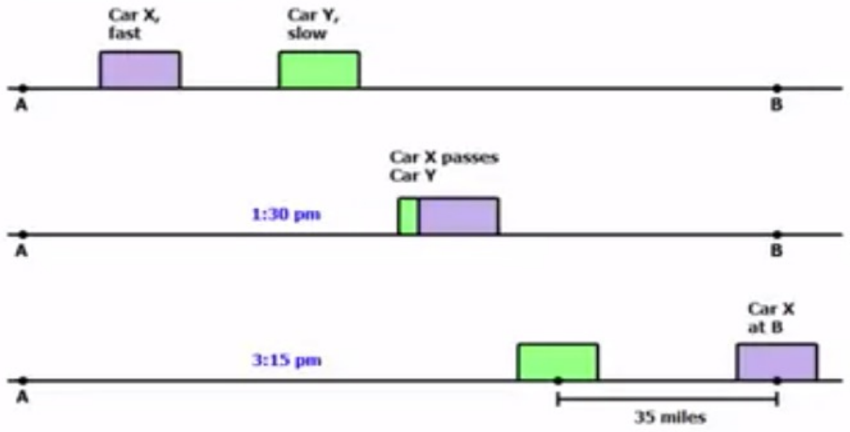
If we figure out the speed at which the gap is shrinking of expending, we can relate the two individual speeds by simple addition of subtraction.

Practice problem

A car and a truck are moving in the same direction on the same highway. The truck is moving at 50 mph, and the car is traveling at a constant speed. At 03:00 pm, the car is 30 miles behind the truck and at 04:30 pm, the car overtakes and passes the truck. What is the speed of the car?

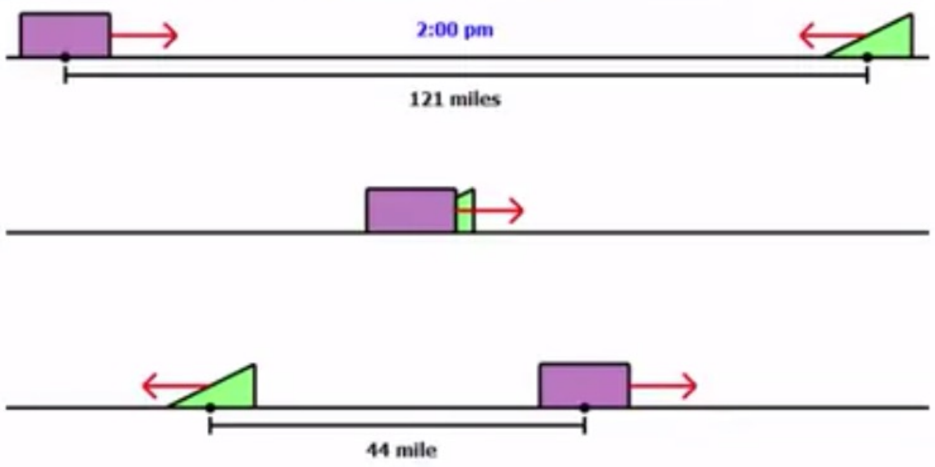
Practice problem

Car X and Y are traveling from A to B on the same route at constant speeds. Car X is initially behind Car Y, but Car X's speed is 1.25 times Car Y's speed. Car X passes Car Y at 1:30 pm. At 03:15 pm, Car X reaches B, and at that moment, Car Y is still 35 miles away from B. What the speed of Car X?



Practice problem

Cars P & Q are approaching each other om the same highway. Car P is moving at 49 mph and Car Q is moving at 61 mph. at 02:00pm, they are approaching each other and 121 mi apart. Eventually they pass each other. At what clock time are they moving away from each other and 44 miles apart?



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Work Questions

Some word problems concern machines or workers and how fast they can get certain jobs done.

The fundamental equation for this is remarkably similar to the D = RT equation for motion problems.

The Work Equation

$$A = RT$$

A = amount of work done

R = the work rate

T = time

The amount, A, can be products manufactured, or houses painted, or pizzas made, or etc. The "Work rate"

As with the D = RT equation, it's important to be able to solve this equation for all three variables.

$\boxed{A = RT}$ is already solved for the amount.

Solve for R:

$$R = \frac{A}{T}$$

Solve for T:

$$T = \frac{A}{R}$$

There are two broad categories of work problems.

One type involves using proportion. Of course, the work rate, like any rate, is a ratio, and can be used to set up a proportion.

The other type involves multiple machines or worker working at different rates, and figuring out how much work theses get done together.

A machine, working at a constant rate, manufactures 36 staplers in 28 minutes. How many staplers does it make in 1 hr 45 min?

1 hr 45 min = 60 + 45 = 105 min

$$\frac{\text{staplers}}{\text{time}} = \frac{36}{28} = \frac{S}{105}$$

$$\frac{9}{7} = \frac{S}{105}$$

$$\frac{9}{1} = \frac{S}{15}$$

S = 9*15 = 135

Example:

To "detail" a car means to clean it thoroughly, inside and out. When Amelia and Bred detail a car together, 1 car take 3 hours. When Amelia details a car alone, 1 car takes 4 hours. How long does it take Bred, working alone, to detail one car?

The secret to problems involving different machines or worker working at different work rates is this:

The combined work rate of people/machines working together is the SUM of the individual work rates.

In other words, we can't add or subtract the times, and we can't add or subtract the different amounts made in different times. (Those are the numbers usually given in the problem.)

To "detail" a car means to clean it thoroughly, inside and out. When Amelia and Bred detail a car together, 1 car take 3 hours. When Amelia details a car alone, 1 car takes 4 hours. How long does it take Bred, working alone, to detail one car?

$$\text{rates} = \frac{\text{cars}}{\text{hours}} \quad , R_{AB} = \frac{1}{3} \, , R_A = \frac{1}{4}$$

$$R_A + R_B = R_{AB}$$

$$R_B = R_{AB} - R_A$$

$$R_B = \frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$$

So, Bred details a car of 12 hours

Pump X takes 28 hours to fill a pool. Pump Y takes 21 hours to fill the same pool. How long does it take them to fill the same pool if they are working simultaneously?

11

Growth and Decay

Some word problems concern populations or samples that are in the process of growth or decay.

Human populations and population of animals or bacteria experience natural growth.

Decay can include radioactive decay, something malting, a bacteria population getting wiped out, etc.

The basic format of this question is you will be given some starting value, and told that the size gets multiplied or divided by a certain quantity in some fixed interval of time. Then you will be asked for the resultant amount after some slightly longer period of time.

Take the calculations one "change interval" at a times. Step by step, figure out the amounts until you arrive at the final amount.

Practice question:

When isotope QXW radioactively decays, it loses exactly half of its mass in each three-day period. Suppose scientists start with a 96 gram sample of pure isotope QXW on a certain day. What will be the remaining mass in 12 days?

Practice question:

Under optimal conditions, the Vericoccus Bacteria multiplies the size of its population by 5/2 every 4 hours. If there are
24 billion at 9:00 a.m., and optimal conditions are maintained, how many are there at 5:00 p.m. of the same day?

12

Double Matrix Method

In some more complicated set problems, every member of the population is classified in two ways at once.

For example, if we were to look at employees at a company, we might group employees by gender (M & F) and education (college only vs. advanced degree).

In that example, every single employee has a gender, either M or F. Also, every single employee has an education level, either "college only" or "advanced degree."

The information the problem could give might be how many females (which would include both education levels) or how many folks with advanced degrees (which would include both genders) or how many employees are females with advanced degrees.

Consider this problem:

In a company of 300 employees, 120 are females. A total of 200 employees have advanced degrees, and the rest have a college degree only. If 80 employees are males with college degree only, hoy many are females with advanced degrees?

For this problem, genders will be the columns and educational level will be the rows.

There are two genders, so we will need THREE columns: one for F, one for M, and one for the totals.

There are also two educational levels, so we will need THREE rows: one for "college only," one for "advanced degree, and one for the totals

	females	males	TOTALS
college only	A	C	E
advanced degree	A	A	F
TOTALS	G	H	T

Boxes A, B, C, and D are the raw data for different individuals.
Boxes E & F are the row totals.
Boxes G & H are the column totals.
Box T is the grand total, everyone in the whole collection.

$G + H = E + F = T$

In a company of 300 employees, 120 are females. A total of 200 employees have advanced degrees, and the rest have a college degree only. If 80 employees are males with college degree only, how many are females with advanced degrees?

	females	males	TOTALS
college only		80	
advanced degree			200
TOTALS	120		300