History of Mathematics

MAI3309

Week 2

Chapter 2
The historic period down to 1000 B.C.

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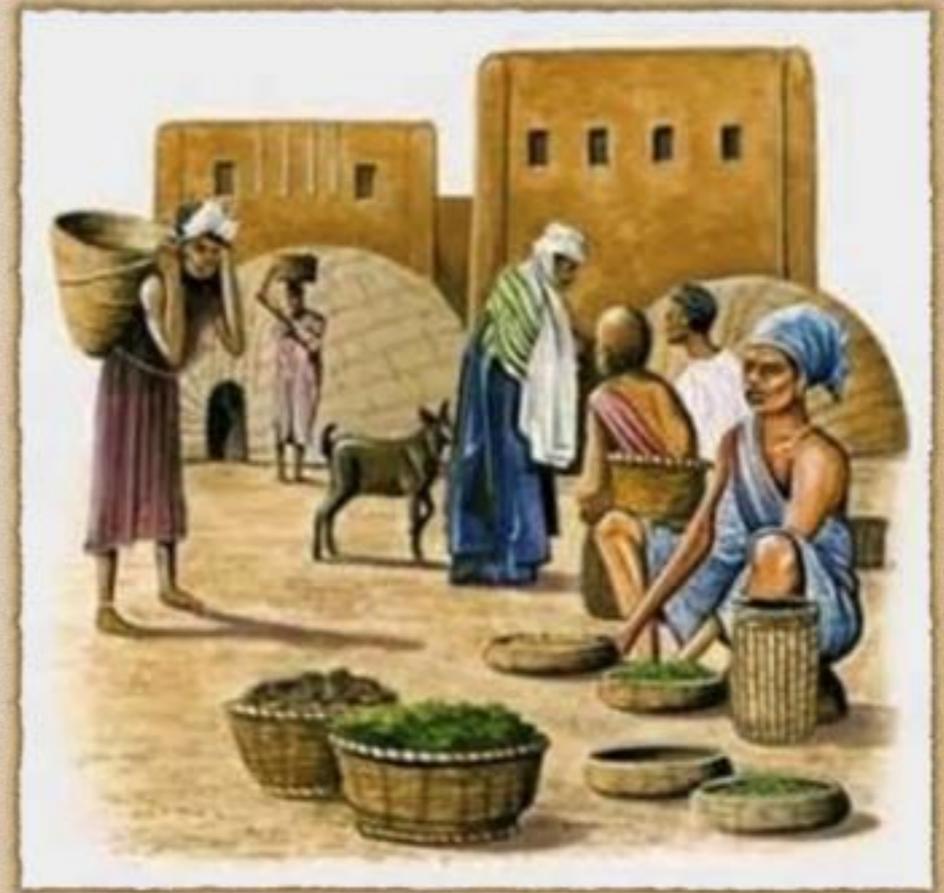
- Egypt

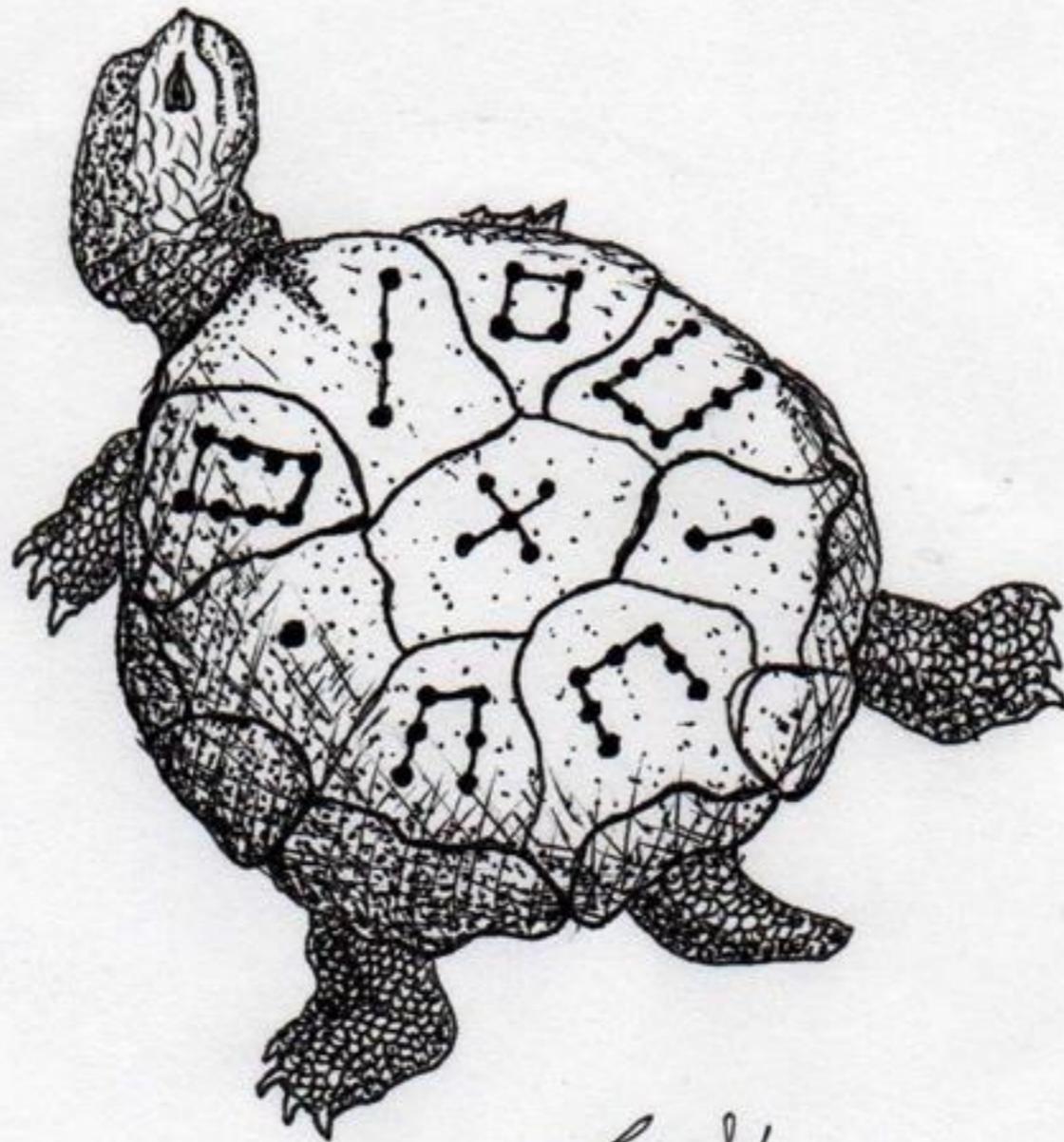# 1. General view

Barter System

Systematic Taxation

Stone masonry

Seagoing

Lo Shu

2. China

Early China Mathematics

1600 B.C. – beginning of Shang Dynasty; the chief of Shang tribe, Tang, led his army and overthrew the Xia Dynasty. Tang established the Shang Dynasty and made Bo the capital city.

1525 B.C. – moved kingdom from Xiang to Geng, then from Geng to Xing

1523 B.C. – first records of Chinese characters

1395 B.C. – the Shang move their capital to Yin (near Anyang)

1200 B.C. – earliest inscribed oracle bones; prosper of Shang oracle bones

1122 B.C. – fall of Shang Dynasty; Battle of Mu Yu; Wu Wang, the "Martial King" of the Zhou defeated the Shang through the use of over 300 chariots

While such claims are generally doubted by competent Sinologists, it is quite likely that the Chinese developed some acquaintance with descriptive astronomy at an early period, and that this development necessitated such knowledge of mathematics as the measure of time and angles and the use of fairly large numbers. Reasonably well-founded tradition gives the probable dates of Fuh-hi,[1] the reputed first emperor of China, as 2852–2738 B.C.,[2] and in his reign there were extensive astronomical observations. In this general period the Chinese are believed to have changed their zodiac into one of twenty-eight animals.

Wu Wang
1122
B.C.

Reign of Huang-ti

Reign of Huang-ti. In the year 2704 B.C.[1] Huang-ti, the Yellow Emperor, began his reign. Under his patronage it is said that Li Shu wrote on astronomy and that Ta-nao established the *Chia-tsŭ*, or sexagesimal system, both of these statements being supported by copies (possibly altered) of ancient records.[2] Even the emperor himself is said to have taken such an interest in mathematics as to write upon astronomy and arithmetic, and in his reign an eclipse of the sun was observed and recorded. Tradition assigns to this period even the decimal system of counting, although it is more likely that some popular work on the subject was written at this time.

East and West

It is this emperor Yau and his successor, the emperor Shun, who, it is said, carried farther to the eastward the dominion established by the Bak tribes which had come from western Asia. These tribes had been under the civilizing influences of the people of Susiana, who in turn had received their civilization from Babylon.[1] If this theory proves to be correct, the similarity between certain early forms of astronomy and mathematics in the East and the West is more easily explained.

Book of permutation

I-king. Of the "Five Canons" (*Wu-king*) of the Chinese probably the third in point of antiquity is the *I-king*, or *Book of Permutations*.[2] In this appear the *Liang I*, or "two principles" (the male, *yang*, ——; and the female, *ying*, — —) and from these were formed the *Sz' Siang*, or "four figures,"

☰ ☳ ☵ ☷

3   2   1   0
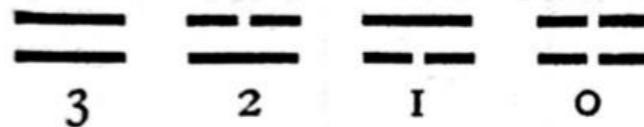
and the *Pa-kua* (eight-kua) or eight trigrams, the eight permutations of two forms taken three at a time, repetitions being allowed. These *Pa-kua* had various virtues assigned to them and have been used from a very early period until the present for purposes of divination. It was probably Wön-wang (1182 –1135 B.C.) who wrote the *I-king*; at any rate it was he who extended the *Pa-kua* into the sixty-four hexagrams now found in this classic.[3]

Eight Trigrams

k'iĕn heaven	tui steam	li fire	chön thunder	sün wind	k'an water	kön mountain	k'un earth
7	6	5	4	3	2	1	0
HEAVEN SKY	COLLECTED WATER	FIRE	THUNDER	WIND WOOD	WATER AS IN RAIN MOON	HILLS	EARTH
S.	S.E.	E.	N.E.	S.W.	W.	N.W.	N.

THE PA-KUA, OR EIGHT TRIGRAMS

From the *I-king*, or *Book of Permutations*. On the ordinary diviner's compass these directions are reversed

Although there is no historical evidence that the Chinese looked upon the *Pa-kua* as numerals, based upon the scale of two, it is true that if we take ——— for one and —— —— for zero, the successive trigrams, beginning at the right, have values which we may represent by our numerals as 000, 001, 010, 011, 100, 101, 110, and 111. If these are considered as numbers written on the scale of two, their respective values are 0, 1, 2, 3, 4, 5, 6, and 7.
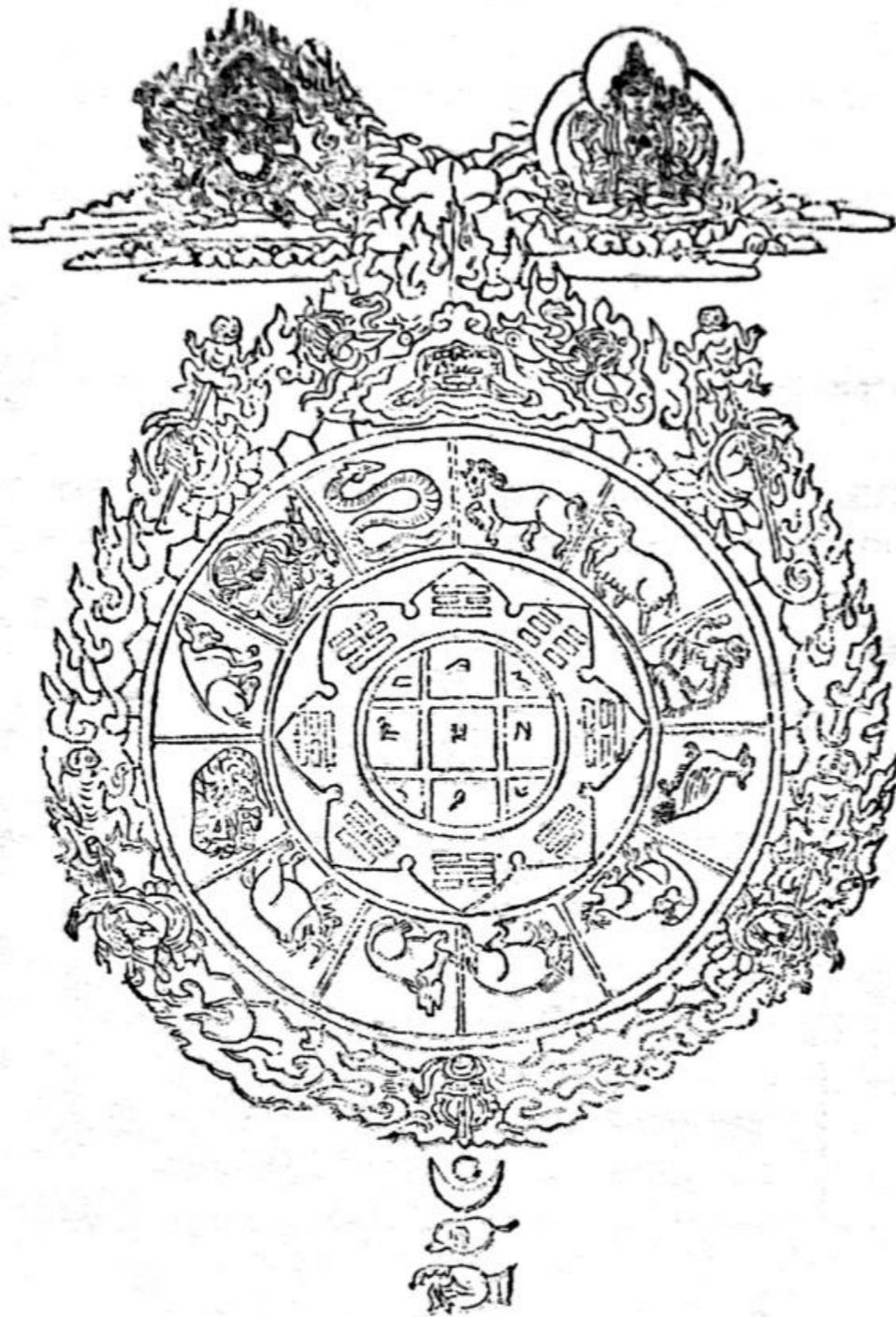
THIBETAN "WHEEL OF LIFE"

From a sheet of block printing done at Lhassa. This portion represents the signs of the zodiac, the *Pa-kua*, and, in the center, a magic square
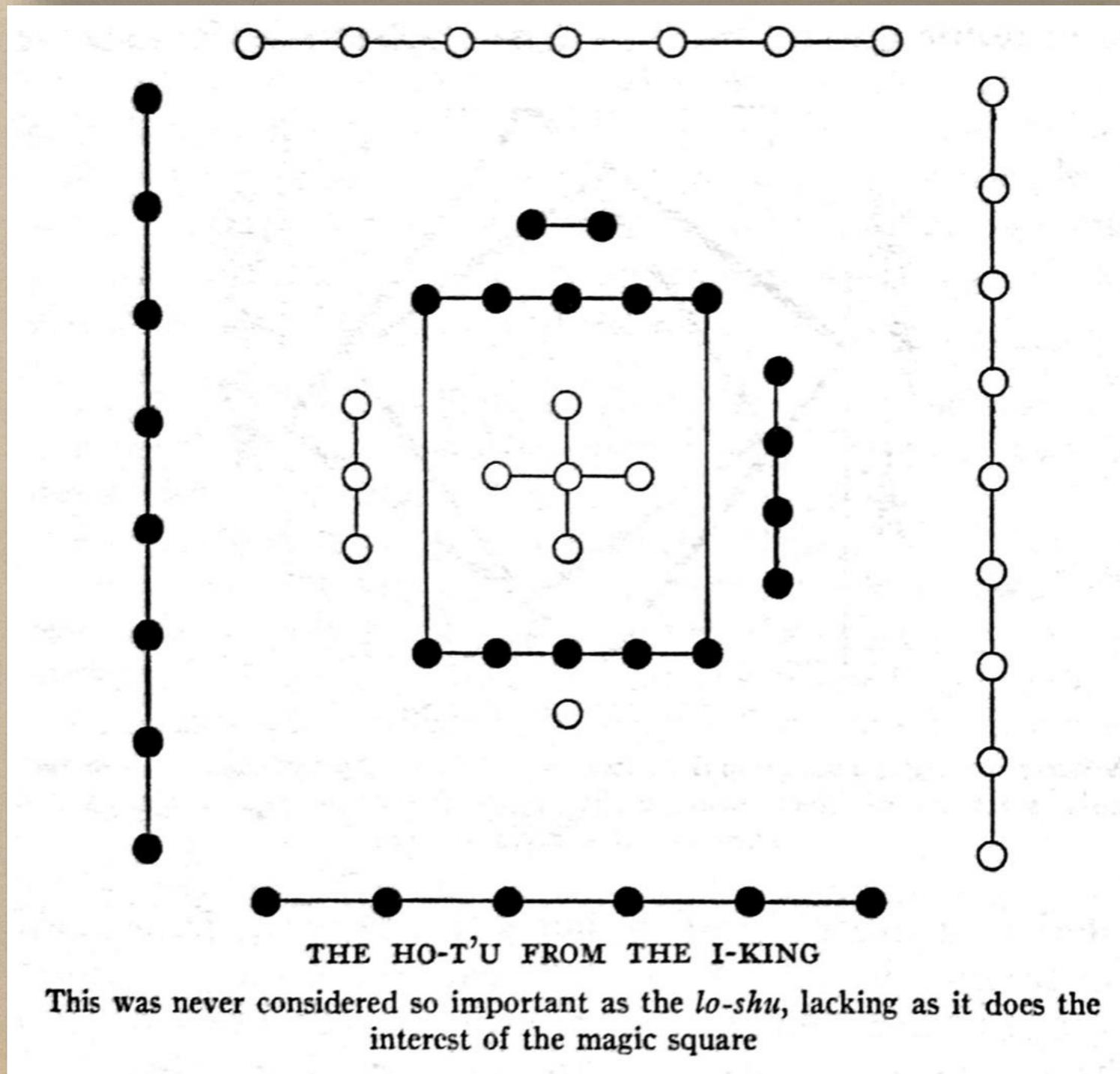
Wheel of life
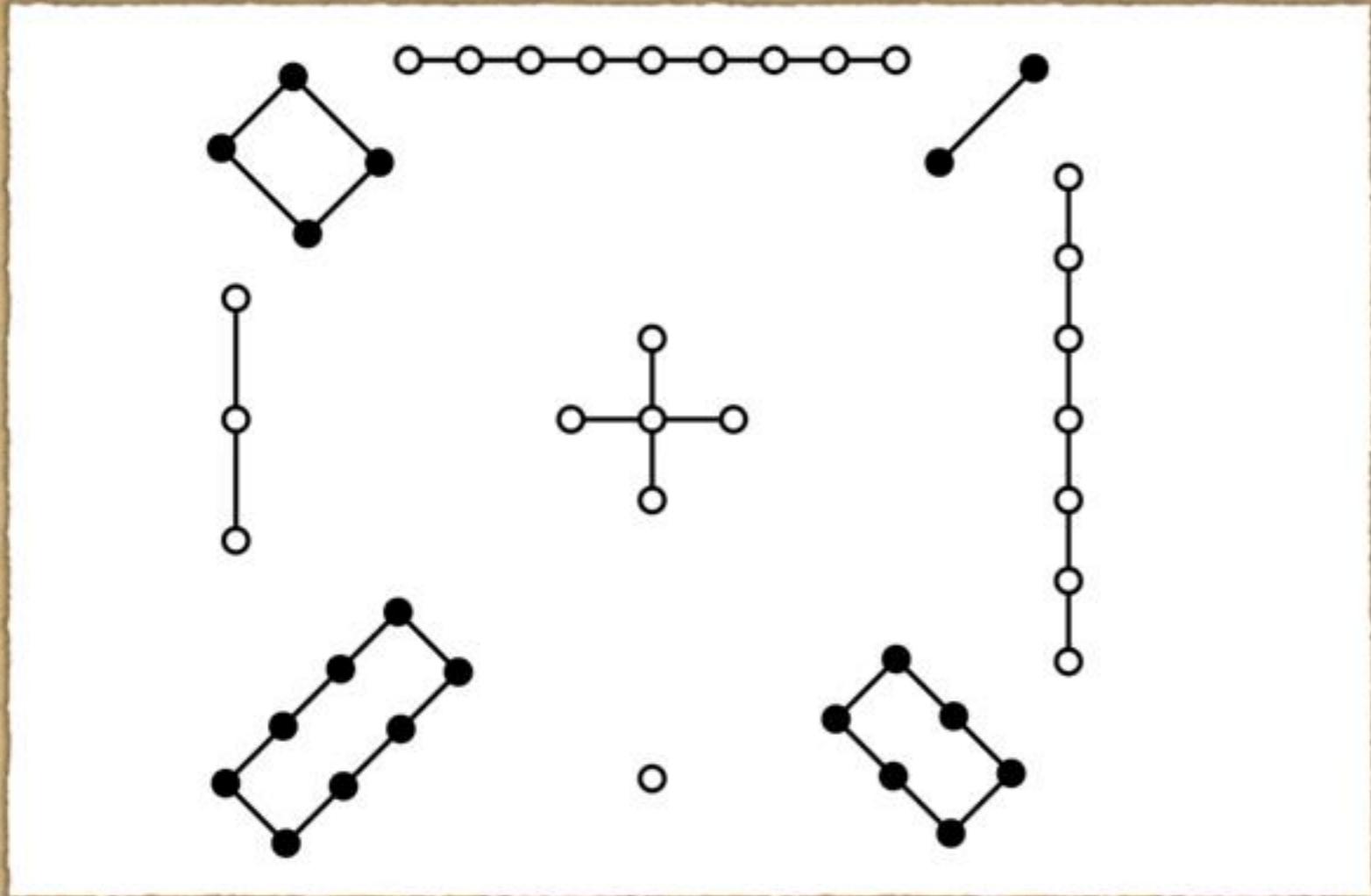
The Lo-shu

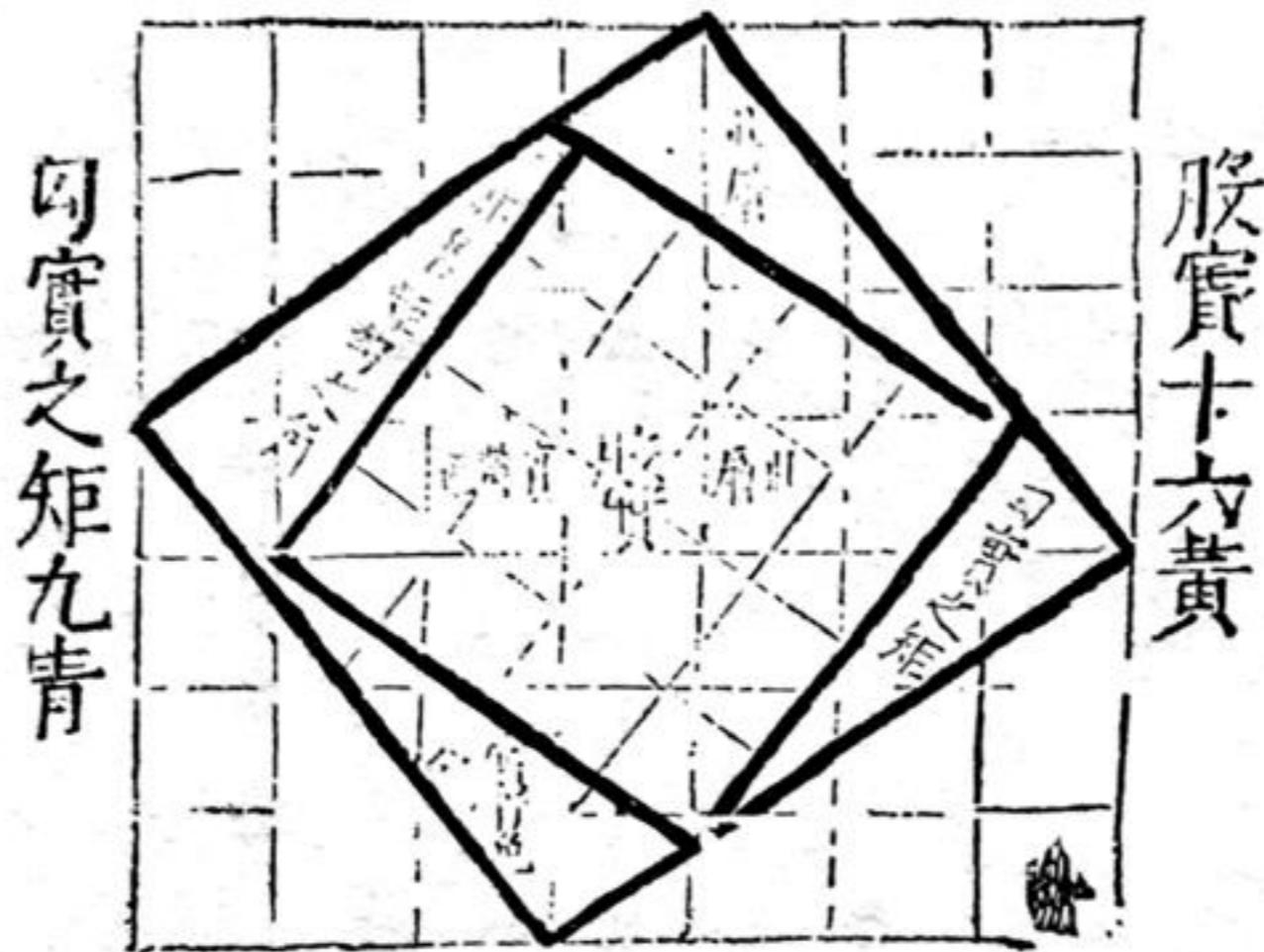| 4	9	2
3	5	7
8	1	6

Magic square

The ho-t'u

THE HO-T'U FROM THE I-KING

This was never considered so important as the *lo-shu*, lacking as it does the interest of the magic square

The chou-pei

CHÓU-PEÏ SUAN-KING

A work written in the second millenium B.C. This illustration is from a very early specimen of block printing. It shows the figure of the Pythagorean Theorem, but gives no proof

The chou-pei idea

The art of numbers is derived from the circle and the square.

Break the line and make the breadth 3, the length 4; then the distance between the corners is 5.[2]

Ah, mighty is the science of number.

Forms are round or pointed; numbers are odd or even. The heaven moves in a circle whose subordinate numbers are odd; the earth rests on a square whose subordinate numbers are even.

One who knows the earth is intelligent, but one who knows the heavens is a wise man. The knowledge comes from the shadow, and the shadow comes from the gnomon.[3]

K'iu-ch'ang Suan-shu

1. *Fang-t'ien* (*Squaring the farm*), relating to surveying, with correct rules for the area of the triangle, trapezium (trapezoid),[2] and circle ($\frac{1}{2} c \cdot \frac{1}{2} d$ and $\frac{1}{4} cd$), and with the circle approximations $\frac{3}{4} d^2$ and $\frac{1}{12} c^2$, where π is taken as 3.

2. *Su-mi* (*Calculating the cereals*), relating to percentage and proportion.

3. *Shuai-fen* (*Calculating the shares*), relating to partnership and the Rule of Three.[3]

4. *Shao-Kuang* (*Finding length*), relating to the finding of the sides of figures, and including square and cube roots.

5. *Shang-kung* (*Finding volumes*), relating to volumes.

6. *Chün-shu*, or *Kin-shu* (*Alligation*), relating to motion problems (couriers, hare and hound) and alligation.

7. *Ying-pu-tsu*, or *Ying-nu* (*Excess and deficiency*), relating to the Rule of False Position,[1] the terms "excess" and "deficiency" relating to two concepts that are used in this rule.

8. *Fang-ch'êng* (*Equation*), relating to simultaneous linear equations, with some idea of determinants.

9. *Kou-ku* (*Right triangle*), relating to the Pythagorean Triangle.
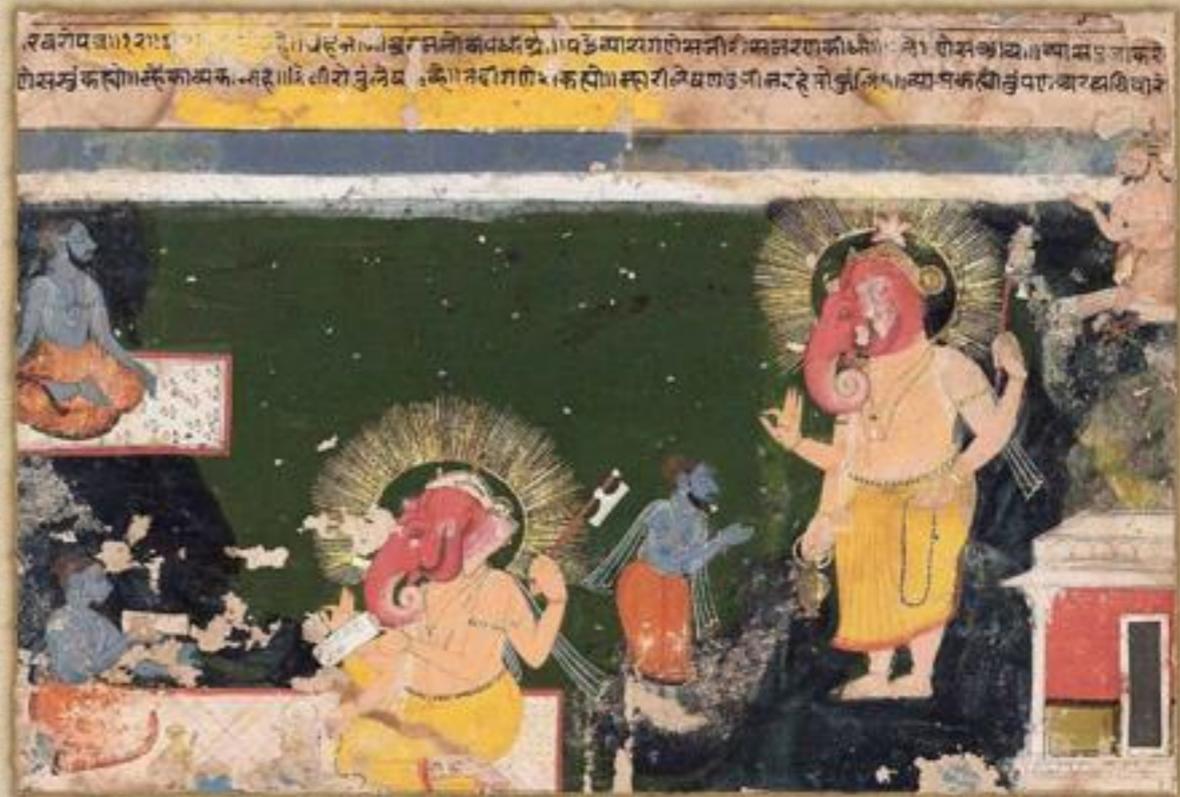
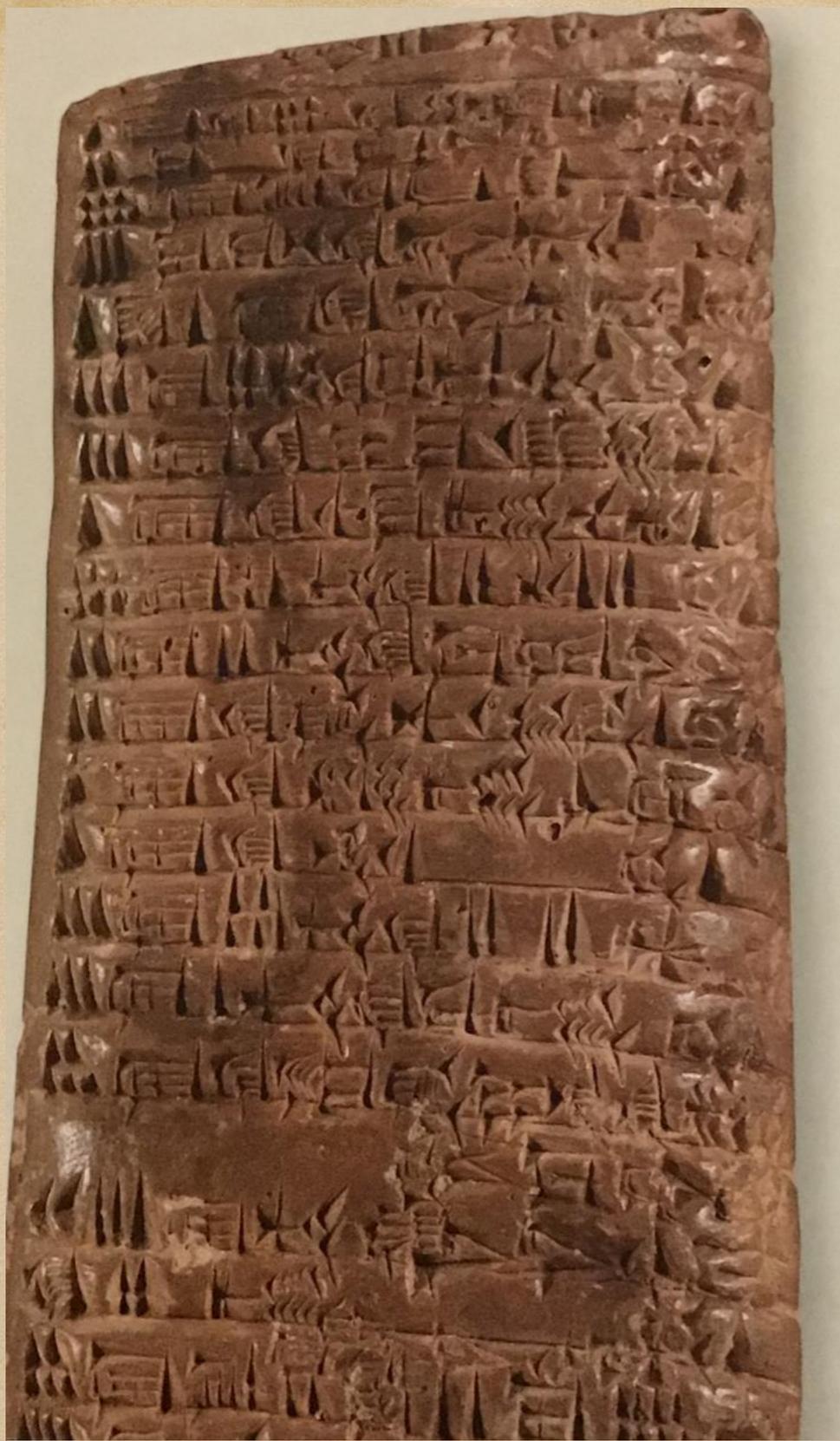Arithmetic in Nine Section

3. India

Mahabharata   Ramayana

Astronomy

The snake sacrifice of Janamejaya

sand years before our era. The Vedas, the sacred writings of India, lead us to understand that in this period some attention was given to astronomy, as was the case in contemporary China, Mesopotamia, and Egypt.

All that we can say, therefore, about this period of Hindu mathematics is that there is some evidence from ancient literature that in very early times India paid attention to astronomy and calculation, just as was the case with other advanced peoples of that period.[2]
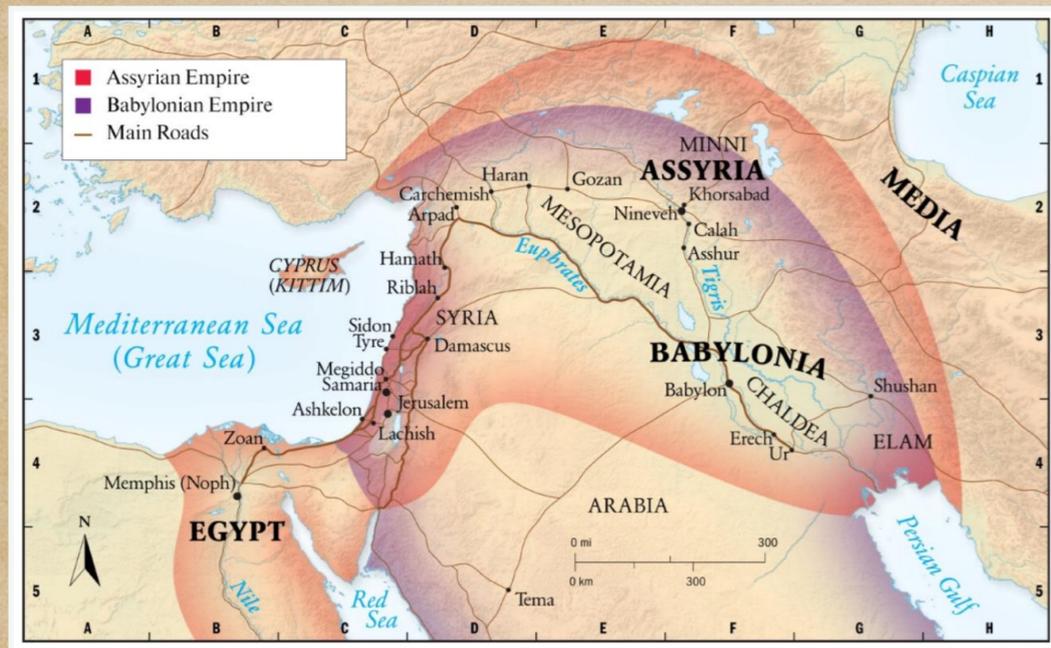
4. Babylon

Map of ancient Assyria

Sumarian Tablet

2800 B.C.

Numerals of Babylon

Scale of Sixty

Scale of Sixty. One peculiarity of Babylonian arithmetic is the constant use of the number 60,—a use which finally suggested the development of sexagesimal fractions and which still survives in our division of degrees, hours, and minutes into sixty sub-units. It is generally thought that the Babylonians, interested as they were in watching the stars, early came to believe that the circle of the year consisted of 360 days. It is also thought that they knew that the side of the regular inscribed hexagon is equal to the radius of the circle, this property suggesting the division of 360 into six equal parts, and 60 being thus looked upon as a kind of mystic number. This may, indeed, be the origin of this use of 60, but we find other nations using 40, 20, and even 15 in somewhat the same way, with no apparent reason, so that all such customs may have developed from racial notions which were started by some leader or sect with no particular reason in mind. It is more probable that 60 was chosen because of its integral divisors 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30, thus rendering work with its fractional parts very simple.

Babylonian Geometry

Babylonian Geometry. The tablets found at Nippur and elsewhere also give us some knowledge of the Babylonian geometry. From these it seems that as early as 1500 B.C. the Babylonians could find the area of a rectangle, including that of a square; the area of a right-angled triangle; the area of a trapezium (trapezoid); and possibly the area of a circle, the volume of a parallelepiped, and the volume of a cylinder. There is ground for the belief that they knew the law of expansion of $(a+b)^2$, although we have no knowledge as to whether this was inferred from a geometric figure or from their extensive study of square numbers. There is also some reason to believe that they knew the abacus, since it has been suggested that one of their signs (ŠID) may have been derived from a pictograph of such an instrument.

Early Calendars

Early Calendars. Some knowledge of mathematics must, however, have long preceded the work recorded on these Sumerian tablets. The old Babylonian year began with the vernal equinox, and the first month was named after the Bull. The calendar must, therefore, have been established at a period in which the sun was in Taurus at this equinox, and such a period began about 4700 B.C. A calendar of any kind presupposes a system of numbers and some form of calculation, so that we may safely say that some kind of arithmetic existed in Babylonia in the 4th or 5th millennium B.C. Indeed, so far as the calendar is concerned, it should be said that the Sumerians celebrated the beginning of the year at the vernal equinox as early as 5700 B.C., and possibly even earlier.[1]

5. Egypt

The great- grandfathers

The first stone masonry wall
Before 3000 B.C.

The great-grandsons

The Great Pyramid of Gizeh
After 2900 B.C.

Egyptian Engineers

Accuracy of Early Engineers. Such was the degree of accuracy secured by these early surveyors that Petrie found the maximum error in fixing the length of the sides of the Great Pyramid to be only 0.63 of an inch, or less than $\frac{1}{14000}$ of the total length, and the angle error at the corners to be 12″, or only $\frac{1}{27000}$ of a right angle.

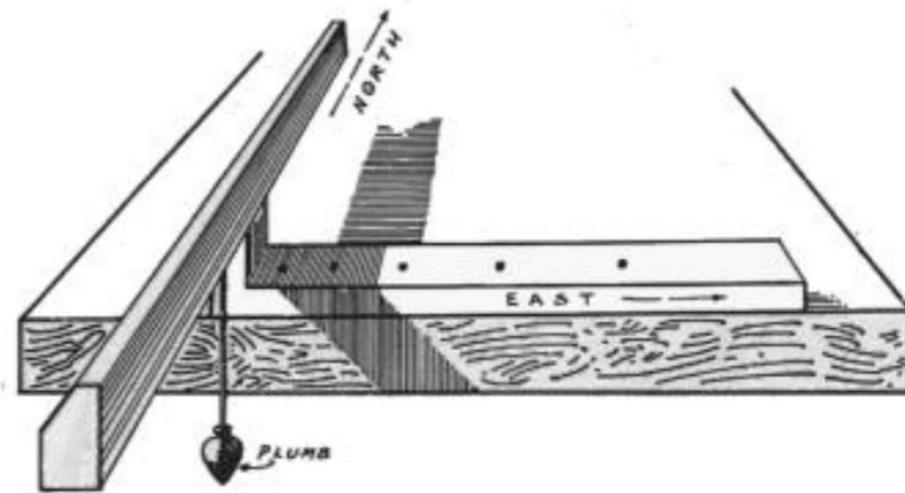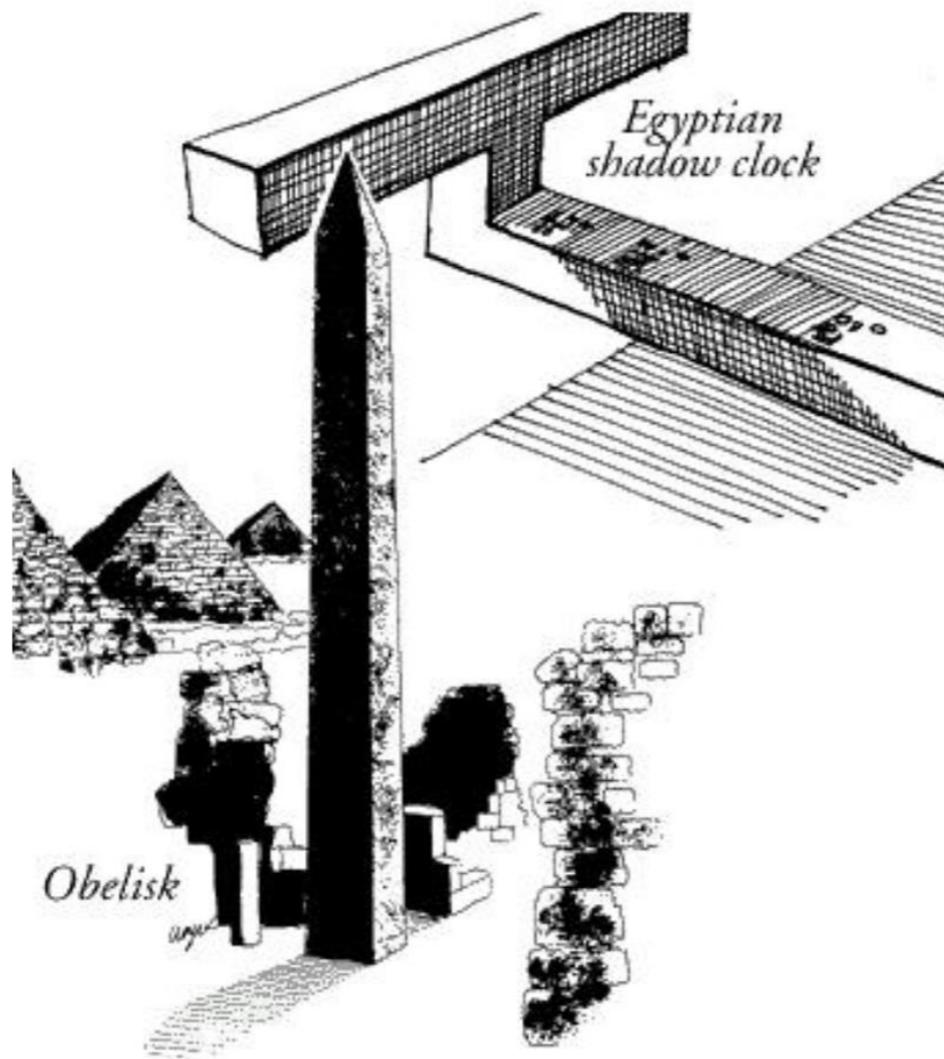# Mathematics of the Feudal Age

Amenemhat III

Ahmes Papyrus : 1650 B.C
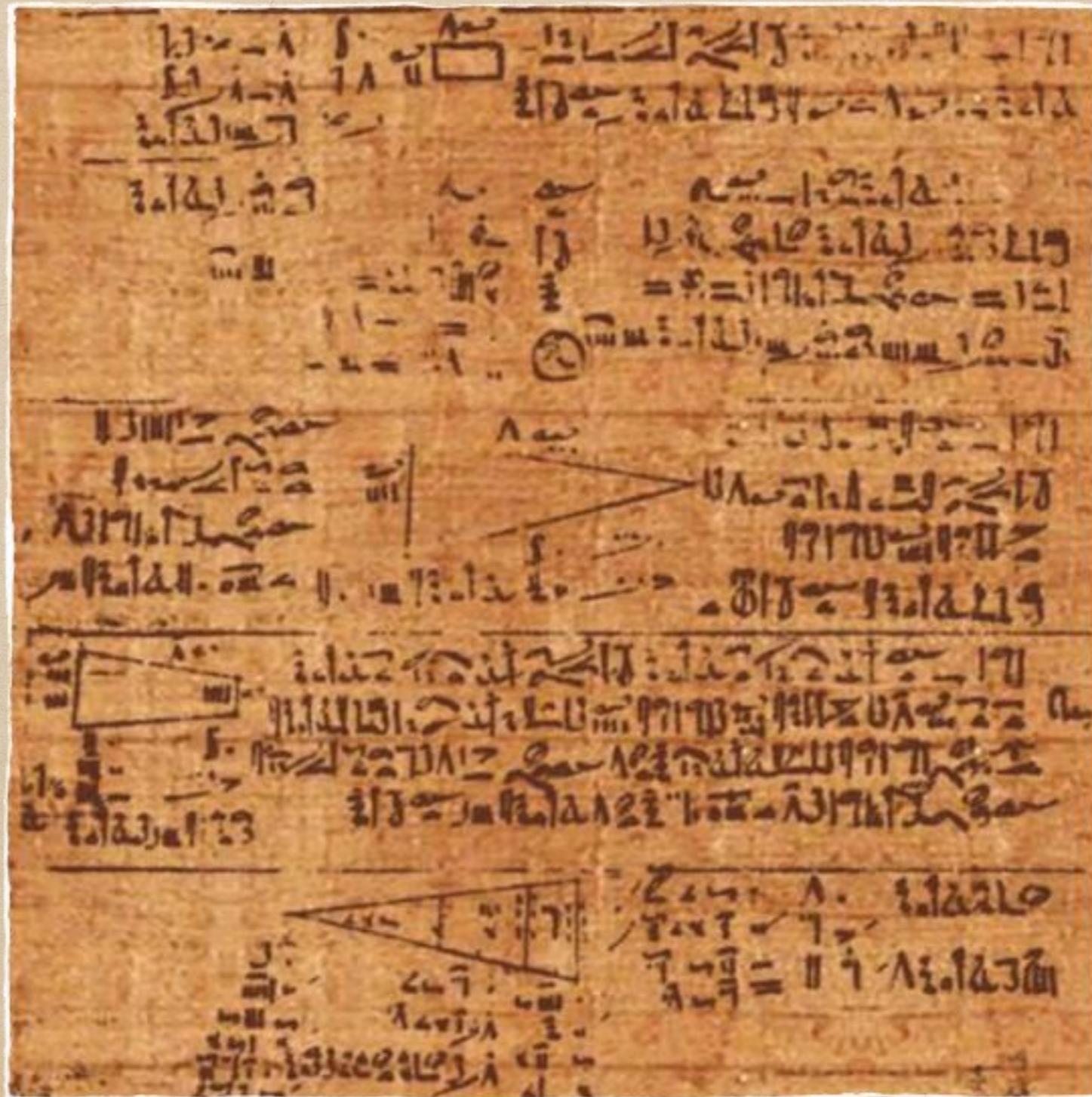
The oldest sundial : 1500 B.C

The oldest dial known c. 1500 B.C. The sundial is considered to be the first scientific instrument.

The practical problems

1601   392,325
together bread 107,893 makes in *ten* 364,371
bread 6121 loaves 1800 *thes* makes in *ten* 21,600
together 385,871
rest 6354
quantity of maize sacks 1601 makes in bread 112,090
makes in *ten* 392,306
brought to the magazine bread 114,064 makes in *ten* 385,971

107,893 loaves weigh	364,371	*ten*
6121 loaves weigh	21,600	*ten*
together they weigh	385,971	*ten*
there is left	6,354	*ten*
the total being	392,325	*ten*

Rameses II

The division of the day into twelve parts
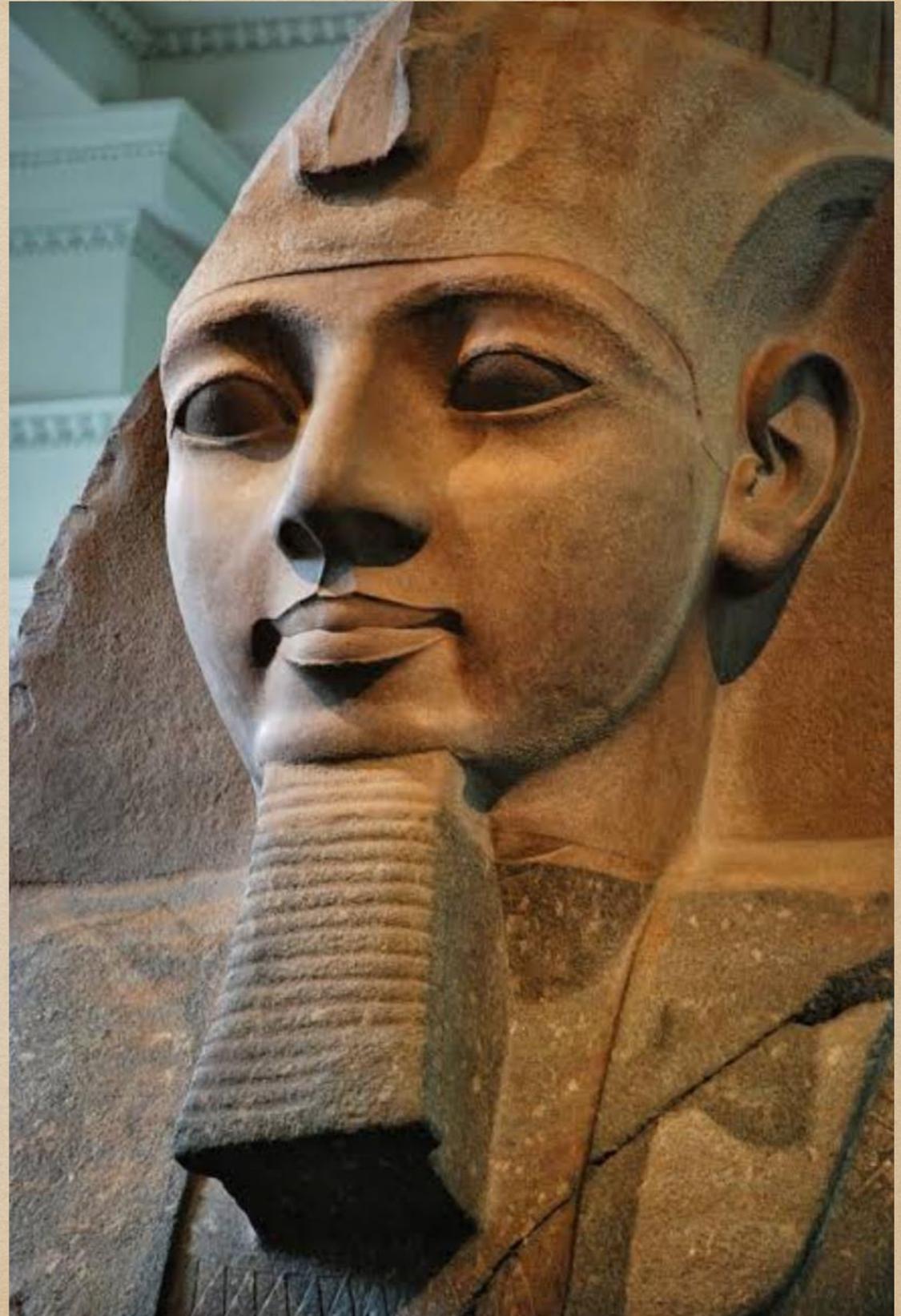
Harris Papyrus : 1167 B.C

Rameses IV

Commercial System

Groups

TOPICS FOR DISCUSSION

1. The countries in which mathematics flourished prior to 1000 B.C., and the reasons for this mathematical activity.

2. Reasons for supposing mathematics to have made some progress in the Late Stone Age, or even earlier.

① 3. Influences leading to an extension in the use of mathematics in the third millennium B.C.

② 4. Probable nature of the earliest mathematics of China, and the influences which developed the study of this science.

5. General nature of the early written mathematical works in China, with approximate dates.

③ 6. The first traces of number mysticism in the East.

7. General period in which the *Nine Sections* was written. Nature of the work.

8. Probable nature of the early Hindu mathematics.

④ 9. Influences that developed Babylonian mathematics and the method of recording the science.

10. General nature of Babylonian mathematics.

11. Evidence of early mathematics in Egypt. General nature of the work in the earliest periods.

12. Mathematics of the Feudal Age in Egypt.

⑤ 13. The Ahmes Papyrus, its origin and general nature.

⑥ 14. Evidences of development of commercial arithmetic between the time of Ahmes and 1000 B.C.

15. Types of problems in arithmetic, algebra, and mensuration that interested the ancient Egyptians.

16. Comparison of the mathematical progress and interests of China, India, Babylonia, and Egypt in early times.

17. A consideration of the reasons why this period was lacking in power to advance its mathematics.

⑦ 18. A study of the evidence of mathematics in Crete and Cyprus before 1000 B.C., and the influence of this mathematics upon Greek science.

19. The evidences of interrelation of mathematical ideas in Mesopotamia, Egypt, and the islands of the Mediterranean Sea.

⑧ 20. Mathematical and astronomical instruments of this period.

21. The degree of accuracy apparently secured by engineers before the year 1000 B.C.

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