Chapter 2
Per cent and Ratios

Outlines

1. Intro to Per cent
2. Working with Per cent
3. Number Sense and Per cent
4. Per cent Increases and Decreases
5. Sequential Per cent Changes
6. Simple and Compound Interest
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1.Intro to Per cent

Percent comes from the Latin per centum

$$X\% \rightarrow \frac{x}{100}$$

Thus. **"per cent"** means **"divided by 100"**

$\therefore$ 53% = $\frac{53}{100}$ or the decimal 0.53

Similarly 0.09% means the fraction

$$\frac{0.09}{100} = \frac{9}{10000} = 0.0009$$

Changing from Per cents to Decimals

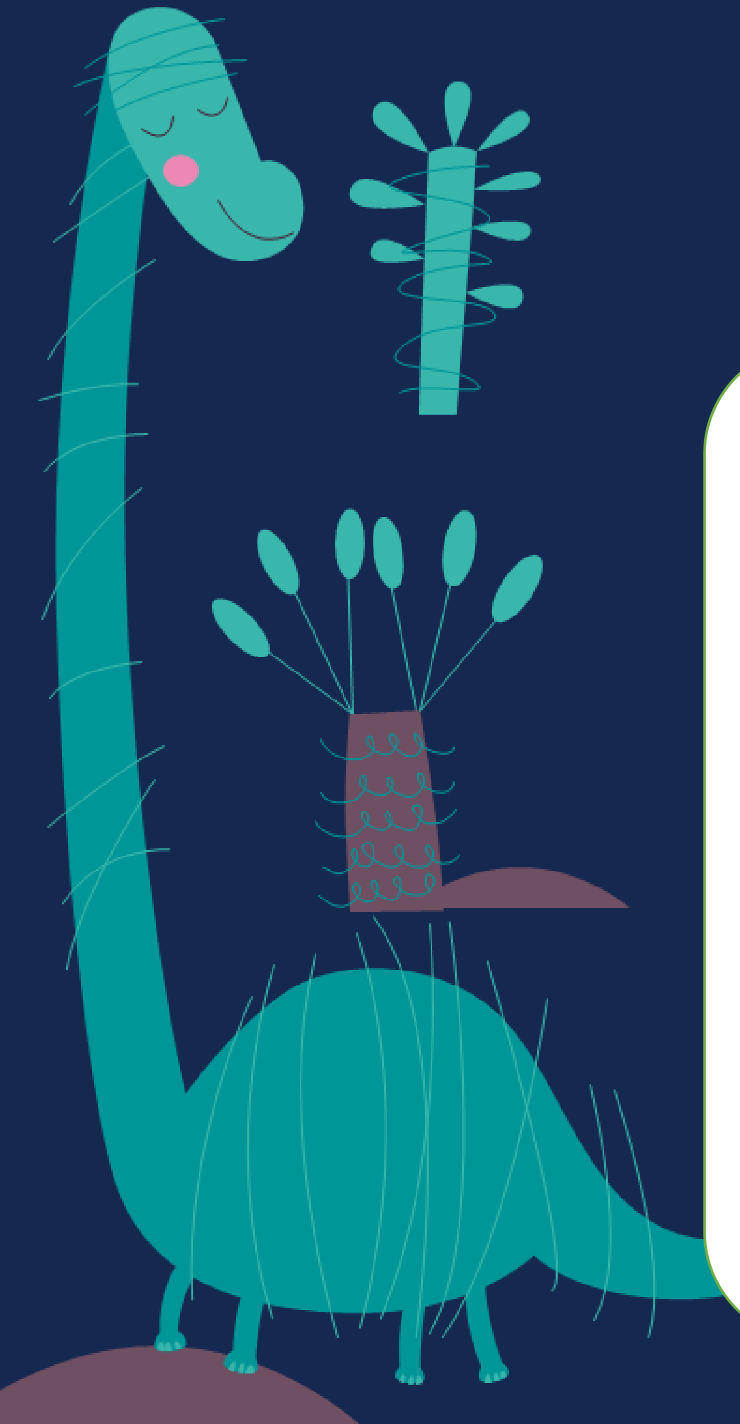
This is divided by 100, so we move decimal point two to the left

$73.8\% = 0.738$

$10\% =$

$0.45\% =$

$0.032\% =$



Changing from Decimals to Per cents

Un-divided by 100 which essentially is multiply by 100. Thus the decimal point is moved two places to the right.

$0.72 = 72\%$

$0.065 =$

$5.3 =$

$2.34 =$

Changing from Per cents to Fractions

We just put per cent over 100, after that, we may have to simplify a bit.

$$50\% = \frac{50}{100} = \frac{1}{2}$$

$$63\% =$$

$$0.65\% =$$

$$0.8\% =$$

Changing from Fractions to Per cents

This is trickier, unless you know the Fraction-to-Decimal conversions, discussed in "Conversions ; Fractions and Decimals"

$\frac{2}{5} = 0.4 \rightarrow \quad 40\%$

$\frac{1}{3} \approx$

$\frac{2}{3} \approx$

Recommendations!!! Often, on the test, we need to approximate percents from fractions or from division.

$$\frac{9}{33} = \frac{27}{99} > \frac{27}{100} = 27\%$$

$$\frac{11}{14} =$$

2.Working with Per cents

What is 55% of 400?

240 is 30% of what number?

A few ways to tackle questions

Method One: Per cents as Multipliers

The **decimal form of a percent** is called the **multiplier** for that per cent. This is because we simply can multiply by this form to take a per cent of a number

1. "is" means "equals"
2. "of" means "multiply"
3. Change any per cent to the multiplier form
4. Replace unknowns with a variable

What is 80% of 200?

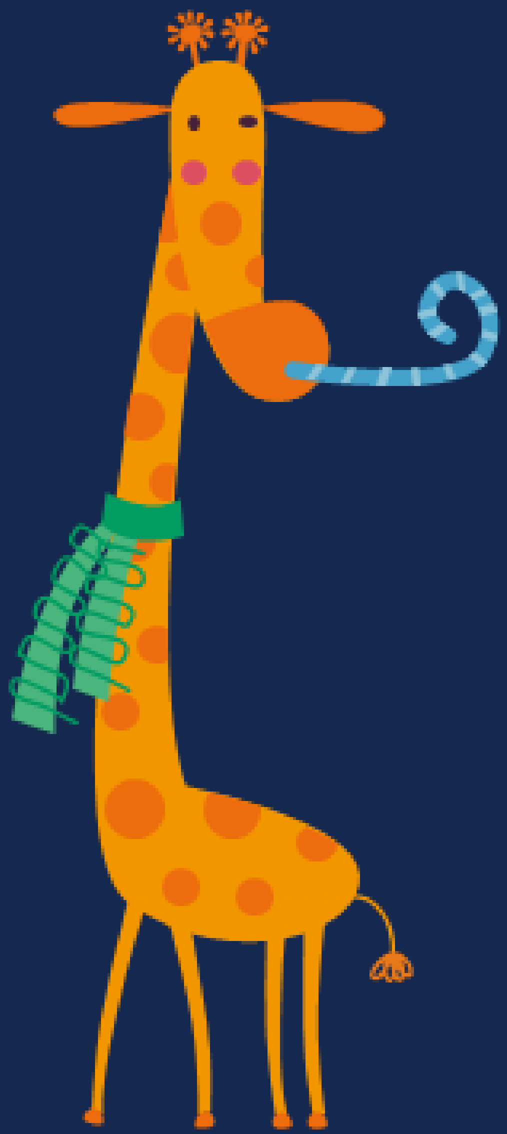
Multiplier is 0.80

Then $x = (0.80)(200) = 160$

240 is 30% of what number?

$240 = (0.30)x$

$$x = \frac{240}{0.30} = \frac{2400}{3} = 800$$



Method Two: Finding the Per cent

56 is what per cent of 800

$$56 = x(800) \qquad (x \text{ is multiplier})$$

$$x = \frac{56}{800} = \frac{7}{100} = 7\%$$

Method Three: Per cent and fractions

Use this approach only if the % is a very easy fraction, for example $\frac{1}{2}$ or $\frac{1}{4}$

What is 75% of 280 ?

$$75\% \times 280 = \frac{3}{4} \times 280 = 210$$

What is 60% of 60 ?
52 is 40% of what number ?
18 is what per cent of 45 ?
What is 50% of 128 ?

3. Number Sense and Per cent

Beginning suggestion : think about 10% of the whole, and work from there. Sometimes it also help to find 1%

What is 80% Of 200?

10% of 200 is clearly 20

We want eight of these -> 8(20) = 160

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240 is 30% of what number?

If 30% is 240, we can divide this by 3 to gel 10%

$\frac{240}{3} = 80$ is 10% that is one-tenth of the whole.

So, the number is 800

56 is what per cent of 800?

10% of 800 = 80

1% of 800 = 8

We need 7 times last piece. Thus 7% of 800 = 56 → **7%**

What is 55% of 400?

50% of 400 = 200

5% Of 400 = 20

Thus 55% of 400 = 200+20 = 220

What is 37% of 700?

10% of 700 = 70 → 30% of 700 = 70*3 = 210

1% of 700 = 7 → 7% of 700 = 7*7 = 49

Thus 37% of 700 = 210 + 49 = 259

Practice these.

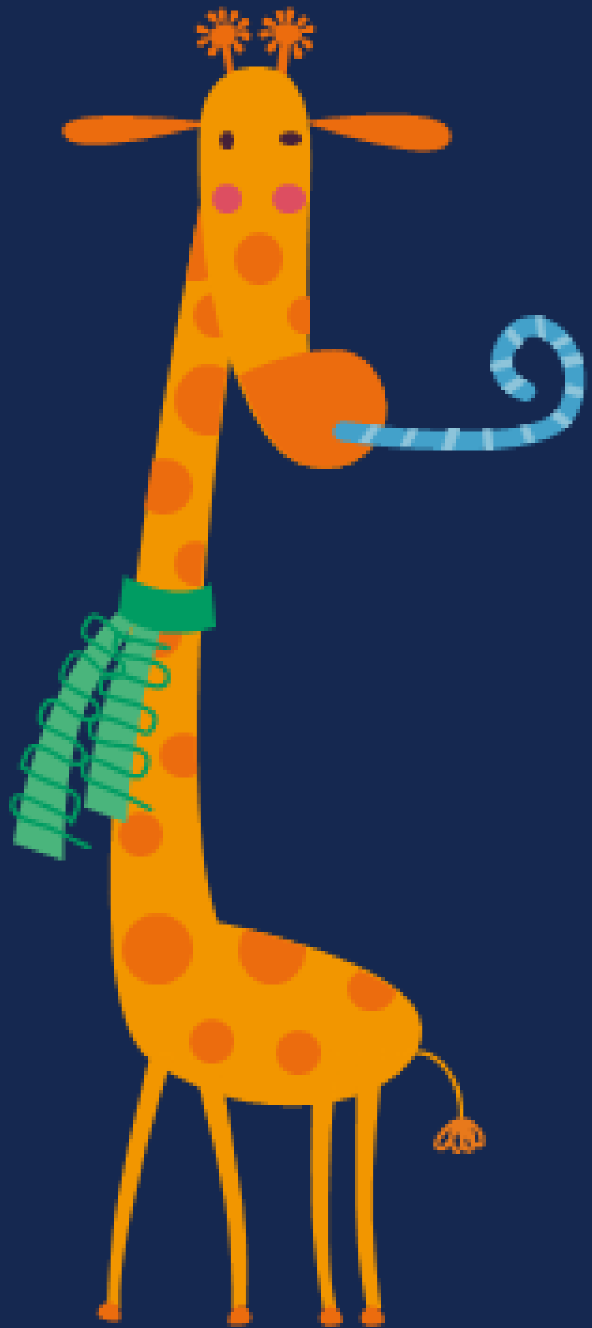
What is 70% of 50?

48 is 30% of what number?

72 is what per cent of 300?

What is 35% of 180?

90 is 15% of what number?

4.Per cent Increases and Decreases

Percent increase and **decrease** problems are highly found in the test.

A $170 item is discounted 30% What is the new price?

The price of an item increased by 35% then decreased by 25%. This is equivalent to what percent change?

Remember the slide in **"working with per cents."** there, we said that the decimal from of a percent was is **multiplier.**

The decimal from of P per cent is the multiplier for finding P per cent of something.

0.30 is the multiplier to find 30% **of** something, but we will use different multiplier for a **30% increase** or a **30% decrease.**

Per cent Increases

This might be phrased as :

Y increased by 30% Or X is 30% greater than Y.

Either way, this is per cent increase for Y. Let's think about this.

If Y increase by 30%, the whole original part of Y is still there, plus

30% increase is : 1 + 0.30 = 1.3

In general, if the problem talks about a P% increase,

(multiplier for a P% increase) = 1 + (P% as a decimal)

Thus, the multiplier for a 46% increase is 1 + 0.46 = 1.46

An item originally cost $800. The price increased by 20%. What is the new price?

800*1.2 = $960

With number sense:
10% is $80, so 20% is $160. That's the increase.

After a 30% increase, the price of something is $78. What was the original price?

Multiplier = 1.3
The unknown original is multiplied by this

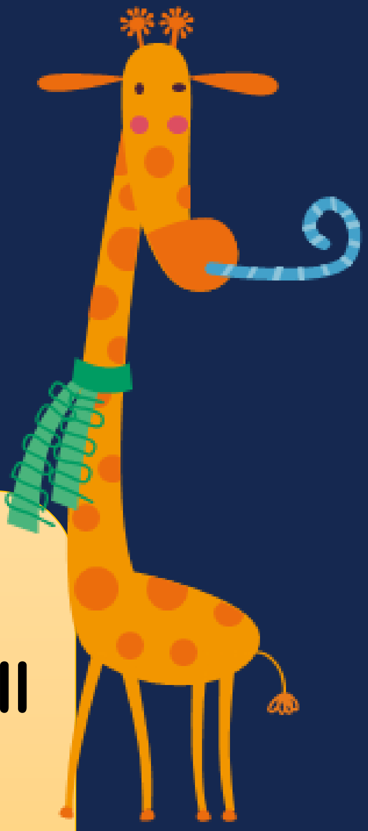
$$\boldsymbol{x \times 1.3 = 78}$$

$$x = \frac{78}{1.3} = \frac{780}{130} = \$60$$

Per cent Decreases

This might be phrased as :

Y decreased by 30% Or X is 30% less than Y



Either way, this is per cent decrease for Y. Let's think about this. If Y decrease by 30% the whole original past of Y is still here, except for a 30% that's now missing. Therefore, the multiplier for a 30% decrease is 1 - 0.30 = 0.7

In general, if the problem talks about a P% decrease

(multiplier for a P% decrease) = 1 – (P% as a decimal)

As $ 170 item is discounted 30%. What is the new price?

Multiplier = 1 - 0.30 = 0.70

170 x (0.70) = $119

I might notice:

10% = 17, so 30% is 3*17 = 51

The price goes down by $ 51 = 50+1

170 down by 50 = 120

120 down by 1 = **119**

After an item discounted 80% the new price is $150. What was the original price?

Notice that:
If 80% is gone , only 20% is left
20% is 150
Therefor
10% = 75
Therefor
100% = 750

Finding the per cent

Some problem give us the starting and ending values,
and ask us to find the per cent of increase or decrease

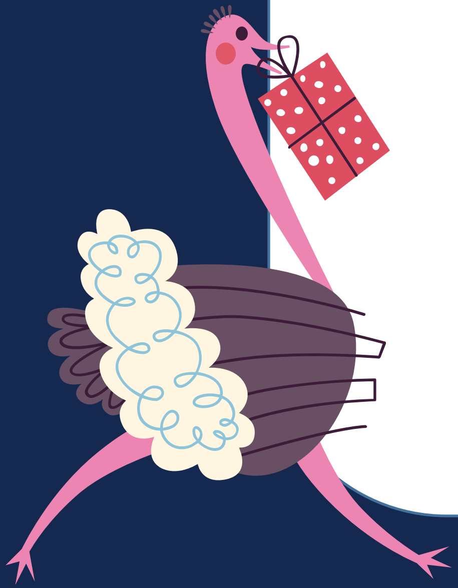
Since (new) = (multiplier)*(old)

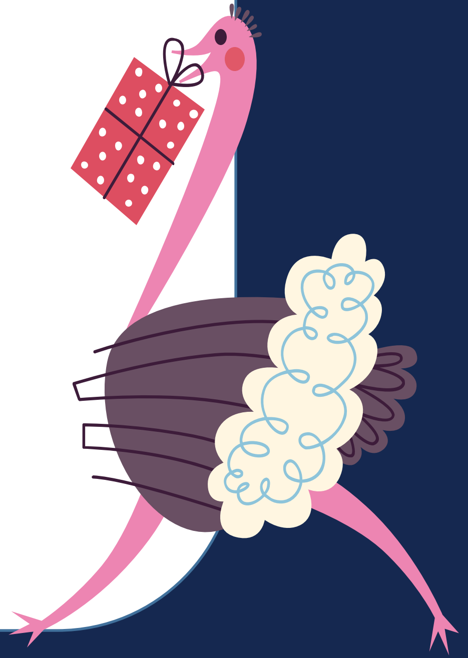
We could say :

$$\text{Multiplier} = \frac{New}{old}$$

**** Remember that ratio gives us the multiplier, and we have to change from that back to a per cent

Finding the per cent





The price of an item increase from $60 to $102. What was the per cent increase?

$$\text{Multiplier} = \frac{new}{old} = \frac{102}{60} = \frac{17}{10} = 1.7$$

That's the multiplier for a 70% increase. 70%

I might notice:
102 - 60 = 42
10% of 60 = 6
70% of 60 = 42
Therefore
102 is **70% greater than 60**

The price of an item increase from $60 to $102.
What was the per cent increase?

$$\text{Multiplier} = \frac{new}{old} = \frac{102}{60} = \frac{17}{10} = 1.7$$

That's the multiplier for a 70% increase. 70%

I might notice:
102 – 60 = 42
10% of 60 = 6
70% of 60 = 42
Therefore
102 is **70% greater** than 60

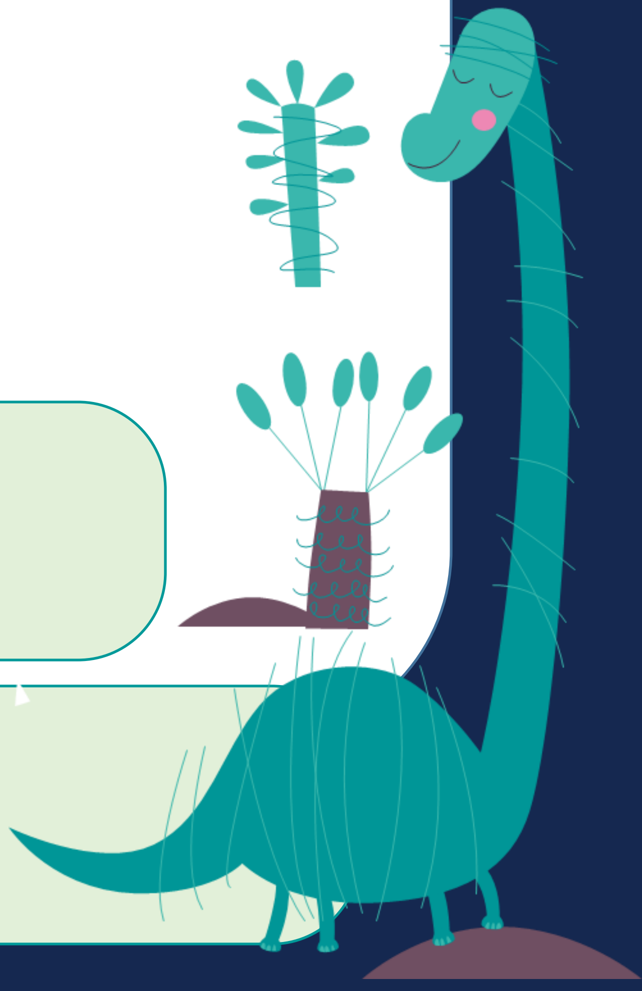
The price of and item decrease from 250$ to 200$. What was the per cent decrease?

$$m = \frac{new}{old} = \frac{200}{250} = \frac{20}{25} = \frac{80}{100} = 0.8$$

That's the multiplier for a **20% decrease**

Notice that price drop $50 is 1/5 of $250
That's a drop of 1/5, or 20%

The price of an item increase from $200 to $800. What was the per cent increase?



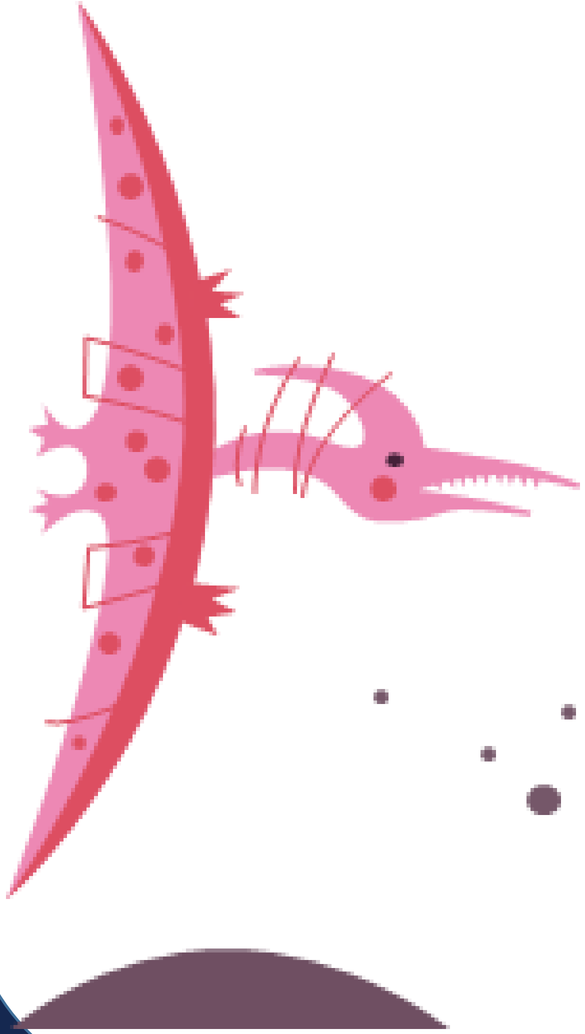
5.Sequential Per cent Changes

There are a few common oversights involving per cent increases and decrease, and the test loves to exploit these common mistakes.

Mistake #1 : an increase by same per cent do NOT get us back to the original starting point. These mistake center on scenarios in which two or more per cent changes follow in sequence (e.g. a% price increase and then a% employee discount)

Consider this question:

An item initially cost $100. At the beginning of the year, the price increase 30%. After the increase, an employee purchased it with a 30% discount. What price did the employee pay?



The mistake : +30% - 30% = 0

The answer is NOT 100$

We answer this using a product of multipliers

30% increase = 1.3 30% decrease = 0.7

$100 \times (1.3) \times (0.7) = \$91$

Here's another :

At beginning of the price of an item increase 30%. After the increase, an employee purchased it with 40% discount. The price the employee paid was what per cent below the original price?

The mistake : +30% - 40% = -10%

Use multipliers -> $(1.3) \times ((0.6) = 0.78$

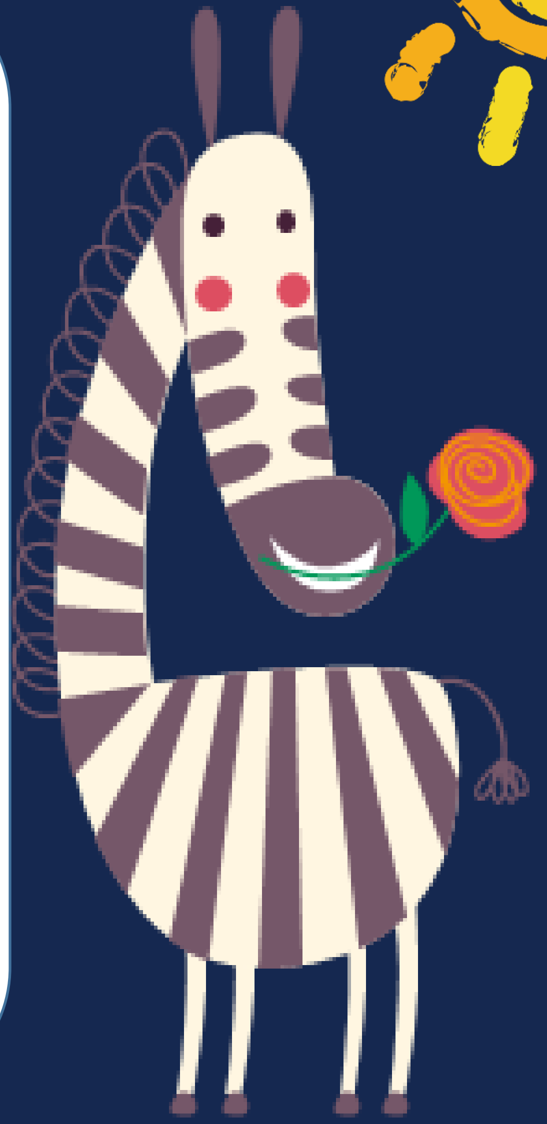
That is the multiplier for a ……………………

Here's another:
The price of stock increase 20% in January, dropped 50% in February, and increase 40% in March . Find the per cent for the three – month period.

The mistake : +20% - 50% + 40% = 10%

Use multiplier : ……………………………

That's the multiplier for a ……….. decrease



6. Simple and Compound Interest

Let's begin with simple interest is a mathematical fiction. Absolutely no one in the real world use simple interest. With simple interest payment is exactly the same each time

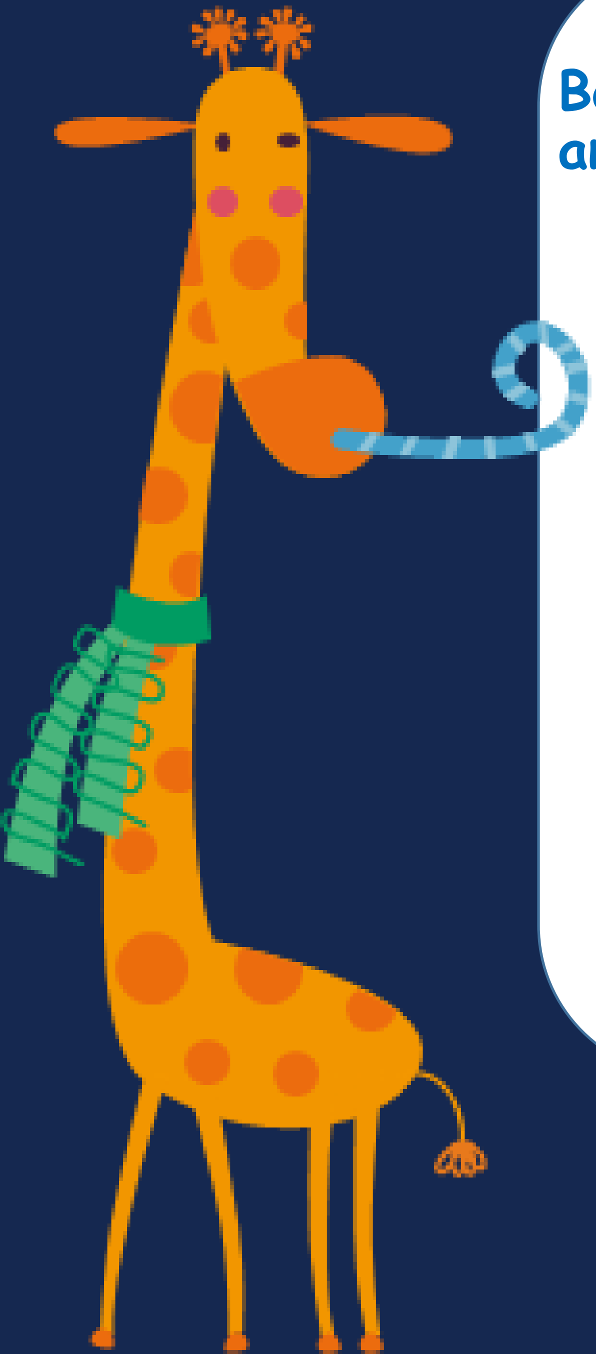
Bob deposits $1000 in an account that yield 5% simple interest annually.

5% of $1000 is $50

1 year : $1000 + $50 = $1050
2 years : $1050 + $50 = $1100
3 years : $1100 + $50 = $1150
4 years : $1150 + $50 = $1200
5 years : $1200 + $50 = $1250
6 years : $1250 + $50 = $1300
7 years : $1300 + $50 = $1350
8 years : $1350 + $50 = $1400

The big idea of compound interest is: interest on interest.

In compound interest, we get the per cent of interest paid on the total amount that has already increased in the account, principal + all previous interest payment. No two successive interest payments are ever the same. The entire amount experiences the same percentage increase in each period in each period



Bob deposits $1000 in an account that yield 5% compound interest annually

Start = $1000

1 year : $1000 x (1.05) = $1050
2 years : $1050 x (1.05) = $1102.5
3 years : $1102.5 x (1.05) = $1157.625
4 years : $1157.625 x (1.05) = $1215.50625

With large amounts of money and/or long periods of time, the difference successive interest payments becomes substantial.

Big idea #1: compound interest always outperforms simple interest, as long as there's more than one year (compounding period)

Notice also: in y years, the principal will be multiplied by the percent increase and multiplier y time.

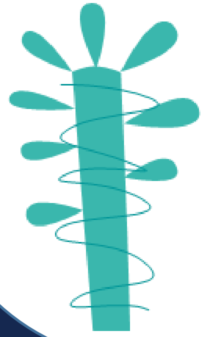
Let P be the principal and r be the multiplier.

The total amount in the account after y year is given by:

$$A = P(r^y)$$

If the annual interest rate is I, then the multiplier is simply

$$r = (1 + \frac{I}{100})$$



$$A = P(r^y) = P(1 + \frac{I}{100})^y$$

Sheila invests $4000 in an account that yield 6% compounding annually for 8 years. What is the total amount after 8 years?

Multiplier for a for 6% increase = ………

The account is multiplied by this multiplier eight times

A = …………

When we change the compound period. Bank always give an annual percent for interest, but they may compound quarterly, or even daily

For any compounding period smaller than a year, we need n, where n is the number of times that compounding period would occur in a year

Quarterly n = 4

Monthly n = 12

Daily n = 365

Suppose the bank pays 5% interest, compounding quarterly. The bank doesn't pay us 5% each quarter : that would be **unrealistically generous.** Instead, the bank pays

$\frac{5}{4}$ = 1.25% each quarter

Thus, in **y** years, there would be **n** compounding periods each year, or (**ny**) in **y** years

$$A = P(r^{y}) = P(1+\frac{I}{100n})^{ny}$$

If Susan invests $1000 in an account that yield 5% annual, compounding quarterly, then how much does she have after 6 years.

Quarterly percent = $\frac{5\%}{4}$ = 1.25% = 0.0125

Thus multiplier = 1.0125

The amount in the account experiences that percent increase 4 times each year, or 6*4 = 24 times in 6 times in 6 years.

$$A = 1000(1.0125^{24})$$

Estimate = simple interest = 50*6 + 1000 = $1300

The compound interest is slightly more than simple interest

How does the size of the compounding period determine the interest earned over time?

For this, we will need a large principal and a long amount of time. Let the annual interest rate is 15%, the principal is $1,000,000, and the time is 20 years. Simple interest: $2,000,000

Compounding annually:	$2,653,297.71
Compounding quarterly:	$2,701,484.9
Compounding monthly:	$2,712,640.29
Compounding daily:	$2,718,095.80
Compounding hourly:	$2,718,274.07

As compounding period decreases, the overall amount of interest earned increases.

7. Intro to Ratios

A ratio is a fraction that may compare **part-to-whole** or **part-to-part**

Suppose is a class the ratio of boys to girls is 3 to 4

$$\frac{\mathbf{3}}{\mathbf{4}}$$

3 → #of boys

4 → #of girls

This is one of the tricky things about ratio. The test will *ALWAYS* give you ratio in their simplest form.

15 boys and 20 girls or
300 boys and 400 girls

21 boys and 28 girls
or 3000 boys and 4000 girl

What is the absolute size of the group?

For some positive integer **n**, we have: (3n) boys and (4n) girls. n is sometimes called the **scale factor**

Notice, if we are given a ratio, 3 to 4, we have **no idea** about the absolute size of either group. That's a big idea, to which we will return.

There are different ways of presenting ratio information.

1. P to q form : "the ratio of boys to girls is 3 to 4."
2. Fraction form : "the ratio of boys to girls is 3/4."
3. Colon from : "the ratio of boys to girls is 3:4."
4. Idiom form : "for every 3 boys, there are 4 girls."

Notice : in all of these, order is important. If we talked about the ratio of "girls to boys", all the numbers would have to switch.

Use **fraction = fraction** to solve the major of ratio problems

An equation of this form is known as a **proportion**

In a class, the ratio of boys to girls is 5:8. If there are 40 girls, how many boys are there?

Let x is a numbers of boys

$$\therefore \frac{x}{40} = \frac{5}{8} \rightarrow x = 25$$

There are 25 boys in a class.

In a class the ratio of boys to girls is 3:7. If there are 32 more girls than boys. How many boys are there?

Approach #1 assign variables, B and G.
We know B/G = 3/7 and G-B = 32. Two equations with two unknown.

Approach #2

Scale factor is the magical link between ratio information and information about full quantities

If we have a ratio term for each part, we can figure out ratios to the whole.

Boys to girls is 3:5. Boys are what fraction of the whole?

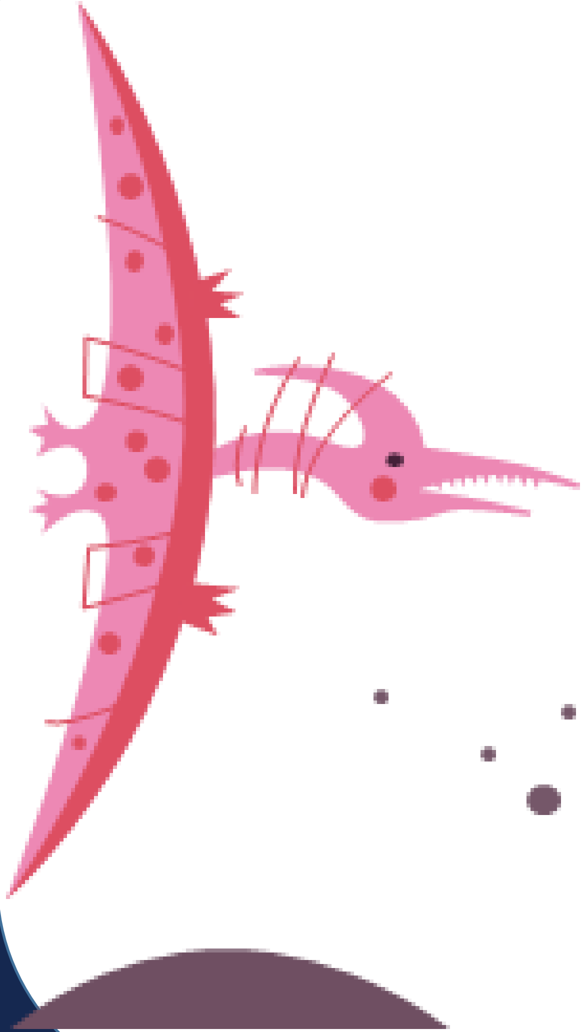
Boys are "three parts" of the class, and girls are "five parts" , so together. There are "eight parts." Thus, boy constitutes "there parts" of the total "eight parts" or **3/8**

This is sometimes called portioning.

Question

General propose concrete using a 1:2:3 ratio of cement to sand to gravel. If we have 150 kilograms of sand available, how many kilograms of concrete can we make. (assume we have more than enough cement and gravel)

8. Combining Ratios



Sometimes between different subgroups in a collection, a problem will give separately expressed ratios, and we will have to rerate these either to the whole or to absolute quantities.

Practice problem

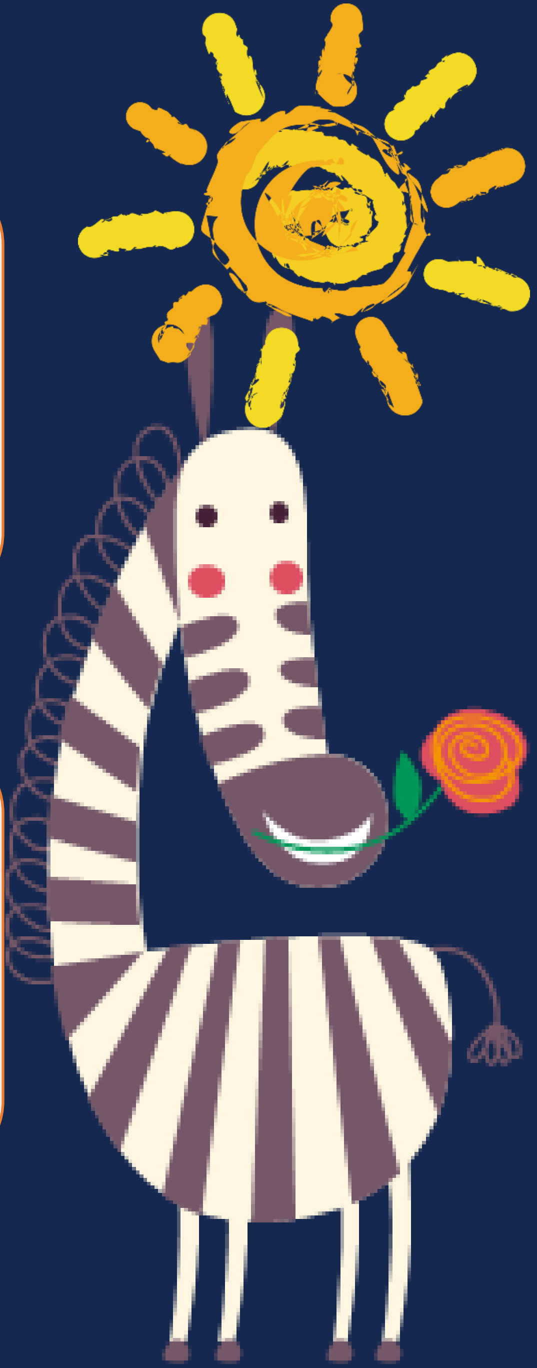
On a certain high school team, the ratio of sophomores to juniors is 2:3, and ratio of juniors to seniors is 5:6, Sophomores are what fraction of the whole team?

Strategy: identify the common element. Find equivalents of both ratios so that the common element is equal.

Problem

In a certain company, the ratio of directors to marketers is 3:8, and ratio of customer service reps(CSRs) to marketers is 2:3. if there are 27 directors, there are how many CSRs ?

Solution



Problem

One cup of butter is enough for 12 of Kathy's cookies, and cup of sugar is enough for 8 of her cookies. If she used five more cups of sugar than of butter, how many cookies did she make?

Solution

9.Ratios and Rates

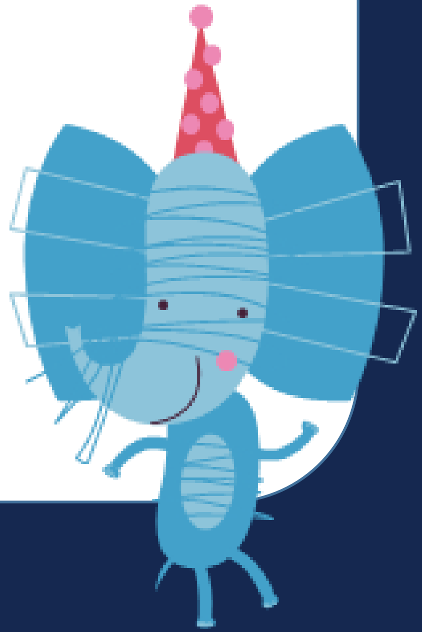
When you see problems with rates, remember you can set up proportions. Always remember to make sure that units of numerators and denominators match.

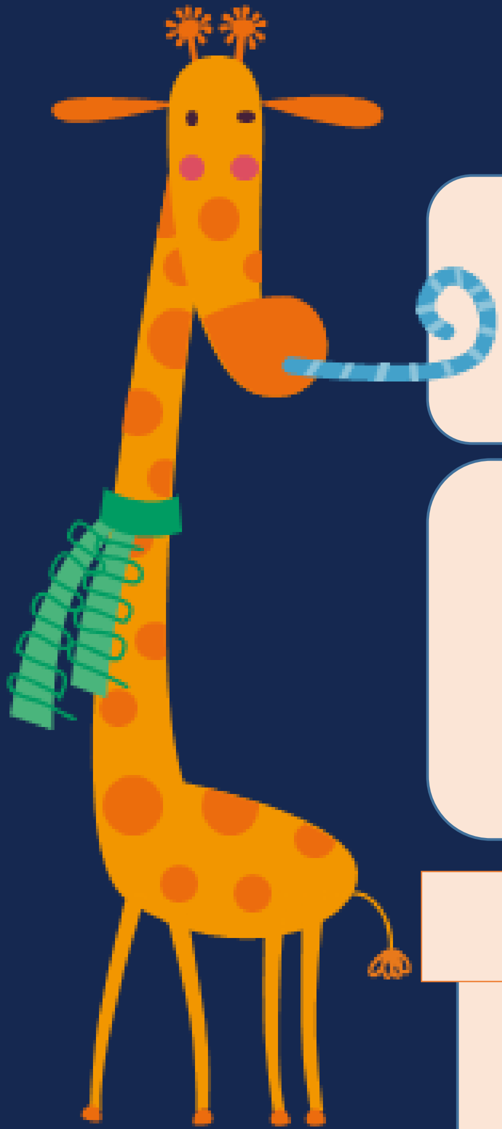
Rates are ratios. Any ratio with different units in the numerator and denominator is called a rate.

To solve most questions involving rates of all kind, all we have to do is to do is to set up an equation of the form

Ratio = ratio -> called a proportion

Rates are often expressed as so many (units) per (unit) such as 60 minutes/hour, 360°/ revolution





Problem

We would set this given rate equal to a fraction with the same units in the numerator and denominator.

A block of ice in a warm room is melting at a rate of 8 grams/hour. If at noon there are 30 grams of ice, when will the block first be entirely melted?

Solution

Problem

A bumblebee's wing flaps 1440 times in 8 second. How many times does it flap in a minute ?

THANKS