Chapter 3:
The Time Value of Money

- Definition of Time Value of Money
- The Importance of Time Value of Money
- Future Value
- Present Value

Definition of Time Value of Money

- The time value of money (TVM) is the concept that a sum of money is worth more now than same sum will be at a future date due to its earnings potential in the interim. The time value of money is a core principle of finance. A sum of money in the hand has greater value than the same sum to be paid in the future. The time value of money is also referred to as the present discounted value.

The Importance of Time Value of Money

- Investment decision
- Future Expected return

Future Value

- Future value (FV) is the value of a current asset at a future date based on an assumed rate of growth. The future value is important to investors and financial planners, as they use it estimate how much an investment made today will be worth in the future.

Future Value of a Single Amount

- Suppose you invest $100 at 5% interest, compounded annually. At the end of one year, your investment would be worth:

$$\$100 + .05(\$100) = \$105$$

or

$$\$100(1 + .05) = \$105$$

- During the second year, you would earn interest on $105. At the end of two years, your investment would be worth:

$$\$105(1 + .05) = \$110.25$$

Future Value of a Single Amount (cont.)

- In General Terms: $FV_1 = PV(1 + i)$

  and

  $FV_2 = FV_1(1 + i)$

- Substituting $PV(1 + i)$ in the first equation for FV_1 in the second equation:

  $$FV_2 = PV(1 + i)(1 + i) = PV(1 + i)^2$$

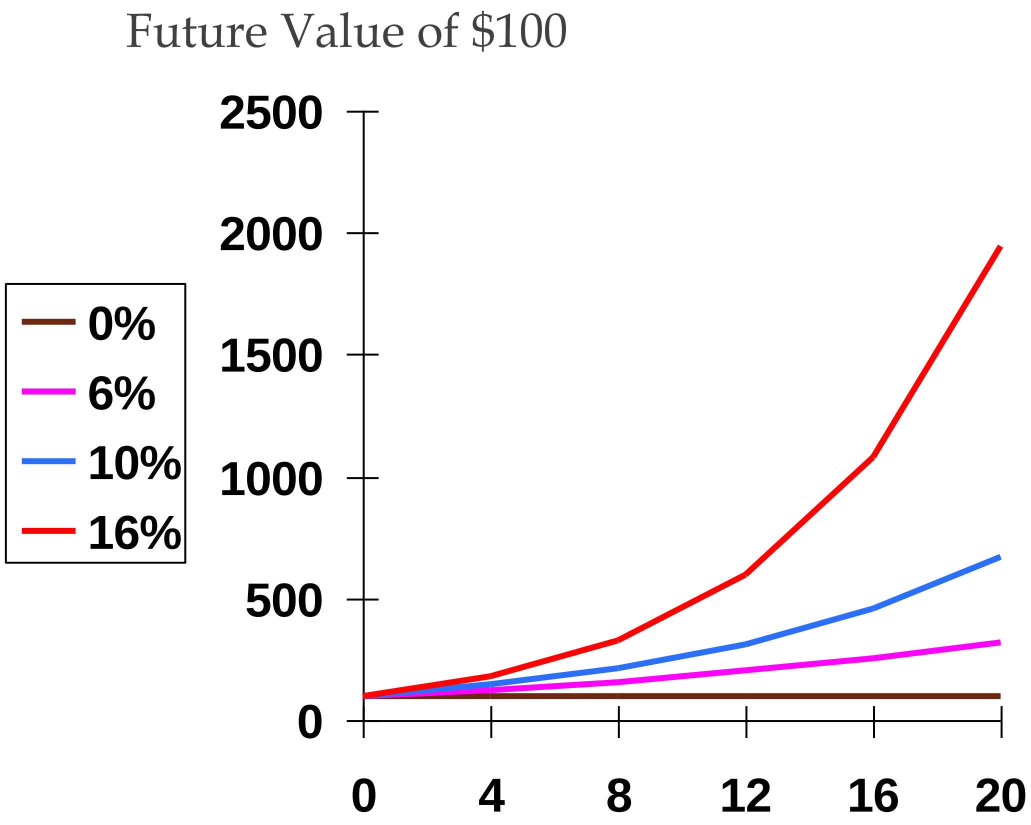
- For (n) Periods: $FV_n = PV(1 + i)^n$
- **Note**: $(1 + i)^n$ is the Future Value of $1 interest factor.
- Example: Invest $1,000 @ 7% for 18 years:

  $$FV_{18} = \$1{,}000(1.07)^{18} = \$1{,}000(3.380) = \$3{,}380$$

Future Value of a Single Amount (Spreadsheet Example)

- **FV(rate,nper,pmt,pv,type)**
- fv is the future value
- Rate is the interest rate per period
- Nper is the total number of periods
- Pmt is the annuity amount
- pv is the present value
- Type is 0 if cash flows occur at the end of the period
- Type is 1 if cash flows occur at the beginning of the period
- **Example**: = fv(7%,18,0,-1000,0) is equal to $3,379.93

Interest Rate, Time, and Future Value



Future Value of an Annuity

- **Ordinary Annuity**: A series of consecutive payments or receipts of equal amount at the end of each period for a specified number of periods.
- Example: Suppose you invest $100 at the end of each year for the next 3 years and earn 8% per year on your investments. How much would you be worth at the end of the 3rd year?

Future Value of an Annuity (cont.)

T_1	T_2	T_3
\$100	\$100	\$100

Compounds for 0 year:

$\$100(1.08)^0 = \$100.00$

Compounds for 1 year:

$\$100(1.08)^1 = \$108.00$

Compounds for 2 year:

$\$100(1.08)^2 = \$116.64$

Future Value of the Annuity \$324.64

Future Value of an Annuity (cont.)

$FV_3 = \$100(1.08)^2 + \$100(1.08)^1 + \$100(1.08)^0$

$= \$100[(1.08)^2 + (1.08)^1 + (1.08)^0]$

= $100[Future value of an annuity of $1 factor for i = 8% and n = 3.]

$= \$100(3.246)$

$= \$324.60$

FV of an annuity of $1 factor in general terms:

$\frac{(1+i)^n - 1}{i}$ (useful when using a non – financial calculator)

Future Value of an Annuity (Example)

- If you invest $1,000 at the end of each year for the next 12 years and earn 14% per year, how much would you have at the end of 12 years?

FV_{12} = $1000(27.271) given i =14% and n = 12

= $27,271

Future Value of an Annuity (Spreadsheet Example)

- **FV(rate,nper,pmt,pv,type)**
- fv is the future value
- Rate is the interest rate per period
- Nper is the total number of periods
- Pmt is the annuity amount
- pv is the present value
- Type is 0 if cash flows occur at the end of the period
- Type is 1 if cash flows occur at the beginning of the period
- **Example**: =fv(14%,12,-1000,0,0) is equal to $27,270.75

Present Value

- Present value (PV) is the current value of a future sum of money or stream of cash flows given a specified rate of return. Future cash flows are discounted at the discount rate, and the higher the discount rate, the lower the present value of the future cash flows. Determining the appropriate discount is the key to properly valuing future cash flows, whether they be earnings or debt obligations.

Present Value of a Single Amount

- Calculating present value (discounting) is simply the inverse of calculating future value (compounding):

$FV_n = PV(1+i)^n$ Compunding

$PV = \frac{FV_n}{(1+i)^n} = FV_n \left[\frac{1}{(1+i)^n}\right]$ Discounting

where: $\left[\frac{1}{(1+i)^n}\right]$ is the PV of $1 interest factor

Present Value of a Single Amount (An Example)

- How much would you be willing to pay today for the right to receive $1,000 five years from now, given you wish to earn 6% on your investment:

$$PV = \$1000\left[\frac{1}{(1.06)^5}\right]$$

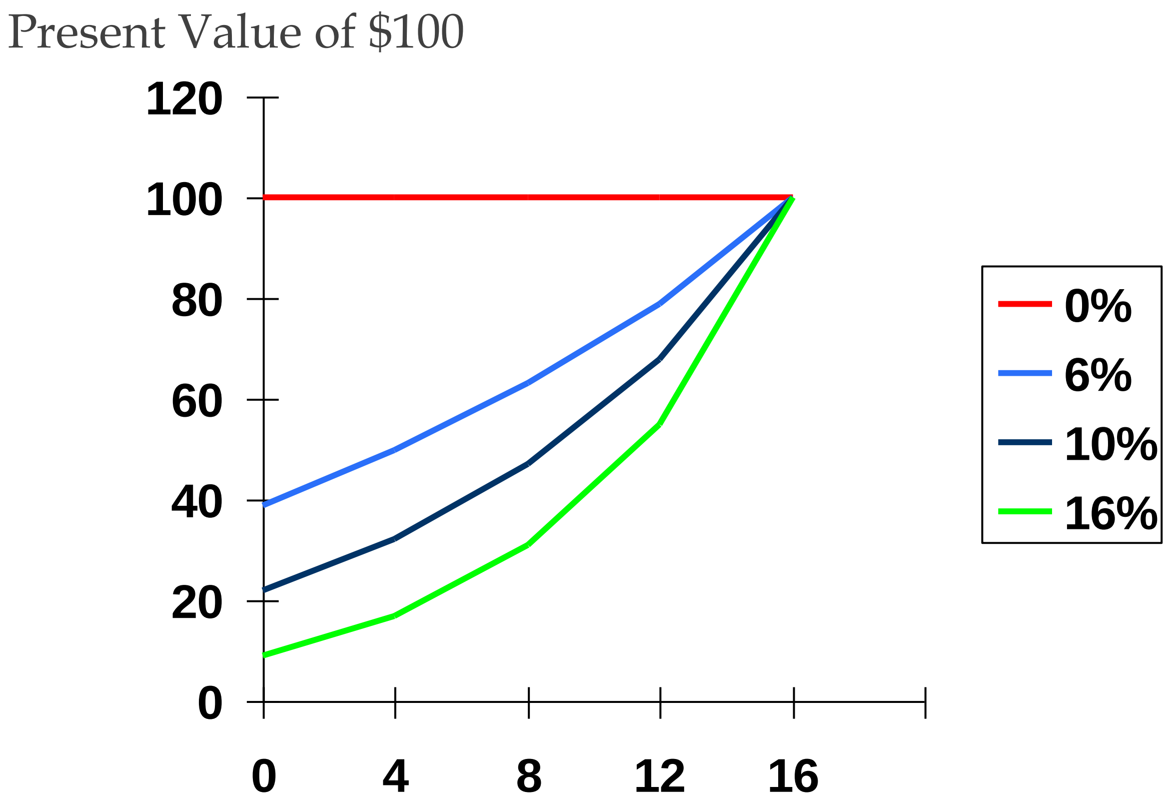
$$= \$1000(0.747)$$

$$= \$747$$

Present Value of a Single Amount (Spreadsheet Example)

- **PV(rate,nper,pmt,fv,type)**
- pv is the present value
- Rate is the interest rate per period
- Nper is the total number of periods
- Pmt is the annuity amount
- fv is the future value
- Type is 0 if cash flows occur at the end of the period
- Type is 1 if cash flows occur at the beginning of the period
- **Example**: =pv(6%,5,0,1000,0) is equal to -$747.26

Interest Rate, Time, and Present Value (PV of $100 to be received in 16 years)



Present Value of an Annuity

- Suppose you can invest in a project that will return $100 at the end of each year for the next 3 years. How much should you be willing to invest today, given you wish to earn an 8% annual rate of return on your investment?

Present Value of an Annuity (cont.)

T_0 T_1 T_2 T_3

$100 $100 $100

Discounted back 1 year:

$\$100[1/(1.08)^1] = \$92.59$

Discounted back 2 years:

$\$100[1/(1.08)^2] = \$85.73$

Discounted back 3 years:

$\$100[1/(1.08)^3] = \underline{\$79.38}$

PV of the Annuity = $257.70

Present Value of an Annuity (cont.)

PV = $\$100[1/(1.08)^1]+\$100[1/(1.08)^2]+\$100[1/1.08)^3]$

= $\$100[1/(1.08)^1+1/(1.08)^2+1/1.08)^3]$

= $100 [Present value of an annuity of $1 factor for i = 8% and n = 3]

= $100(2.577)

= $257.70

PV of an annuity of $1 factor in general terms:

$\frac{1-\frac{1}{(1-i)^n}}{i}$ (useful with non – financial calculator)

Present Value of an Annuity (An Example)

- Suppose you won a state lottery in the amount of $10,000,000 to be paid in 20 equal annual payments commencing at the end of next year. What is the present value (ignoring taxes) of this annuity if the discount rate is 9%?

PV = $500,000(9.129) given I = 9% and n = 20

= $4,564,500

Present Value of an Annuity (Spreadsheet Example)

- **PV(rate,nper,pmt,fv,type)**
- pv is the present value
- Rate is the interest rate per period
- Nper is the total number of periods
- Pmt is the annuity amount
- fv is the future value
- Type is 0 if cash flows occur at the end of the period
- Type is 1 if cash flows occur at the beginning of the period
- **Example**: =pv(9%,20,-500000,0,0) is equal to $4,564,272.83

Summary of Compounding and Discounting Equations

- In each of the equations above:
  - Future Value of a Single Amount
  - Future Value of an Annuity
  - Present Value of a Single Amount
  - Present Value of an Annuity

there are four variables (interest rate, number of periods, and two cash flow amounts). Given any three of these variables, you can solve for the fourth.