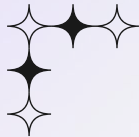


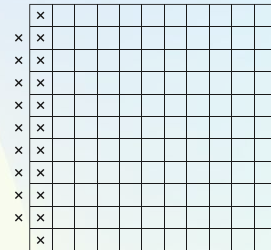
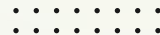
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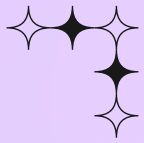
First-Order Ordinary Differential Equations

Part 1: Separable Form



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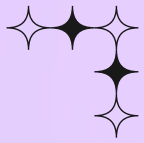




There are 4 types of the first order ODE

- 1. Variables separable form**
- 2. Homogenous form**
- 3. Exact form**
- 4. Linear form**





Characteristic of Separable ODE

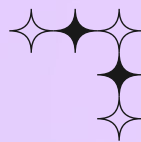
A separable ODE is an ODE that can be written as

$$f(x)dx = g(y)dy$$

- $f(x)$ is a function of x
- $g(y)$ is a function of y

Remark 1 In this form, the variables x and y are separable.





Arrangement of Separable ODE

Remark 2 Normally, general ODEs are not in the form $f(x)dx = g(y)dy$. Hence, one have to reformulate the ODE in the first step of solving th ODE

Example 1 Reformulate the following ODE to find the functions $f(x)$ และ $g(y)$

$$(1 + y)^2 dx = \frac{1}{2x} dy$$

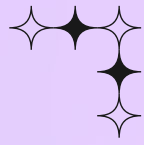
Solution

$$f(x) = 2x, \quad g(y) = (1 + y)^{-2}$$

Technique

Move the term connecting with dx to the left hand side and move dy of the right.





rrangement of Separable ODE

Remark 3 Normally, general ODEs are in the form of y' . Hence, we have to replace y' with dy/dx and then reformulate the equation.

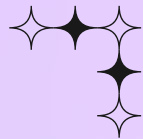
Example 2 Reformulate the following ODE to find the functions $f(x)$ and $g(y)$

Solution

$$y' = \frac{x}{y}$$

$$y' = \frac{x}{y} \implies \frac{dy}{dx} = \frac{x}{y}$$





Solution of Separable ODE

- For ODE equation in the form

$$f(x)dx = g(y)dy$$

We can find the solution by integrating both sides of equation

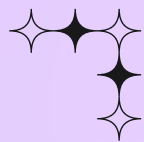
$$\int f(x)dx = \int g(y)dy + c$$

where c is an arbitrary constant.

- When the integrals are obtained, we will reformulate

$$y = \dots\dots$$





Solution of Separable ODE

Example 3 Find a general solution of

$$(1 + y)^2 dx = \frac{1}{2x} dy$$

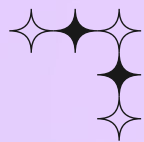
Solution From Example 1, we have

$$f(x)dx = g(y)dy \quad \text{where} \quad f(x) = \frac{1}{2x}, \quad g(y) = (1 + y)^{-2}$$

Hence,

$$\int f(x)dx = \int g(y)dy =$$





Solution of Separable ODE

Example 3 Find a general solution of

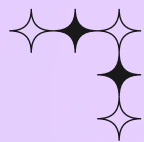
$$(1 + y)^2 dx = \frac{1}{2x} dy$$

Solution (con.) Hence, from

$$\int f(x) dx = \int g(y) dy + c ,$$

we have...





Solution of Separable ODE

Example 4 Find a general solution of

$$y' = \frac{y}{x}$$

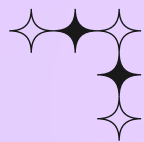
Solution From Example 2, we obtain

$$f(x)dx = g(y)dy \quad \text{where } f(x) = \frac{1}{x}, \quad g(y) = y$$

Hence,

$$\int f(x)dx = \int g(y)dy =$$





Solution of Separable ODE

Example 4 Find a general solution of

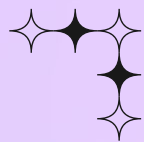
$$y' = \frac{y}{x}$$

Solution (con.) Hence it follows from

$$\int f(x)dx = \int g(y)dy + c$$

that





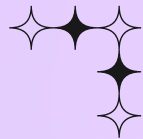
Solution of Separable ODE

Example 5 Find a general solution of $y' = \frac{y^2 + 1}{x^2}$

Solution In the first step, reformulate the ODE in the form

$$f(x)dx = g(y)dy$$





Solution of Separable ODE

Example 5 Find a general solution of $y' = \frac{y^2 + 1}{x^2}$

Solution (con.) From $f(x) =$ and $g(y) =$

$$\int f(x)dx = \int g(y)dy =$$

Hence, $=$ + c





References

Adkins, William A., and Mark G. Davidson Ordinary differential equations. Springer Science & Business Media, 2012.

