แบบฝึกหัด 5.1

จงหาอินทิกรัลของแต่ละข้อต่อไปนี้

1. $\int (2x+3)\,dx$
2. $\int (x^2-\sqrt{x})\,dx$
3. $\int (3x-1)^{234}\,dx$
4. $\int (2-7t)^{2/3}\,dt$
5. $\int \sqrt{2+5y}\,dy$
6. $\int \frac{dx}{(3x+2)^2}$
7. $\int \frac{3r}{\sqrt{1-r^2}}\,dr$
8. $\int \sqrt{2x^2+1}\,x\,dx$
9. $\int t^2(1+t^3)^{\frac{-2}{3}}\,dt$
10. $\int \frac{y}{\sqrt{2y^2+1}}\,dy$
11. $\int \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$
12. $\int \frac{z+1}{\sqrt[3]{z^2+2z+2}}\,dz$
13. $\int \frac{x^2+2x}{\sqrt[3]{x^3+3x^2+1}}\,dx$
14. $\int (x^2+1)^3\,x^3\,dx$
15. $\int \frac{\sin 3\theta}{\sqrt{1+\cos 3\theta}}\,d\theta$
16. $\int \frac{\cos 3x}{(1+\sin 3x)^5}\,dx$
17. $\int \left(\frac{4x^2-2\sqrt{x}}{x}\right)dx$
18. $\int \frac{x^2}{x+2}\,dx$
19. $\int \frac{dx}{x+1}$
20. $\int \frac{x}{3-x^2}\,dx$
21. $\int \frac{x}{(3-x^2)^2}\,dx$
22. $\int \left(\frac{1}{x-a}-\frac{1}{x-b}\right)dx$
23. $\int x\left(\frac{1}{x^2-a^2}-\frac{1}{x^2-b^2}\right)dx$
24. $\int \frac{\sqrt{x}}{1+x\sqrt{x}}\,dx$

25. $\int e^{ax+b}\,dx$

26. $\int xe^{-x^2}\,dx$

27. $\int \frac{e^{1/x}}{x^2}\,dx$

28. $\int (e^{-x}-1)^2\,dx$

29. $\int \frac{4}{\sqrt{e^x}}\,dx$

30. $\int \frac{e^x}{\sqrt{e^x+1}}\,dx$

31. $\int \frac{2e^x}{\sqrt[3]{e^x+1}}\,dx$

32. $\int \frac{e^{2x^2}}{e^{2x}+3}\,dx$

33. $\int \frac{xe^{ax^2}}{e^{ax^2}+1}\,dx$

34. $\int \frac{e^{2x}-1}{e^{2x}+3}\,dx$

35. $\int \frac{\ln x^3+\ln^3 x}{x}\,dx$

36. $\int \frac{x^2\ln(4+x^3)}{4+x^3}\,dx$

37. $\int \frac{\ln\tan x}{\sin 2x}\,dx$

38. $\int \frac{\sec^3 x\tan x}{(1+\sec^3 x)^2}\,dx$

39. $\int \frac{\sin x\, e^{\sec x}}{\cos^2 x\,(e^a+e^{\sec x})}\,dx$

40. $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$

41. $\int (e^x-x^e)\,dx$

42. $\int \frac{e^x\cos ec^2 e^x}{\cot e^x-1}\,dx$

43. $\int \frac{\cot x}{\ln(\sin x)}\,dx$

44. $\int 5^{\frac{\ln y}{y}}\left(\frac{1-\ln y}{y^2}\right)dy$

45. $\int \frac{\sin x}{1+\cos x}\,dx$

46. $\int \frac{\cos x}{\sqrt{1-\sin x}}\,dx$

47. $\int \frac{\cos 2x}{2-\sin 2x}\,dx$

48. $\int \frac{\sec^2 x}{1-\tan x}\,dx$

คำตอบแบบฝึกหัด 5.1

1. x^2+3x+c
2. $\frac{x^3}{3}-\frac{2}{3}x^{3/2}+c$
3. $\frac{(3x-1)^{235}}{705}+c$
4. $\frac{-3}{35}(2-7t)^{5/3}+c$
5. $\frac{2}{5}(2+5y)^{3/2}+c$
6. $\frac{-1}{9x+6}+c$
7. $-3\sqrt{1-r^2}+c$
8. $\frac{(2x^2+1)^{3/2}}{6}+c$
9. $\frac{1}{2}(1+2t^3)^{1/3}+c$
10. $\frac{1}{2}\sqrt{2y^2+1}+c$
11. $\frac{2}{3}x^{3/2}+2x^{1/2}+c$
12. $\frac{3}{4}(z^2+2z+2)^{\frac{2}{3}}+c$
13. $\frac{1}{2}(x^3+3x^2+1)^{2/3}+c$
14. $\frac{1}{40}(x^2+1)^4(4x^2-1)+c$
15. $\frac{-2}{3}\sqrt{1+\cos 3\theta}+c$
16. $\frac{-1}{12(1+\sin 3x)^4}+c$
17. $2x^2-4\sqrt{x}+c$
18. $\frac{x^2}{2}-2x+4\ln|x+2|+c$
19. $\ln|x+1|+c$
20. $\frac{-1}{2}\ln|3-x^2|+c$
21. $\frac{1}{2(3-x^2)}+c$
22. $\ln\left|\frac{x-a}{x-b}\right|+c$
23. $\frac{1}{2}\ln\left|\frac{x^2-a^2}{x^2-b^2}\right|+c$
24. $\frac{2}{3}\ln|1+x\sqrt{x}|+c$
25. $\frac{1}{a}e^{ax+b}+c$
26. $\frac{-1}{2}e^{-x^2}+c$
27. $-e^{1/x}+c$
28. $\frac{-1}{2}e^{-2x}+2e^{-x}+x+c$
29. $-8e^{\frac{-x}{2}}+c$
30. $2\sqrt{e^x+1}+c$
31. $3(e^x+1)^{\frac{2}{3}}+c$
32. $\frac{1}{2}\ln(e^{2x}+3)+c$
33. $\frac{1}{2a}\ln(e^{ax^2}+1)+c$
34. $\ln(e^{2x}+3)^{2/3}-\frac{1}{3}x+c$
35. $\frac{3}{2}\ln^2 x+\frac{1}{4}\ln^4 x+c$
36. $\frac{1}{6}\ln^2(4+x^3)+c$
37. $\frac{1}{4}\ln^2|\tan x|+c$
38. $\frac{-1}{3(1+\sec^3 x)}+c$
39. $\ln(e^a+e^{\sec x})+c$
40. $\ln|e^x+e^{-x}|+c$
41. $e^x-\frac{x^{e+1}}{e+1}+c$
42. $-\ln|\cot e^x-1|+c$
43. $\ln|\ln(\sin x)|+c$
44. $\frac{5^{\frac{\ln y}{y}}}{\ln 5}+c$
45. $-\ln|1+\cos x|+c$
46. $-2\sqrt{1-\sin x}+c$
47. $\frac{-1}{2}\ln|1-\sin 2x|+c$
48. $-\ln|1+\tan x|+c$

แบบฝึกหัด 5.2

จงหาอินทิกรัลของแต่ละข้อต่อไปนี้

1. $\int \cos(3x-1)\,dx$
2. $\int 3\sin^2 x\cos x\,dx$
3. $\int \sec 2x\tan 2x\,dx$
4. $\int e^{-\sin x}\cos x\,dx$
5. $\int x^{\frac{-1}{2}}\sin x^{\frac{1}{2}}\,dx$
6. $\int \cos^2 \pi x\sin \pi x\,dx$
7. $\int x^2\sin(a^3x^3+b^3)\,dx$
8. $\int \tan^5 x\sec^2 x\,dx$
9. $\int \frac{\csc^2(3-2x)}{\cot^4(3-2x)}\,dx$
10. $\int \frac{\cos^3 x}{1-\sin x}\,dx$
11. $\int \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{8}{3}} x}\,dx$
12. $\int \frac{dx}{1+\sec x}$
13. $\int \frac{x\tan\sqrt{x^2+1}}{\sqrt{x^2+1}}\,dx$
14. $\int x\left(\sec^2 x^2\right)\left(\tan^3 x^2\right)dx$
15. $\int \frac{\cot 3x}{\sin 3x}\,dx$
16. $\int \tan 3x\sqrt{\sec^2 3x+\sec^3 3x}\,dx$
17. $\int \frac{dx}{1+\cos x}$
18. $\int \frac{\sqrt{5+2\tan\theta}}{\cos^2\theta}\,d\theta$

คำตอบแบบฝึกหัด 5.2

1. $\frac{1}{3}\sin(3x-1)+c$
2. $\sin^3 x+c$
3. $\frac{1}{2}\sec 2x+c$
4. $-e^{-\sin x}+c$
5. $-2\cos x^{\frac{1}{2}}+c$
6. $\frac{-1}{3\pi}\cos^3 \pi x+c$
7. $\frac{-1}{3a^2}\cos(a^3x^3+b^3)+c$
8. $\frac{1}{6}\tan^6 x+c$
9. $\frac{-1}{6}\tan^3(3-2x)+c$
10. $\sin x+\frac{1}{2}\sin^2 x+c$
11. $\frac{-3}{5}\cot^{5} x^{3}+c$
12. $x+\cot x-\csc x+c$
13. $\ln\left|\sec\sqrt{x^2+1}\right|+c$
14. $\frac{1}{8}\tan^4 x^2+c$
15. $\frac{-1}{3}\csc 3x+c$
16. $\frac{2}{9}(1+\sec 3x)^{3/2}+c$, $\sec 3x>0$
17. $\csc x-\cot x+c$
18. $\frac{(5+2\tan x)^{3/2}}{3}+c$

แบบฝึกหัด 5.3

จงหาอินทิกรัลของแต่ละข้อต่อไปนี้

1. $\int \frac{dx}{5+x^2}$
2. $\int \frac{1}{1+4x^2}\,dx$
3. $\int \frac{dx}{x^2+2x+5}$
4. $\int \frac{dx}{4x^2+4x+2}$
5. $\int \frac{dr}{r^2-2r-3}$
6. $\int \frac{dx}{3x^2+4x-7}$
7. $\int \frac{dy}{3-2y-y^2}$
8. $\int \frac{1+2x}{1+x^2}\,dx$
9. $\int \frac{2x-5}{3x^2-2}\,dx$
10. $\int \frac{\cos\theta}{4-\sin^2\theta}\,d\theta$
11. $\int \frac{2e^x}{1-e^{2x}}\,dx$
12. $\int \frac{2x^4-x^2}{2x^2+1}\,dx$
13. $\int \frac{\sin 8x}{9+\sin^4 4x}\,dx$
14. $\int \frac{dx}{(e^x+e^{-x})^2-1}\,dx$
15. $\int \frac{x^2}{9x^6-3x^3-1}\,dx$
16. $\int \frac{5x-6}{2x^2-4x+3}\,dx$

คำตอบแบบฝึกหัด 5.3

1. $\frac{\sqrt{5}}{5}\arctan\left(\frac{\sqrt{5}}{5}x\right)+c$
2. $\frac{1}{2}\arctan 2x+c$
3. $\frac{1}{2}\arctan\left(\frac{x+1}{2}\right)+c$
4. $\frac{1}{2}\arctan(2x+1)+c$
5. $\frac{1}{4}\ln\frac{r-3}{r+1}+c$
6. $\frac{1}{10}\ln\left|\frac{3x-3}{3x+7}\right|+c$
7. $\frac{1}{4}\ln\left|\frac{3+y}{1-y}\right|+c$
8. $\arctan x+\ln|1+x^2|+c$
9. $\frac{1}{3}\ln|3x^2-2|-\frac{5\sqrt{6}}{12}\ln\left|\frac{\sqrt{3}x-\sqrt{2}}{\sqrt{3}x+\sqrt{2}}\right|+c$
10. $\frac{1}{4}\ln\left|\frac{2+\sin\theta}{2-\sin\theta}\right|+c$
11. $\ln\left|\frac{1+e^x}{1-e^x}\right|+c$
12. $\frac{1}{3}x^3-x+\frac{\sqrt{2}}{2}\arctan(\sqrt{2}\,x)+c$
13. $\frac{1}{12}\arctan\left(\frac{\sin^2 4x}{3}\right)+c$
14. $\frac{1}{\sqrt{3}}\arctan\left(\frac{2e^{ex}+1}{\sqrt{3}}\right)+c$
15. $\frac{1}{9\sqrt{5}}\ln\left|\frac{6x^3-1-\sqrt{5}}{6x^3-1+\sqrt{5}}\right|+c$
16. $\frac{5}{4}\ln|2x^2-4x+3|-\frac{\sqrt{2}}{2}\arctan[\sqrt{2}(x-1)]+c$

แบบฝึกหัด 5.4

จงหาอินทิกรัลของแต่ละข้อต่อไปนี้

1. $\int \frac{dx}{\sqrt{4x^2-25}}$
2. $\int \frac{dx}{\sqrt{x^2-4x+13}}$
3. $\int \frac{dx}{\sqrt{4-(x-1)^2}}$
4. $\int \frac{2}{\sqrt{2+x-x^2}}\,dx$
5. $\int \frac{x+1}{\sqrt{4-x^2}}\,dx$
6. $\int \frac{5-4x}{\sqrt{12x-4x^2-8}}\,dx$
7. $\int \frac{x}{\sqrt{27+6x-x^2}}\,dx$
8. $\int \frac{4x+5}{\sqrt{3x-x^2}}\,dx$
9. $\int \sqrt{1-4x^2}\,dx$
10. $\int \sqrt{16-9x^2}\,dx$
11. $\int \sqrt{4+25x^2}\,dx$
12. $\int \sqrt{9x^2-1}\,dx$
13. $\int \sqrt{2x-x^2}\,dx$
14. $\int \sqrt{10-4x+4x^2}\,dx$
15. $\int \sqrt{5-4x-x^2}$
16. $\int \sqrt{5+2x+x^2}\,dx$
17. $\int \frac{dx}{x\sqrt{4-9\ln^2 x}}$
18. $\int \sqrt{x^2-8x}\,dx$

คำตอบแบบฝึกหัด 5.4

1. $\frac{1}{2}\ln\left|2x+\sqrt{4x^2-25}\right|+c$
2. $\ln\left|x-2+\sqrt{x^2-4x+13}\right|+c$
3. $\arcsin\frac{x-1}{2}+c$
4. $2\arcsin\frac{2x-1}{3}+c$
5. $-\sqrt{4-x^2}+\arcsin\frac{x}{2}+c$
6. $\sqrt{12x-4x^2-8}-\frac{1}{2}\arcsin(2x-3)+c$
7. $-\sqrt{27+6x-x^2}+3\arcsin\frac{x-3}{6}+c$
8. $-4\sqrt{3x-x^2}+11\arcsin\frac{2x-3}{3}+c$
9. $\frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\arcsin 2x+c$
10. $\frac{x}{2}\sqrt{16-9x^2}+\frac{8}{3}\arcsin\frac{3x}{4}+c$
11. $\frac{x}{2}\sqrt{4+25x^2}+\frac{2}{5}\ln\left|5x+\sqrt{4+25x^2}\right|+c$
12. $\frac{x}{2}\sqrt{9x^2-1}-\frac{1}{6}\ln\left|3x+\sqrt{9x^2-1}\right|+c$
13. $\frac{x-1}{2}\sqrt{2x-x^2}+\frac{1}{2}\arcsin(x-1)+c$
14. $\frac{2x-1}{4}\sqrt{10-4x+4x^2}+\frac{9}{4}\ln\left|2x-1+\sqrt{10-4x+4x^2}\right|+c$
15. $\frac{x+2}{2}\sqrt{5-4x-x^2}+\frac{9}{2}\arcsin\frac{x+2}{3}+c$
16. $\frac{x+1}{2}\sqrt{5+2x+x^2}+2\ln\left|x+1+\sqrt{5+2x+x^2}\right|+c$
17. $\frac{1}{3}\arcsin\left(\frac{3}{2}\ln x\right)+c$
18. $\frac{x-4}{2}\sqrt{x^2-8x}-8\ln\left|x-4+\sqrt{x^2-8x}\right|+c$

แบบฝึกหัด 5.5

1. $\int xe^{-x}\,dx$
2. $\int x^2e^{-2x}\,dx$
3. $\int xe^{3x}\,dx$
4. $\int x\sin 2x\,dx$
5. $\int x^2e^{x}\,dx$
6. $\int x\cos 3x\,dx$
7. $\int x^2\cos x\,dx$
8. $\int x^2\sin x\,dx$
9. $\int \sqrt{x}\ln x\,dx$
10. $\int \ln^2 x\,dx$
11. $\int \frac{\ln x}{\sqrt{x}}\,dx$
12. $\int \ln(2x+3)dx$
13. $\int \ln(x^2+4)dx$
14. $\int \arcsin x\,dx$
15. $\int \arccos 2x\,dx$
16. $\int \arctan 2x\,dx$
17. $\int x\arctan 2x\,dx$
18. $\int e^{x}\sin x\,dx$
19. $\int e^{-3t}\sin 3t\,dt$
20. $\int e^{2t}\cos 2t\,dt$
21. $\int \sin(\ln x)dx$
22. $\int \cos(\ln x)dx$
23. $\int x\sec^2 x\,dx$
24. $\int x\tan^2 x\,dx$
25. $\int x^3e^{x^2}\,dx$

เฉลยแบบฝึกหัด 5.5

1. $-xe^{-x}-e^{-x}+C$
2. $-\frac{1}{2}x^2e^{-2x}-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}+C$
3. $\frac{1}{3}xe^{3x}-\frac{1}{9}e^{3x}+C$
4. $\frac{1}{4}\sin 2x-\frac{1}{2}x\cos 2x+C$
5. $x^2e^x-2xe^x+2e^x+C$
6. $\frac{1}{9}\cos 3x+\frac{1}{3}x\sin 3x+C$
7. $x^2\sin x-2\sin x+2x\cos x+C$
8. $-x^2\cos x+2\cos x+2x\sin x+C$
9. $\frac{2}{3}x^{3/2}\ln x-\frac{4x^{3/2}}{9}+C$
10. $\ln^2(x)x-2\ln(x)x+2x+C$
11. $2\sqrt{x}\ln x-4\sqrt{x}+C$
12. $\frac{1}{2}\ln(2x+3)(2x+3)-x-\frac{3}{2}+C$
13. $x\ln\left(x^2+4\right)-2x+4\arctan\left(\frac{x}{2}\right)+C$
14. $x\arcsin x+\sqrt{1-x^2}+C$
15. $x\arccos 2x-\frac{\sqrt{1-4x^2}}{2}+C$
16. $x\arctan 2x-\frac{1}{4}\ln\left(1+4x^2\right)+C$
17. $\frac{1}{2}x^2\arctan 2x-\frac{x}{4}+\frac{1}{8}\arctan 2x+C$
18. $-\frac{1}{2}e^x+\frac{1}{2}e^x\sin x+C$
19. $-\frac{1}{6}e^{-3t}\cos 3t-\frac{1}{6}e^{-3t}\sin 3t+C$
20. $\frac{1}{4}e^{2t}\cos 2t+\frac{1}{4}e^{2t}\sin 2t+C$
21. $-\frac{1}{2}\cos(\ln x)x+\frac{1}{2}\sin(\ln x)x+C$
22. $\frac{1}{2}\cos(\ln x)x+\frac{1}{2}\sin(\ln x)x+C$
23. $x\tan x+\ln(\cos x)+C$
24. $-\frac{x^2}{2}+x\tan x-\frac{1}{2}\ln\left(1+\tan^2 x\right)+C$
25. $\frac{1}{2}x^2e^{x^2}-\frac{1}{2}e^{x^2}+C$