Variable Step Size Complex Direct Frequency Estimation Algorithm

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Abstract

A variable step size complex direct frequency estimation (VSCDFE) algorithm for linear prediction for a complex sinusoidal frequency estimation is proposed in this paper. The step size of a conventional CDFE algorithm is adjusted so as to improve the convergence properties by using an accumulative gradient technique. Simulation results have been drawn to demonstrate the validation of the proposed VSCDFE.

Keywords: Adaptive algorithm, Gradient Algorithm, Variable Step size, Direct Frequency estimation

1. Introduction

A complex sinusoidal frequency estimation based on a linear predictor using the CDFE adaptive algorithm [1] is very attractive due to low complexity and simplicity of implementation. Frequency estimation is an interesting problem and is found in applications in many fields, such as communications [2], [3], smart grids [4], radar [5], and so on. There are many studies concerning frequency estimation, but we will only be focusing on the CDFE [1] in this article. As has been shown in Fig. 8 in [1], the CDFE is worse than the modified complex plain gradient (MCPG) [6] in terms of convergence speed; the convergence properties of the CDFE must be further improved. The CDFE sets step size as a constant value, making it impossible to control the convergence properties. This problem is therefore the motivation for this work.

To improve the convergence properties of the CDFE, we present a technique to adapt the value of the step size. In the early state of adaptation, it is set to have a high value, and after convergence, it is set to have a low value. By doing this, the proposed VSCDFE will converge faster and provide more accurate solutions.

2. Proposed Technique

It is assumed that a single-tone sinusoidal signal is of the form

$$x_n = Ae^{j(\Omega_0 n+\phi)} + v_n,\ \ n = 0,1,... \tag{1}$$

where A, Ω_0, and ϕ are, respectively, the signal amplitude, frequency and phase. For the sake of simplicity, A and Ω_0 are treaded as the unknown constants and θ is uniformly distributed over $[-\pi,\pi]$. $v_n = v_{r,n} + jv_{i,n}$ is a complex zero mean Gaussian noise whose variance of σ^2. $v_{r,n}$ and $v_{i,n}$ are, respectively, real and imaginary parts of the noise whose identical variance of $\sigma^2/2$. An input signal to noise ratio is calculated by $SNR = A^2/\sigma^2$. To estimate Ω_0 of the observation x_n, a linear predictor [1] is utilized. The linear prediction error signal is given by

$$e_n = x_n - e^{j\hat{\Omega}_0} x_{n-1} \tag{2}$$

where $\hat{\Omega}_0$ is an estimate of Ω_0 and to be adjusted by the CDFE algorithm (see [1], Eqs. (3) - (11)), which can be described as follows:

$$\hat{\Omega}_{0,n+1} = \hat{\Omega}_{0,n} + \mu g_n \tag{3}$$

$$g_n = \mathrm{Re}\{e_n s_n^*\} \tag{4}$$

$$s_n = jx_{n-1}e^{j\hat{\Omega}_0} \tag{5}$$

where Re{x} is the real value of x, the asterisk '*' means the complex conjugation, and μ is a step size whose value is fixed at some constant.

In this work, the step size μ is adjusted based on accumulative gradient signal, which can be described by

$$\mu_{n+1} = \alpha\mu_n + (1-\alpha)p_n^2 \tag{6}$$

$$p_n = \alpha p_{n-1} + (1-\alpha)g_n, \tag{7}$$

where $0 << \alpha < 1$ is the tuning parameter and g_n is known as a gradient signal. Eq. (7) is an accumulative value of the gradient signal g_n which is used to adjust the step size in Eq. (6). Ultimately, the proposed VSCDFE is derived as

$$\hat{\Omega}_{0,n+1} = \hat{\Omega}_{0,n} + \mu_n g_n \tag{8}$$

In the next section, the performance of the proposed VSCDFE will be demonstrated.

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3. Numerical Results

In this section, the performances of the proposed VSCDFE are demonstrated and compared with those of CDFE and MCPG. Fig. 1 shows the averaged values of the proposed time-varying step size parameter for 0 and 10 dB of SNR. The parameters setting herein include $\Omega_0 = 0.3\pi$, $\phi \in [-\pi, \pi]$, $\mu(0) = 0$, $\hat{\Omega}_0(0) = 0$, $\alpha = 0.93$, data length L=500 and 1000 computer runs. It is seen from Fig. 1 that at the early stage of adaptation, the step size is high. After converging, its value is low as desired. Therefore, it ensures that the convergence of the CDFE is improved. In Figs. 2 and 3, the proposed and comparative algorithms are assigned to estimate the input signal frequency under the following conditions: SNR ={0, 10} dB, $\phi \in [-\pi, \pi]$, $\Omega_0 = 0.3\pi$, $\hat{\Omega}_0(0) = 0$, $\mu(0) = 0$, @0dB $\mu_{cdfe} = 0.02$, $\mu_{mcpg} = 0.025$ @10dB, $\mu_{cdfe} = 0.025$, $\mu_{mcpg} = 0.0125$ data length L=500 and 1000 computer runs. The step size μ of the CDFE and MCPG are individually adjusted to reach the same simulated mean square error (MSE) as those of the proposed VSCDFE. Note that, a general form of the MSE is defined by

$$\text{MSE} = \text{var}(\hat{\Omega}_0) + \text{bias}^2 \tag{9}$$

where $\text{var}(\hat{\Omega}_0)$ is the variance of $\hat{\Omega}_0(n)$, bias $= \Omega_0$ - $E[\hat{\Omega}_0(n)]$. To obtain the estimated MSE, the 1000 independent runs of each algorithm are ensemble averaged. The results are shown in Figs. 2 and 3 for 0 and 10 of SNR, respectively. It is seen that at a low SNR scenario (0 dB), the performance of the VSCDFE and CDFE is almost the same, whereas the speed of the MCPG is better than the counterparts. This is not surprise because the filter structure of MCPG is IIR but they are FIR for VSCDFE and CDFE. However, for a high value of SNR (10 dB) as shown in Fig. 3, the convergence speed of the proposed VSCDFE is better than those of the CDFE and MCPG. This is due to the effect of the proposed time-varying step size proposed in Eqs. (6) and (7). Note for the proposed VSCDFE, to obtain the desired performance, the user can tune the parameter α independently. The recommended range is $\alpha \in [0.9, 0.95]$.

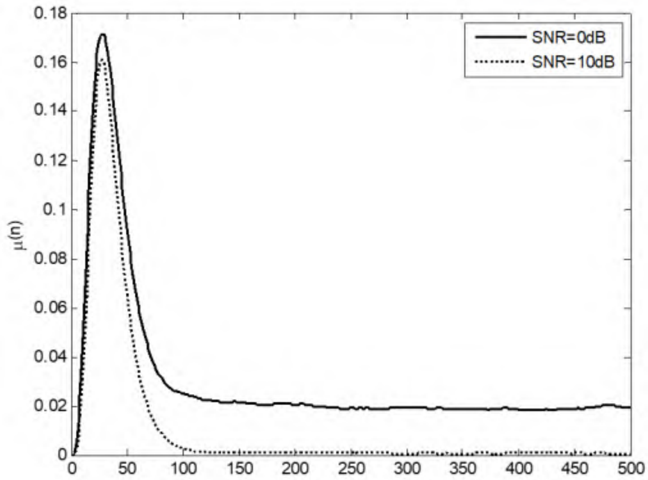

Fig. 1. Proposed time-varying step size parameter for 0 and 10 dB of SNR.

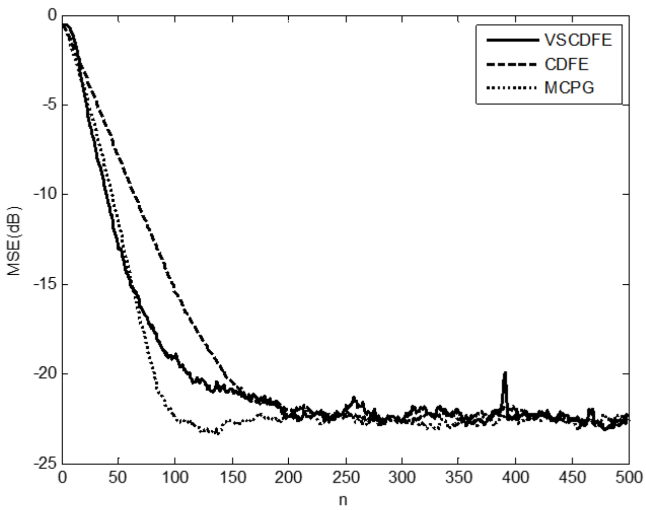

Fig. 2. Convergence speed comparison at the same value of MSE for 0 dB of SNR.

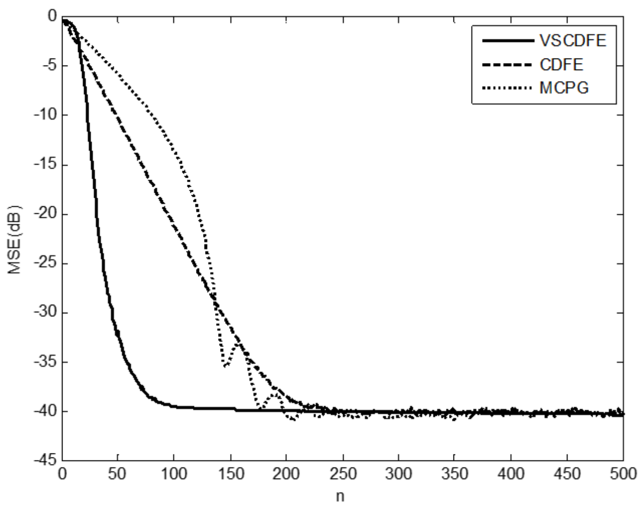

Fig. 3. Convergence speed comparison at the same value of MSE for 10 dB of SNR.

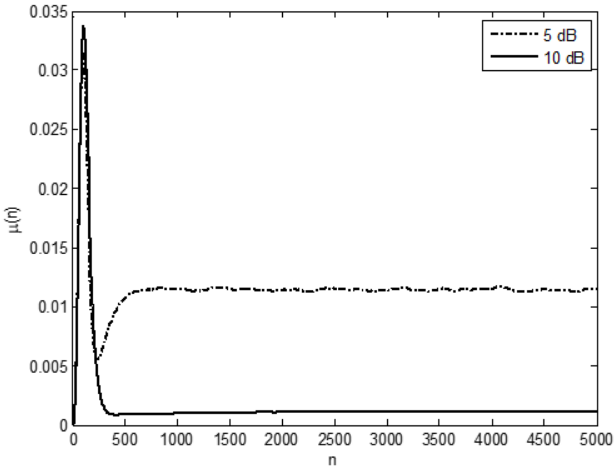

Fig. 4. Proposed time varying step-size parameter for 5 and 10 dB of SNR

4. Conclusions

A VSCDFE adaptive algorithm for a complex linear predictor for a complex single-tone frequency estimation is proposed. As compared to the conventional CDFE and MCPG, VSCDFE shows better convergence speed in a high SNR value scenario. Steady state analysis of the proposed VSCDFE in an interesting topic and will be done in the further work.

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