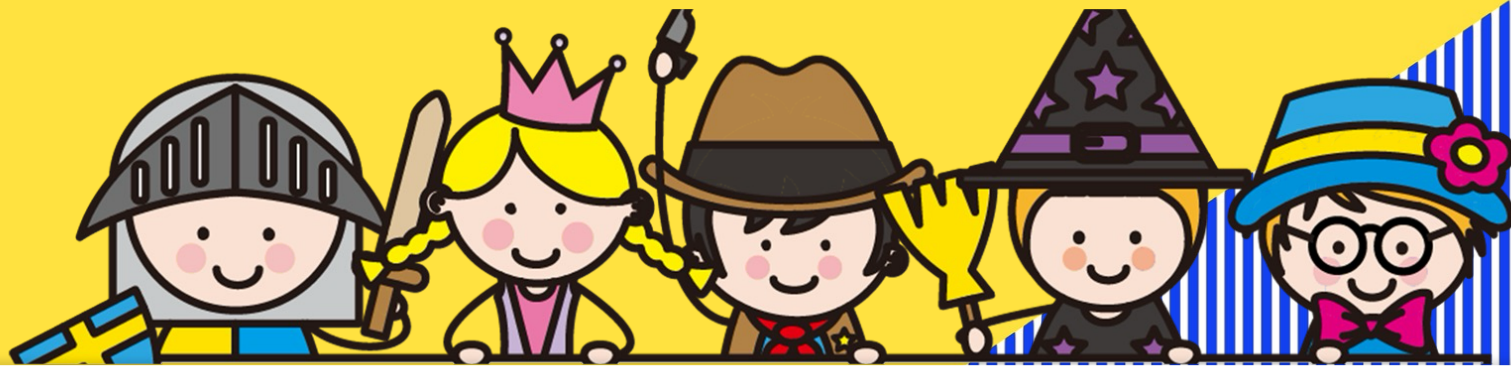




Chapter 5

Word Problems

By Wariya Puttapatimok



สัปดาห์ 3/09/68

แบ่ง 053 จำนวน 040

เทอม 052

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34 90%

สัปดาห์

Outlines

1

Word Problems

กำหนดตัวแปร

2

Assigning Variables

3

Writing Equations

4

Number of Variables

เชื่อมกัน

5

Age Questions

อายุปัจจุบัน / อดีต



Outlines

6

Intro to Motion Questions

การเคลื่อนที่

$$D = RT$$

Dirt

7

Average Speed

ความเร็วเฉลี่ย

A → B → C

8

Multiple Traveler Questions

การเดินทางหลายต่อ

↓ → ← ↓ →

9

Shrinking and Expanding Gaps

การเคลื่อนที่

↓ หด

↓ ขยาย

10

Work Questions

* ทำงานร่วมกัน

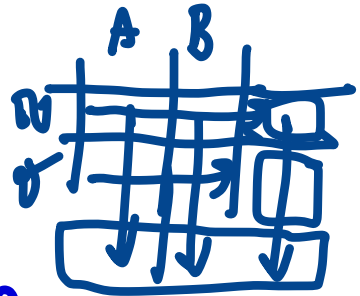
$$D = RT$$

$$A = R \downarrow T$$

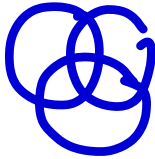
อัตราการทำงาน

Outlines

11 ^{เติบโต} Growth and Decay ^{เลือก 1 ของ A หรือ B ที่ดีกว่า}

~~12 ~~Sets and Venn Diagrams~~~~ 

~~13~~¹² Double Matrix Method ^{ตาราง}

~~14 ~~Three Criteria Venn Diagrams~~~~ 

~~15 ~~Intro to Sequences~~~~ ^{ลำดับ}



Outlines

~~16 Arithmetic Sequence~~ ลำดับเลขคณิต

~~17 Reclusive Sequence~~

~~18 Inclusive Sequence~~

~~19 Sum of Sequence~~

ลำดับ

~~20 Consecutive Integers~~ → จำนวนเต็มที่อยู่ติดกัน



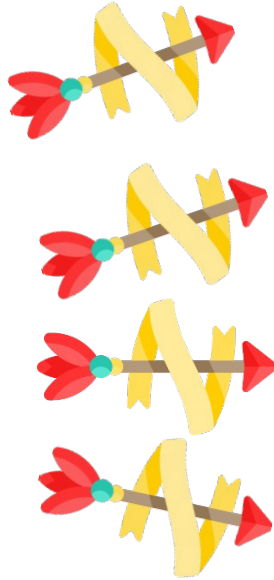
1

Word Problems





Word Problems



age questions

motion questions

work questions

growth & decay questions



Typically, questions state the information in words and do not provide variables. ตัวแปรไม่มาให้

We have to

1) assign variables

and

2) write equations

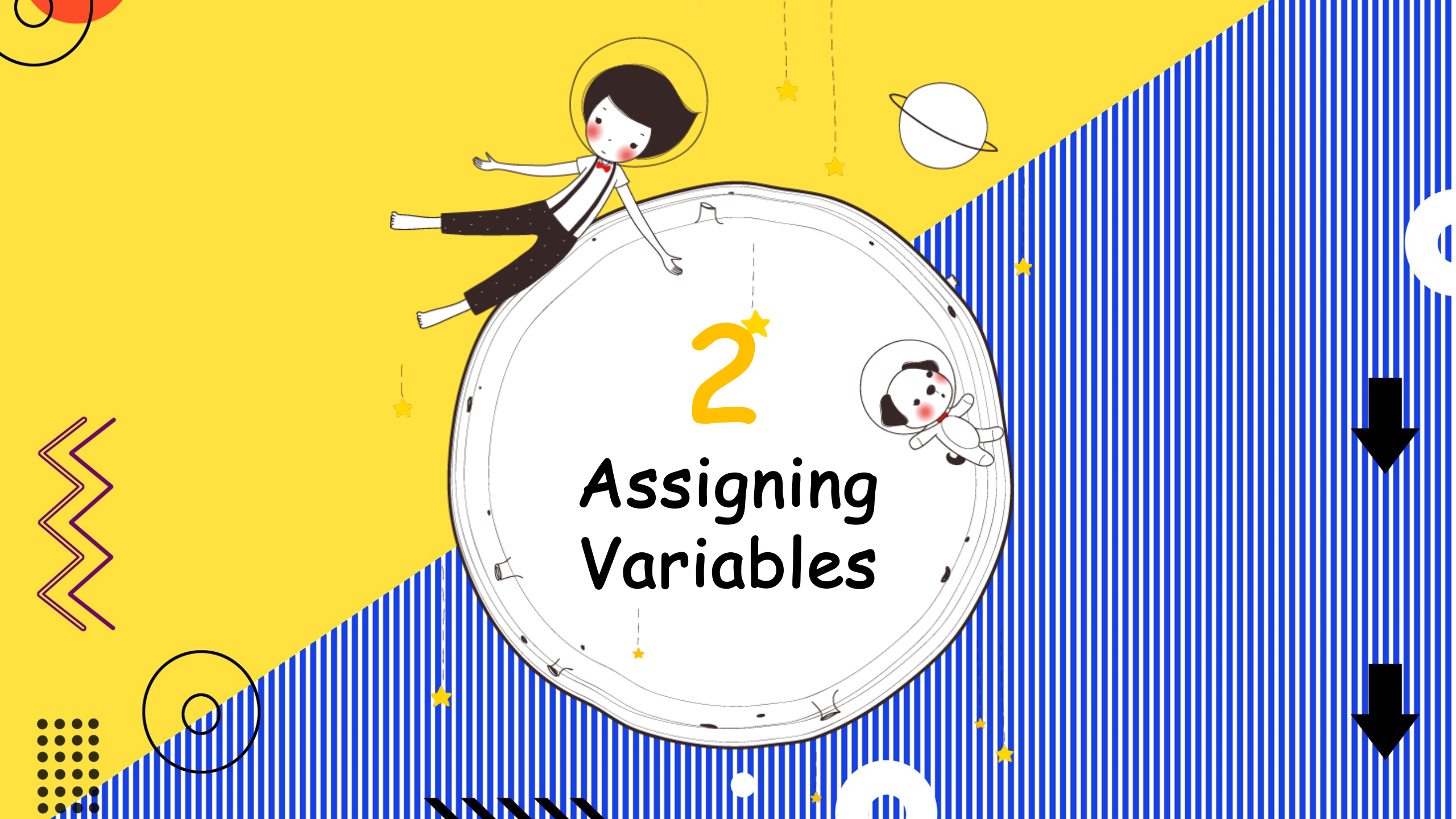
เขียนสมการ

กำหนดตัวแปร

We often have to consider whether to use just one variable or more than one.

2

Assigning Variables



To solve a word problem algebraically, we need to **introduce variables**.

The very first thing I will say is this: be thoughtful in the choice of letter for your variable.



x, y



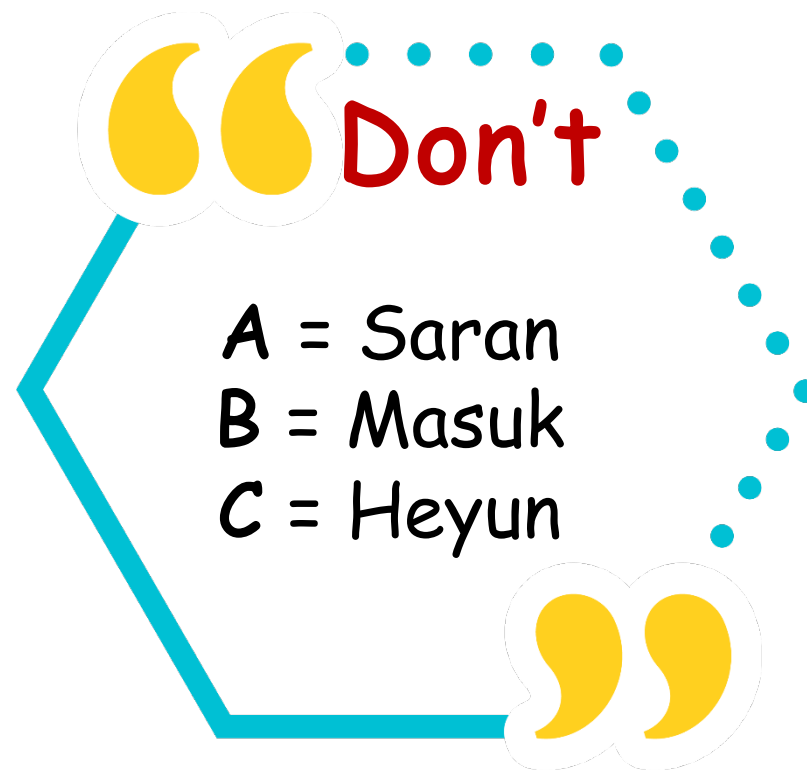
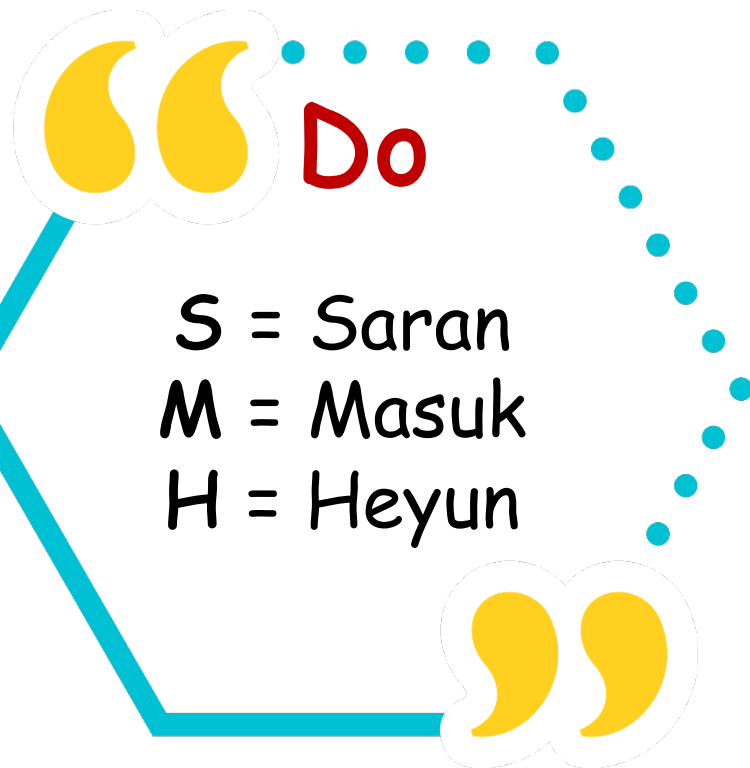
Don't pick plain old x .



Beginning of a problem:

Each week, Saran makes \$500 more than Masuk, and Heyun makes \$700 more than Masuk. If the sum of their weekly salaries is ...

If you pick x for the salary of one of the women, then you may correctly solve for it, but when you get that number for x , how will you know whether it answers the question?



Other considerations in choosing variables:

1) It may be helpful to assign a variable for the smallest value, and then express everything in terms of that.

2) It may be helpful to assign a variable for the target value, what the question is asking.

3) If all the quantities in the problem relate to a single quantity, make this conceptually central quantity the variable.

กำหนดตัวแปรที่เล็กที่สุดก่อน

ตัวแปรอื่น

ค่าเป้าหมาย

โจทย์ถาม?

กำหนดเป็นตัวแปร

ที่เกี่ยวข้อง

ตัวแปร

นั้น

ที่เล็ก

ที่สุด

ปริมาณ

Best

central quantity = Fine
 $\Rightarrow F$

Practice problem

Sam's car was fined for parking when he gave Joe and Peter a ride, so they decided to help Sam pay the fine. Joe paid \$3 more than 1/4 of the fine and Peter paid \$3 less than 1/3 of the fine, leaving Sam \$4 less than 1/2 the fine to complete the payment. How much did Sam pay?

$$J = \frac{1}{4}F + 3$$

$$P = \frac{1}{3}F - 3$$

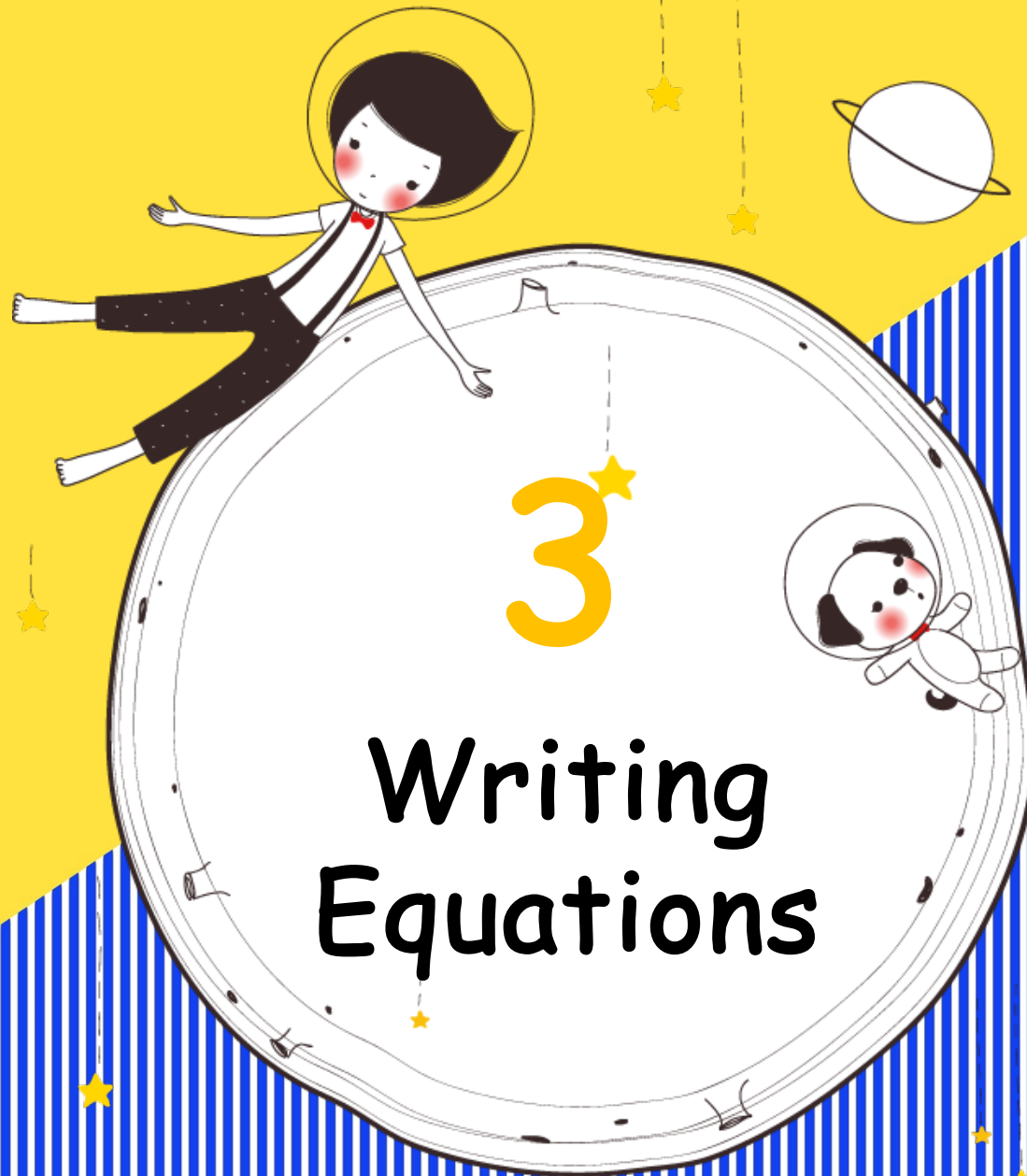
$$S = \frac{1}{2}F - 4$$

$$F = J + P + S = \frac{1}{4}F + 3 + \frac{1}{3}F - 3 + \frac{1}{2}F - 4$$

$$F = \frac{13}{12}F - 4$$

$$F = 48 \rightarrow S = \frac{1}{2}F - 4 = \frac{1}{2}(48) - 4 = 20$$

$\therefore$ Sam paid the fine \$20



Writing Equations



Translating from Words to Math

- 1) The words "is/are" means equal sign. $=$
- 2) The word "of" means multiply. $\times$
- 3) The words "more than" or "greater than" mean addition. $+$ $+$
- 4) The word "less than" means subtraction. $-$



The words "is" or "are" in the text correspond to the equals sign. =

If we have "A is 50 more than B": $A = B + 50$
 $A - B = 50$

If we have "A is 50 less than B":

$$A = B - 50$$

A ខ្ពស់ជាង B ៥០



$$A =$$

$$3B - 100$$

Suppose we have "Twice A is 100 less than three times B":

$$2A = 3B - 100$$

$$= 50\% (x) B$$

Suppose we have "A is 50% of B":

$$A = 0.5B$$

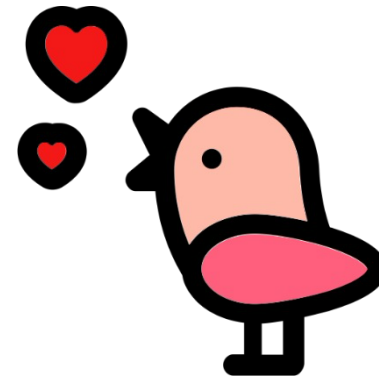


$$2A = B \quad \checkmark$$

A ၁၀၀% B ၏ ၅၀%

Suppose we have "A is 50% greater than B":

$$A = B + \frac{50}{100} B = 1.5 B \rightarrow A = 1.5 B \quad \checkmark$$





Alice and Bruce each bought a refrigerator, and the sum of their purchases was \$900. If twice of what Alice paid was \$75 more than what Bruce paid, what did Alice pay for her refrigerator? $A = 9$

$$A + B = 900 \text{ --- (1)}$$

$$2A = B + 75$$

$$B = 2A - 75 \text{ --- (2)}$$

$$\text{sub B in (1)}$$

$$A + (2A - 75) = 900$$

$$3A = 900 + 75$$

$$A = \frac{975}{3} = 325$$

$\therefore$ Alice paid \$325 for her refrigerator.

Sale





HW

In a certain company, 50% of Ed's salary is 20% of Ruth's salary. If the difference between their salaries is \$90,000, what is Ruth's salary?

→ 150,000

sale



A very simple word problem:

Frank has \$13 more dollars than Glenda does, and together they have \$81. How much does Frank have? $F = G + 13$

Approach #1: two variables



$$F + G = 81$$

A very simple word problem:

Frank has \$13 more dollars than Glenda does, and together they have \$81. How much does Frank have? $(G + 13) + (G) = 81$

Approach #2: one variable





$$M = L + 3$$

L = central variable

or ตัวแปรที่มีค่าที่คงตัว

A relatively simple word problem:

$$P = 3L - 3$$

Mark is 8 years older than Lisa, and Peter's age is 3 years less than three times Lisa's age. If the sum of their ages is 80, what is Peter's age?

$$L + (L + 3) + (3L - 3) = 80$$

$$5L + 6 = 80$$

$$L = 15$$

$$\text{from } P = 3L - 3$$

$$= 3(15) - 3$$

$$= 42$$

∴ Peter's age is 42 years old

✘

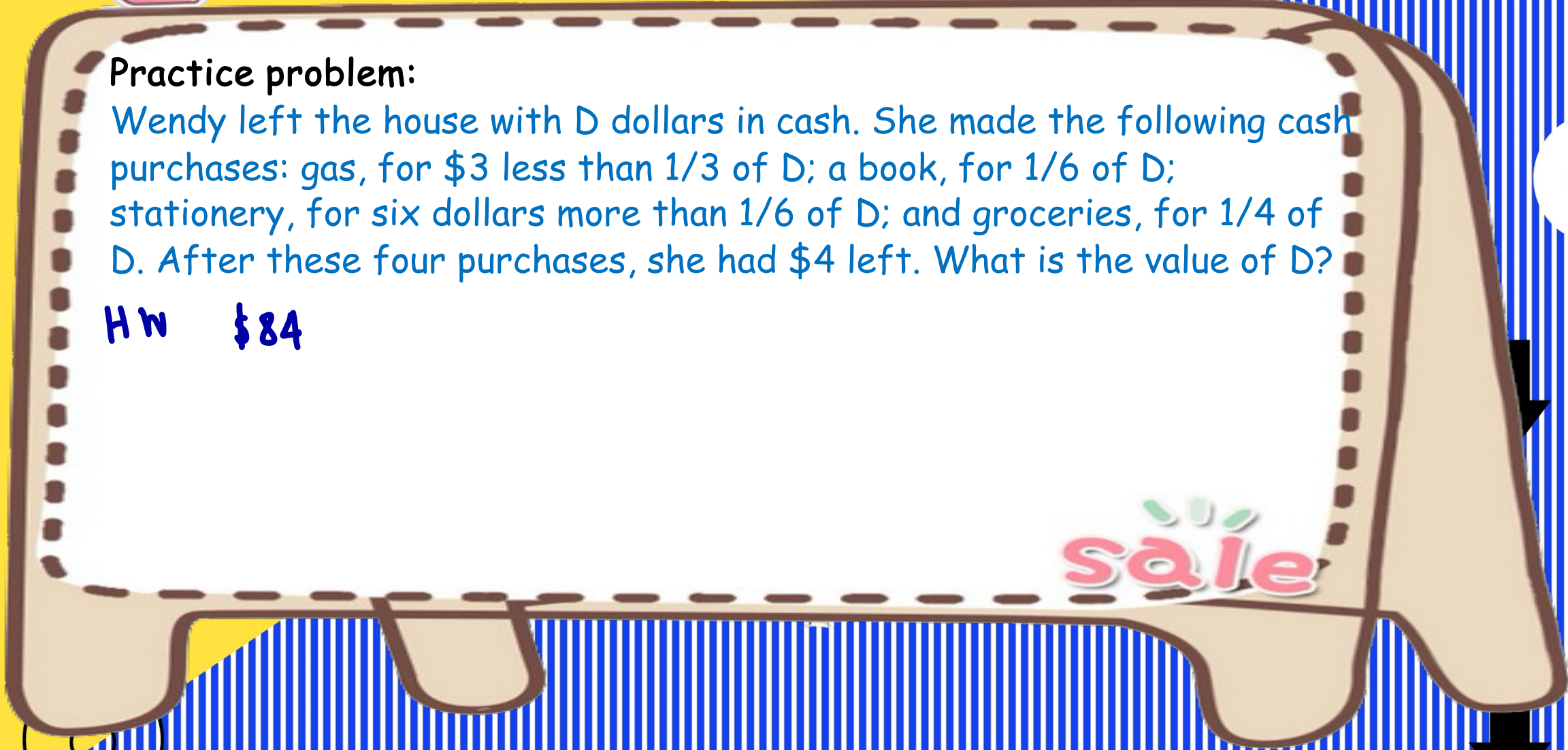
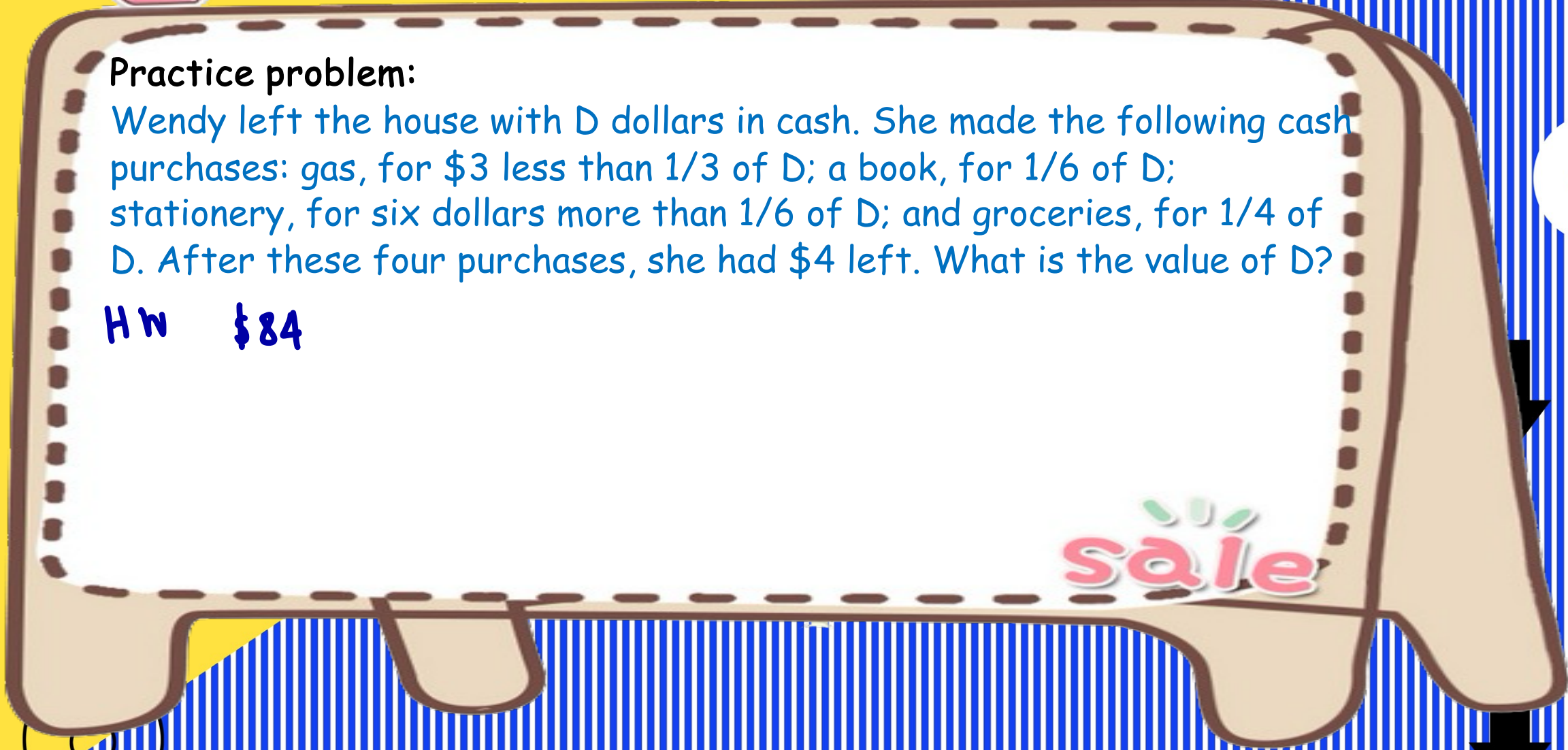
Sale

Practice problem:

Wendy left the house with D dollars in cash. She made the following cash purchases: gas, for \$3 less than $\frac{1}{3}$ of D ; a book, for $\frac{1}{6}$ of D ; stationery, for six dollars more than $\frac{1}{6}$ of D ; and groceries, for $\frac{1}{4}$ of D . After these four purchases, she had \$4 left. What is the value of D ?

HW \$84

Sale





One category of word problems concerns people of different ages.

อายุที่ต่างกัน



What's tricky about these questions is that mathematical relationships can be specified at two different times.

ความสัมพันธ์ของ 2 ช่วงเวลา



Practice problem:

Right now, Steve's age is half of Tom's age. In eight years, twice Tom's age will be five more than three times Steve's age. How old is Tom right now?

$$\checkmark \checkmark 2(T+8) = 3(S+8) + 5 \checkmark \checkmark$$

The big mistake associated with this problem is thinking of the ages as fixed numbers.

✓ The first equation is $S = (1/2)*T$, but the second equation is NOT

$$2T = 3S + 5. \text{ ผิดเพราะ เวลา 1}$$

✗

เป็น เวลา 1 ปี

เช่น 1 ปี



We will use these expressions to create equations that correspond to statements about age relationships in the future or past.

... and in seven years, Edgar will be twice as old as Frieda ...

$$(E + 7) = 2*(F + 7)$$



It is usually to pick the **variable** to represent the **age right now**.

~~*~~ *	Age five years ago	Age right now	Age seven years later
Frieda	$F - 5$	F	$F + 7$
Jannie (Jannie is twice as old as Frieda)	$2(F - 5)$	$2F$	$2(F + 7)$

~~*~~ *





Practice problem:

Right now, Steve's age is half of Tom's age. In eight years, twice Tom's age will be five more than three times Steve's age. How old is Tom right now? $\rightarrow T = ?$

$$S = T_{/2} \text{ --- ①}$$

$$2(T+8) = 3(S+8) + 5 \text{ --- ②}$$

แทน S ใน ② ให้อ

$$2T + 16 = 3\left(\frac{T}{2} + 8\right) + 5$$

$$2T + 16 = \frac{3T}{2} + 24 + 5$$

$$2T - \frac{3T}{2} = 29 - 16$$

$$\frac{T}{2} = 13$$

$$T = 26$$

$\therefore$ Tom's age right now is 26 years old.



6

Intro to Motion Questions





Perhaps the largest category of word problems concerns moving things and the related quantities distance, rate (speed), and time.

Distance = Rate $\times$ Time



A pink rectangular box with a green cloud on top and motion lines on the right side.
$$D = RT$$

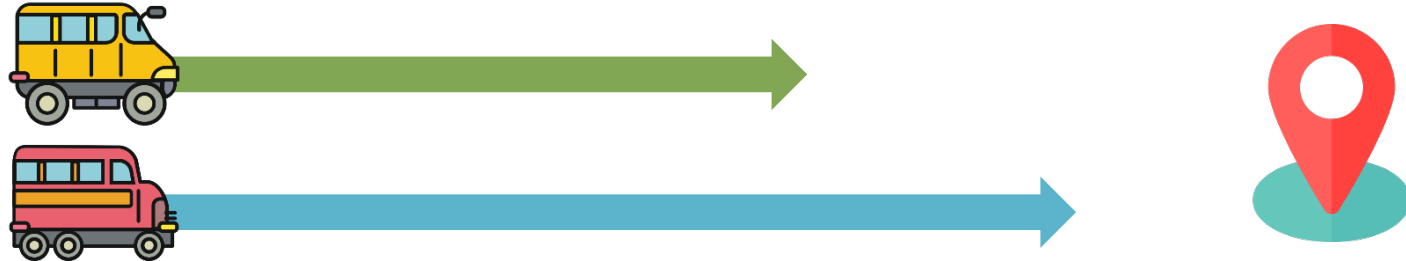
An oval frame with a dashed yellow border.
$$R = \frac{D}{T} \text{ or } T = \frac{D}{R}$$

Types of motion word problems

- 1 Two objects going in opposite directions.



- 2 Both objects going in the same direction, but one goes further.



- 3 One object going and returning at different rates.



Notice that $D = RT$ is already solved for D , and we can easily solve for R or T .

Divide both sides by T to solve for R

$$RT = D \rightarrow R = \frac{D}{T}$$

Divide both sides by R to solve for T

$$RT = D \rightarrow T = \frac{D}{R}$$

It's important to be comfortable moving back and forth between any of the three forms.



$$D = RT \rightarrow R = \frac{D}{T}$$

Let's do a couple simple calculations for practice.

1) What is the speed of someone who covers 240 miles in 6 hours at a constant speed? $R = \frac{240}{6} = 40 \text{ m/h}$

$$D = ?$$

2) How far does someone moving at 8 m/s move in 40 seconds? $D = RT = 8 \times 40 = 320 \text{ m/s}$

HW

3) How much time does it take to move 300 feet at a speed of 20 ft/s?

What if the units given in the prompt and/or the units for which the prompt asks are inconsistent?

!! เปลี่ยนหน่วยให้สัมพันธ์กัน

Changing the units, using a unit conversion, such as (1 hour = 60 minutes) or (1 foot = 12 inches). Often, the test will give you the conversions for any that aren't very common.



Practice question:

A lichen advances 4 cm each year across a rock slab.
If this rate remains constant over time, how many years
will it take to cross 30 meters? (1 m = 100 cm)

$$T = \frac{D}{R} = \frac{30 \times 100}{4} = 750$$

∴ It will take to cross 30 meters for 750 years.

Sale



Practice question: HW

A car moving at 72 km/hr moves how many meters in one second? (1 km = 1000 m)

Sale





Average Speed

70 mph

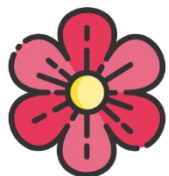


50 mph

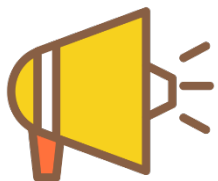


$$\uparrow 10\% + \downarrow 10\% = 0\%$$

$$\begin{aligned}\text{Average Speed} &= \frac{70 + 50}{2} \\ &= \frac{120}{2} \\ &= 60 \text{ mph.}\end{aligned}$$



$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$



You always need to apply

$$D = RT$$



Be careful!!! You will get the wrong answer if you add the two speeds and divide the answer by two.



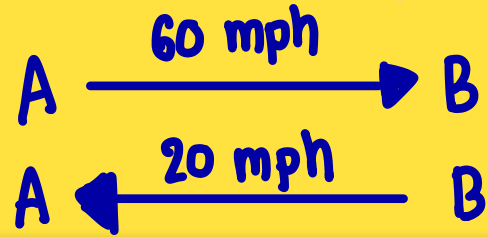
Sometimes, the problem won't give you all the numbers.

For example:

Cassandra drove from A to B at a constant 60 mph speed. She then returned, on the same route, from B to A, at a constant speed of 20 mph. What was her average speed?

Trap answer = 40 mph

$$\frac{60 + 20}{2}$$



Casdra drove from A to B at a constant 60 mph speed. She then returned, on the same route, from B to A, at a constant speed of 20 mph. What was her average speed?

Alternate solution: pick a value for the distance.

Let distance = 60 miles

A-to-B takes 1 hour; B-to-A takes 3 hours

$$D = RT$$

$$R_{\text{avg}} = \frac{D_{\text{tot}}}{T_{\text{tot}}}$$

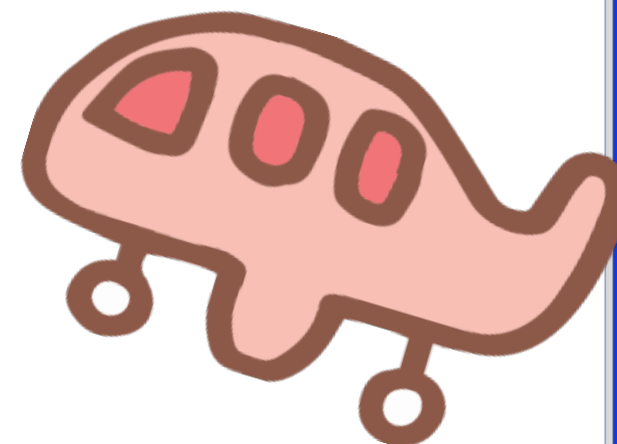
$$R_{\text{avg}} = \frac{60+60}{4} = \frac{120}{4} = 30 \text{ mph}$$

HW

Practice question:

An airplane has a 3600 mile trip. It covers the first 1800 miles of a trip at 400 mph. Which of the following is the closest to the constant speed the plane would have to follow in the last 1800 miles so that the average speed of the whole trip is 450 mph?

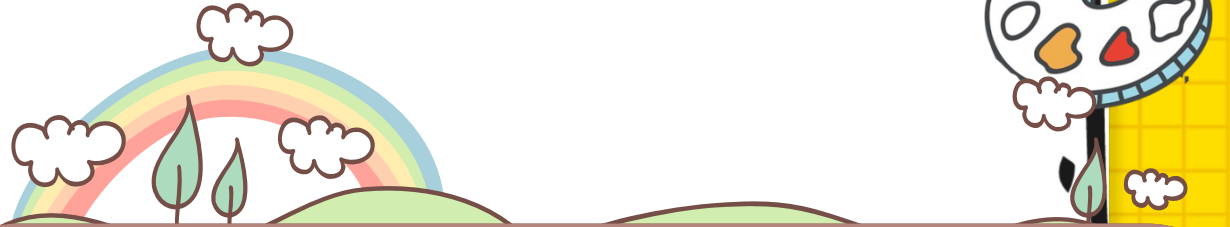
- (A) 450 mph
- (B) 455 mph
- (C) 500 mph
- ~~(D) 514 mph~~
- (E) 600 mph







Some motion-based word problems involve more than one traveler and/or trip of more than one segment.



We already saw a little of this in the previous video on average speed. The basic strategy is that each traveler and each trip gets its own $D = RT$ equation.



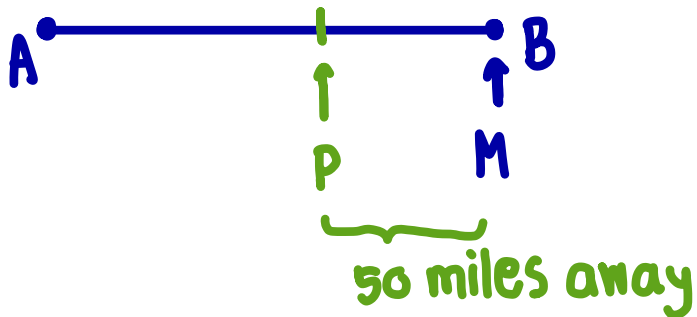


$$D = RT \rightarrow T = \frac{D}{R}$$



Practice problem

Martha and Paul started traveling from A to B at the same time. Martha traveled at a constant speed of 60 mph, and Paul at a constant speed of 40 mph. When Martha arrived at B, Paul was still 50 miles away. What is the distance between A and B?



$$R_M = 60 \text{ mph}, R_P = 40 \text{ mph}$$

$$T_M = T_P$$

$$\frac{AB}{60} = \frac{(AB - 50)}{40}$$

$$2AB = 3AB - 150$$

$$AB = 150$$

$\therefore$ The distance between A and B is 150 miles

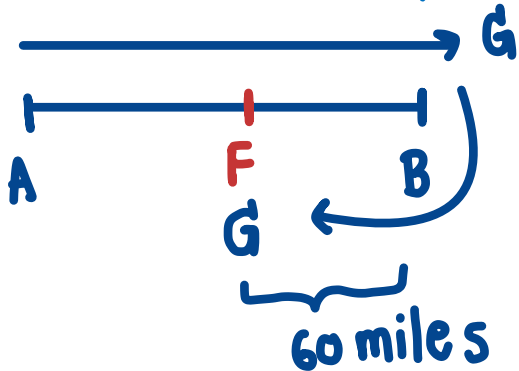




Practice problem

$$R_G = 1.5 R_F$$

Frank and Georgia started traveling from A to B at the same time. Georgia's constant speed was 1.5 times Frank's constant speed. When Georgia arrived at B, she turned around immediately and returned by the same route. She crossed paths with Frank, who was coming toward B, when they were 60 miles away from B. How far away are A and B?



$$T_G = T_F$$
$$\left(\frac{D}{R}\right)_G = \left(\frac{D}{R}\right)_F$$

$$AB = 300$$

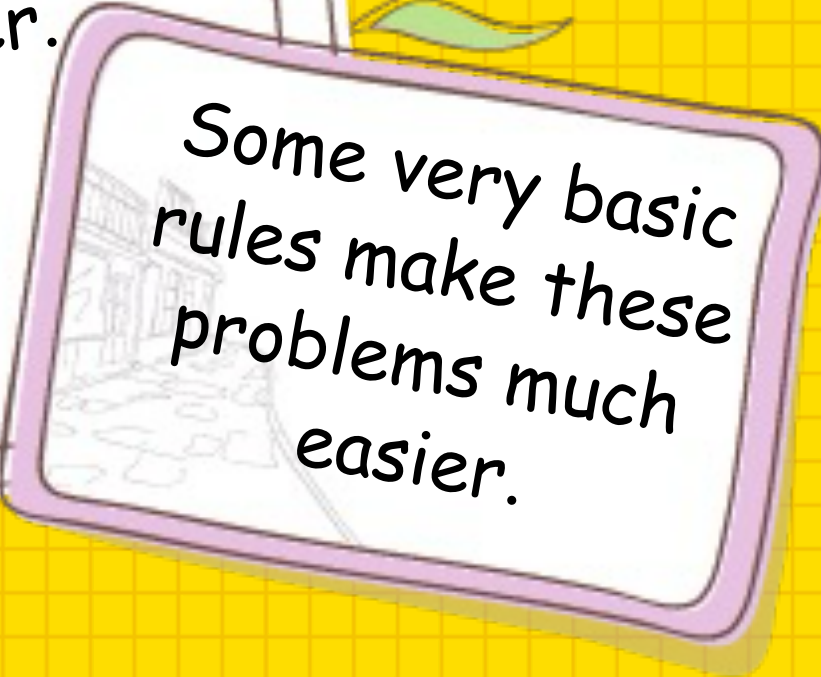



9 Shrinking and Expanding Gaps





Some motion-based word problems involve two travelers moving toward or away from each other.



Some very basic rules make these problems much easier.





Two travelers in opposite directions

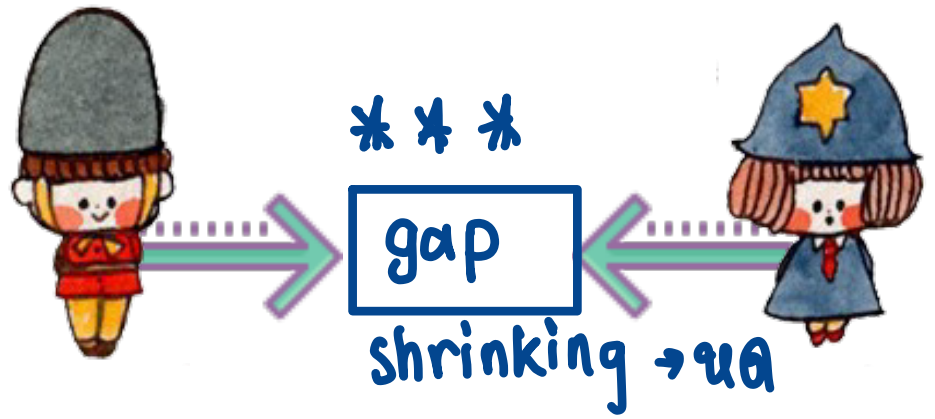
ทิศทางตรงกันข้าม

ความเร็วรวม
บวกกัน



$D = RT$
gap $\rightarrow$ gap

We always add the speeds.



Approaching
the sum of the speeds is the speed at which the gap is shrinking



Receding
the sum of the speeds is the speed at which the gap is expanding.





Two travelers in the same direction

We always subtract the speeds (bigger minus smaller).

ความเร็วลบกัน



1



slow



fast

faster traveler is in front the difference in speeds is the speed at which the gap is expanding.

2



fast



slow

slower traveler is in front the difference in speeds is the speed at which the gap is shrinking.





In a problem in which a gap is obviously shrinking or expanding, we could set a $D = RT$ for each traveler, but sometimes, it saves time to set up a $D = RT$ for the gap itself.

If we figure out the speed at which the gap is shrinking or expanding, we can relate the two individual speeds by simple addition or subtraction.





Practice problem

ทิศทางเดียวกัน speed - กัน

A car and a truck are moving in the same direction on the same highway. The truck is moving at 50 mph, and the car is traveling at a constant speed. At 03:00 pm, the car is 30 miles behind the truck and at 04:30 pm, the car overtakes and passes the truck. What is the speed of the car?



3.00 pm

C

T

30 miles

4.30 pm

T

C

$D_{\text{gap}} = 30 \text{ miles}$, $T_{\text{gap}} = 1.5 \text{ hours}$, $R_T = 50 \text{ mph}$, $R_C = ?$

$$D = RT \quad (\text{gap})$$

$$30 = R(1.5)$$

$$R = 20$$

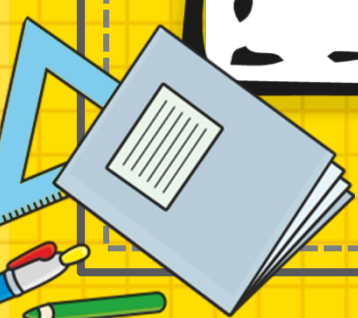
$$R_C - R_T = 20$$

$$R_C - 50 = 20$$

$$\therefore R_C = 70$$

$\therefore$ The car is moving at
70 mph

✳



Practice problem

Same direction

⊖ speed

Car X and Y are traveling from A to B on the same route at constant speeds. Car X is initially behind Car Y, but Car X's speed is 1.25 times Car Y's speed. Car X passes Car Y at 1:30 pm. At 03:15 pm, Car X reaches B, and at that moment, Car Y is still 35 miles away from B. What the speed of Car X? $R_x = ? \rightarrow R_x = 1.25 R_y \rightarrow R_y = R_x / 1.25$

$$D_y = 35 \quad T_y = 1.75$$

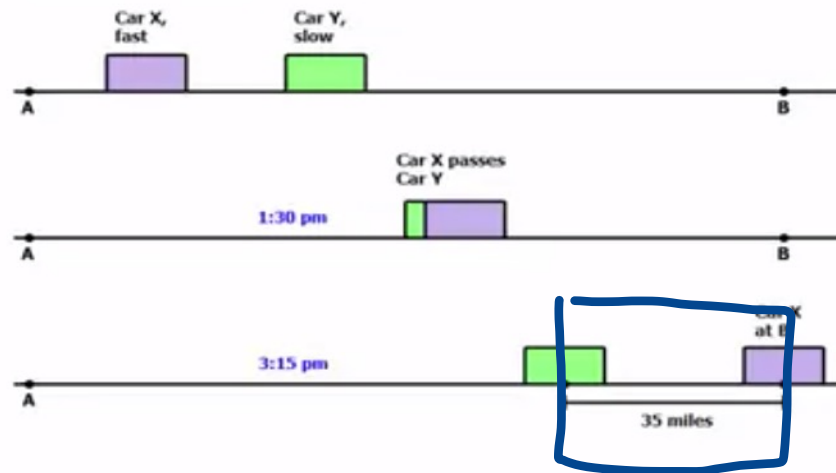
$$D = RT$$

$$35 = R(1.75)$$

$$R = 20$$

$$\therefore R_x - R_y = 20$$

$$R_x - \frac{R_x}{1.25} = 20$$



$$R_x = 100$$

$\therefore$ The speed of Car X is 100 mph

Practice problem

ทิศทางตรงกัน $\oplus$ speed

Cars P & Q are approaching each other on the same highway. Car P is moving at 49 mph and Car Q is moving at 61 mph. at 02:00pm, they are approaching each other and 121 mi apart. Eventually they pass each other. At what clock time are they moving away from each other and 44 miles apart?

$$R = R_P + R_Q = 49 + 61 = 110$$

$$D = RT$$

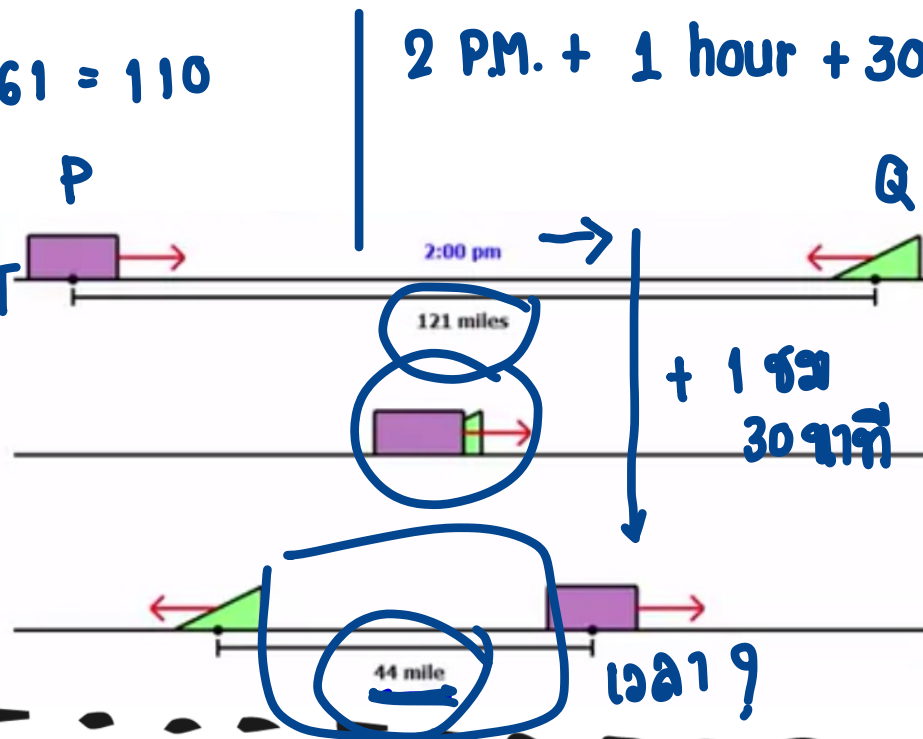
$$(121 + 44) = (R_P + R_Q)T$$

$$165 = 110T$$

$$T = 1.5$$

$$2 \text{ PM.} + 1 \text{ hour} + 30 \text{ minutes} = 3.30$$

Q $\therefore$ The time clock that they moved away from each other and 44 miles apart is at 3:30 pm.





10

Work Questions





Some word problems concern machines or workers and how fast they can get certain jobs done.

The fundamental equation for this is remarkably similar to the $D = RT$ equation for motion problems.





The Work Equation


$$A = RT$$

A = amount of work done

R = the work rate

T = time



The amount, A , can be products manufactured, or houses painted, or pizzas made, or etc. The "Work rate"





As with the $D = RT$ equation, it's important to be able to solve this equation for all three variables.

$A = RT$ is already solved for the amount.



Solve for R:

$$R = \frac{A}{T}$$



Solve for T:

$$T = \frac{A}{R}$$



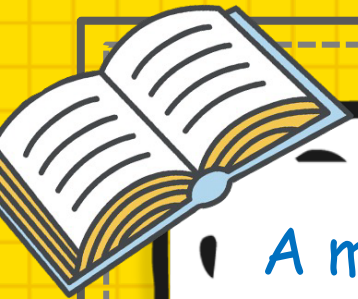
There are two broad categories of work problems.

One type involves using proportion. Of course, the work rate, like any rate, is a ratio, and can be used to set up a proportion.



The other type involves multiple machines or worker working at different rates, and figuring out how much work theses get done together.





A machine, working at a constant rate, manufactures 36 staplers in 28 minutes. How many staplers does it make in 1 hr 45 min?

$$1 \text{ hr } 45 \text{ min} = 60 + 45 = 105 \text{ min}$$

$$\frac{\text{staplers}}{\text{time}} = \frac{36}{28} = \frac{S}{105}$$

$$\frac{9}{7} = \frac{S}{105}$$

$$\frac{9}{1} = \frac{S}{15}$$

$$S = 9 \times 15 = 135$$





Example:

To "detail" a car means to clean it thoroughly, inside and out. When Amelia and Bred detail a car together, 1 car take 3 hours. When Amelia details a car alone, 1 car takes 4 hours. How long does it take Bred, working alone, to detail one car?

ละเอียด ทำใน รถ

$W = RT \rightarrow$ หมายเหตุ T_B

$$R_A = \frac{1}{4}, R_{AB} = \frac{1}{3}$$

$$\therefore R_{AB} = R_A + R_B$$

$$\frac{1}{3} = \left(\frac{W}{T}\right)_A + \left(\frac{W}{T}\right)_B$$

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{T_B}$$

$$T_B = 12$$

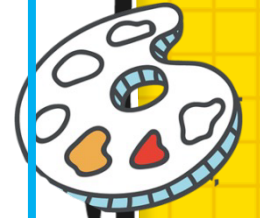
$\therefore$ Bred takes 12 hours to detail one car.





The secret to problems involving different machines or worker working at different work rates is this:

The combined work rate of people/machines working together is the **SUM** of the individual work rates.



In other words, we can't add or subtract the times, and we can't add or subtract the different amounts made in different times. (Those are the numbers usually given in the problem.)





To "detail" a car means to clean it thoroughly, inside and out. When Amelia and Bred detail a car together, 1 car take 3 hours. When Amelia details a car alone, 1 car takes 4 hours. How long does it take Bred, working alone, to detail one car?



$$\text{rates} = \frac{\text{cars}}{\text{hours}} \quad , R_{AB} = \frac{1}{3} , R_A = \frac{1}{4}$$

$$R_A + R_B = R_{AB}$$

$$R_B = R_{AB} - R_A$$

$$R_B = \frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$$

So, Bred details a car of 12 hours





Pump X takes 28 hours to fill a pool. Pump Y takes 21 hours to fill the same pool. How long does it take them to fill the same pool if they are working simultaneously?

HW

12 hours







Some word problems concern populations or samples that are in the process of growth or decay.



Human populations and population of animals or bacteria experience natural growth.

Decay can include radioactive decay, something melting, a bacteria population getting wiped out, etc.





The basic format of this question is you will be given some starting value, and told that the size gets multiplied or divided by a certain quantity in some fixed interval of time. Then you will be asked for the resultant amount after some slightly longer period of time.



Take the calculations one "**change interval**" at a times. Step by step, figure out the amounts until you arrive at the final amount.





Practice question:

When isotope QXW radiactively decays, it loses exactly half of its mass in each three-day period. Suppose scientists start with a 96 gram sample of pure isotope QXW on a certain day. What will be the remaining mass in 12 days?

Handwritten notes: 3 → 12, 3 periods, 4 periods

1. $96 \rightarrow 48$

2. $48 \rightarrow 24$

3. $24 \rightarrow 12$

4. $12 \rightarrow 6$

$\therefore$ The remaining mass in 12 days will be 6 grams.

~~X~~

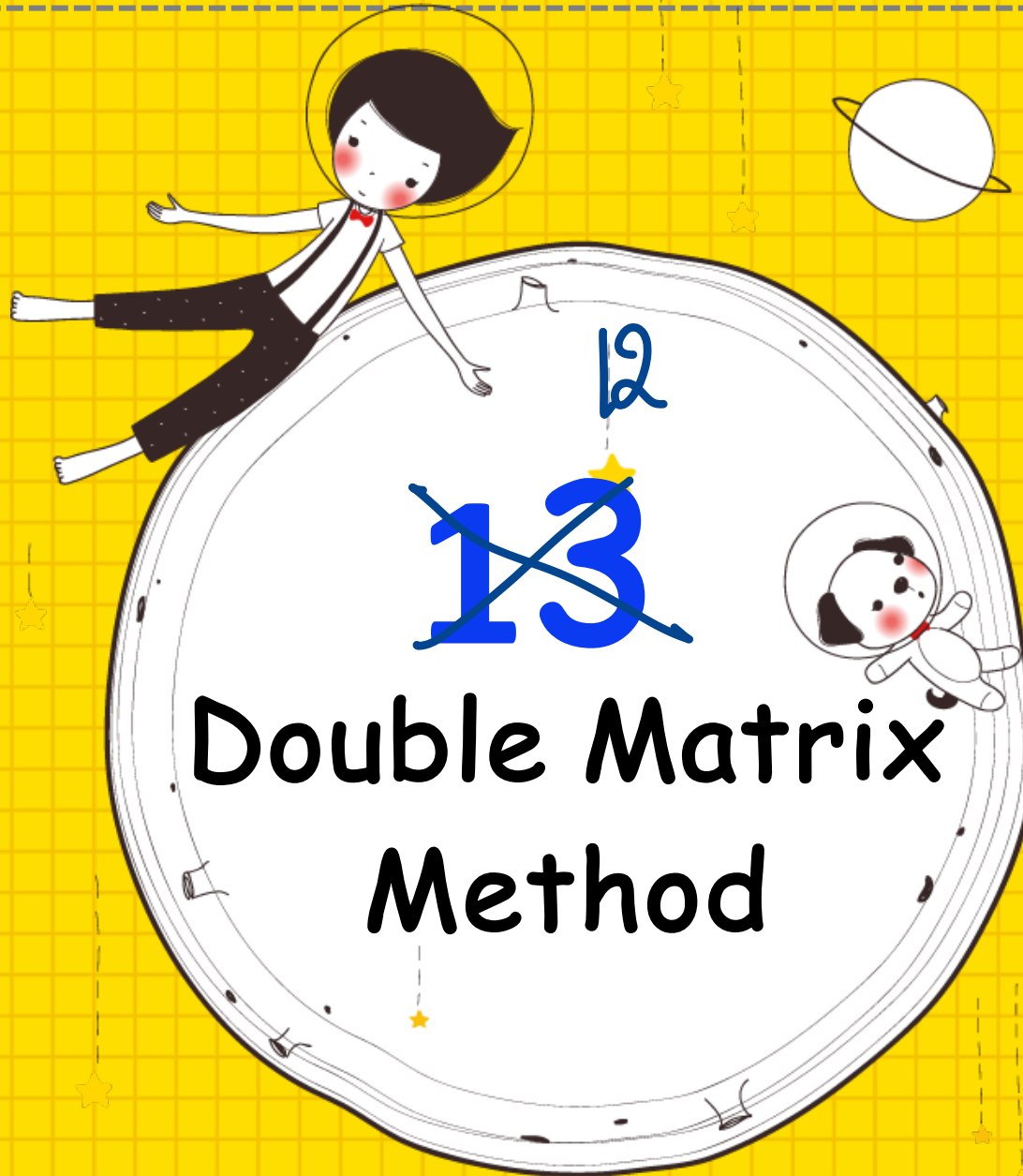




Practice question: HW

Under optimal conditions, the Vericoccus Bacteria multiplies the size of its population by $\frac{5}{2}$ every 4 hours. If there are 24 billion at 9:00 a.m., and optimal conditions are maintained, how many are there at 5:00 p.m. of the same day?





Double Matrix Method



In some more complicated set problems, every member of the population is classified in two ways at once.

For example, if we were to look at employees at a company, we might group employees by gender (M & F) and education (college only vs. advanced degree).

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In that example, every single employee has a gender, either M or F. Also, every single employee has an education level, either "college only" or "advanced degree."



The information the problem could give might be how many females (which would include both education levels) or how many folks with advanced degrees (which would include both genders) or how many employees are females with advanced degrees.





Consider this problem:

In a company of 300 employees, 120 are females. A total of 200 employees have advanced degrees, and the rest have a college degree only. If 80 employees are males with college degree only, how many are females with advanced degrees?





For this problem, genders will be the columns and educational level will be the rows.



There are two genders, so we will need **THREE** columns: one for F, one for M, and one for the totals.

There are also two educational levels, so we will need **THREE** rows: one for "college only," one for "advanced degree, and one for the totals





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ج. ل. م.
١٥٩

college
only
advanced
degree
TOTALS
٥٦٩

	١٥٧ females	١٧٦ males	TOTALS ٥٦٩
college only	A ↓	C ↓	E
advanced degree	A ↓	A ↓	F
TOTALS ٥٦٩	G	H H	T

I



Boxes A, B, C, and D are the raw data for different individuals.

Boxes E & F are the row totals.

Boxes G & H are the column totals.

Box T is the grand total, everyone in the whole collection.

$$G + H = E + F = T$$





In a company of 300 employees, 120 are females. A total of 200 employees have advanced degrees, and the rest have a college degree only. If 80 employees are males with college degree only, how many are females with advanced degrees?



	females	males	TOTALS
college only	20	80	100
advanced degree	100	100	200
TOTALS	120	180	300

∴ A total of
100 females with
advanced degrees.





In a certain school, there are 80 freshmen, 100 sophomores, and 220 upperclassmen, drawn from three cities: A, B, and C. Sixty percent of students are from A, 30% from B, and the rest from C. All the students from C are freshmen. Half the students from B are upperclassman and the rest are split evenly between the other two grades. How many sophomores are from A?

HW





Practice question: **HW**

What is the sum of all the multiples of 20 from 160 to 840 inclusive?

