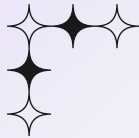
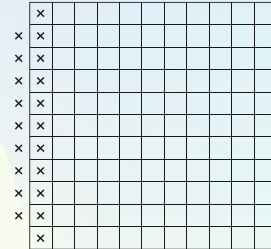
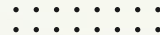


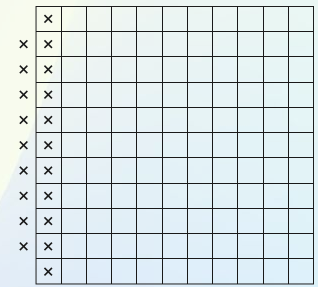
# FOE1003

# Introduction to Ordinary Differential Equations



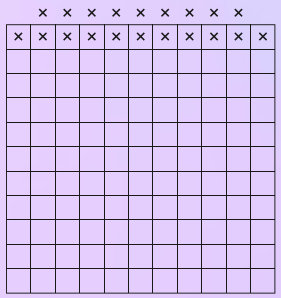
Tadchanon Chuman  
Department of Electrical Technology, SSRU

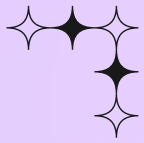




**01**

# Meaning and Types of ODEs





# Meaning of ODEs

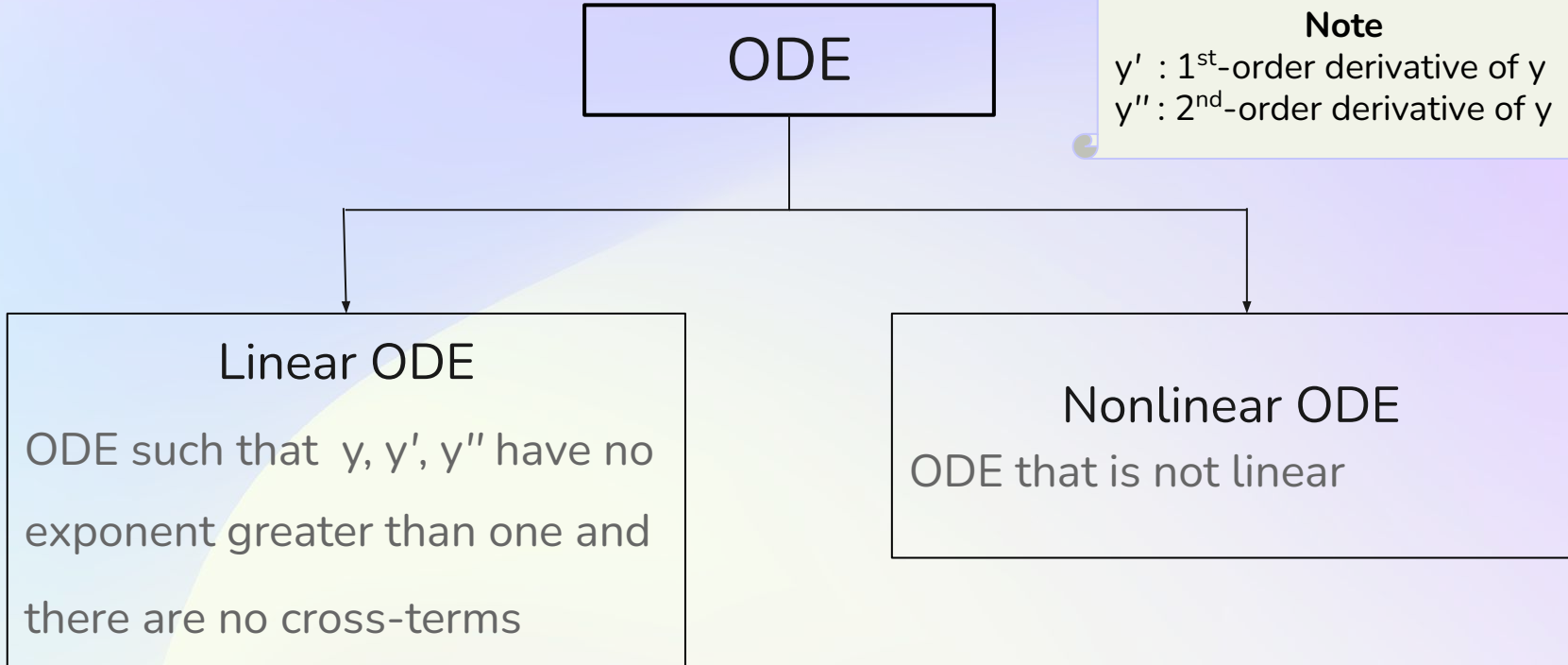
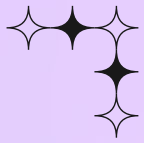
Ordinary differential equations (ODEs) consists of derivatives of some functions, e. g.,

$$\frac{dx}{dy} = f(x) \quad \text{or} \quad y' = f(x)$$

- $y$  is a function of  $x$ ,
- $x$  is an independent variable,
- $y$  is a dependent variable,
- $dy/dx$  (or  $y'$ ) denote the derivative of  $y$  with respect to  $x$

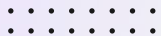


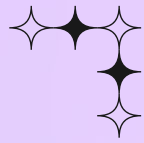
# Types of ODEs: Linearity



## Two Notations of Derivatives

$\frac{dy}{dx} = y'$	1 <sup>st</sup> -order derivative
$\frac{d^2y}{dx^2} = y''$	2 <sup>nd</sup> -order derivative
$\frac{d^3y}{dx^3} = y''' = y^{(3)}$	3 <sup>rd</sup> -order derivative
$\frac{d^4y}{dx^4} = y^{(4)}$	4 <sup>th</sup> -order derivative

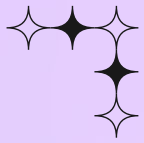




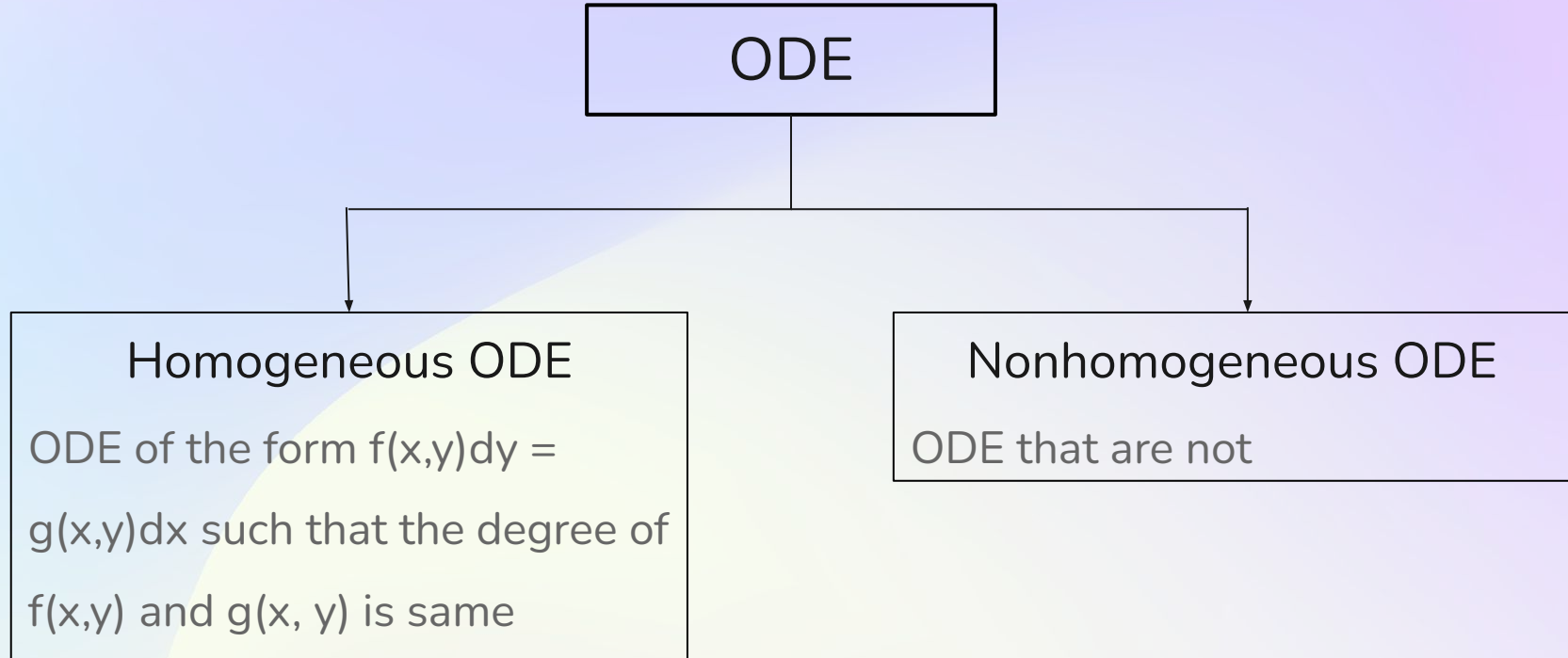
## Example: Linearity

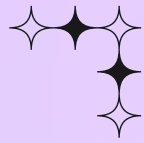
$\frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = \sin(x)$	This is a linear ODE because the terms $d^2y/dx^2$ , $dy/dx$ และ $y$ are all linear.
$y'' + 4yy' + 2y = \sin(x)$	This is a nonlinear ODE because....
$y'' + \cos(y) = 0$	This is a...
$(y'')^2 + 2y = \cos(x)$	This is a...





# Types of ODEs: Homogeneity

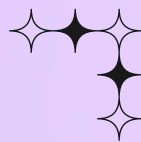




## Example: Homogeneity

$y'' + 4xy' + 2y = 0$	This is a homogeneous ODE because that is no forcing function.
$y'' + 4xy' + 2y = \sin(x)$	This is a nonhomogeneous ODE because ...
$y'' \cos(x) + 4y' + 2xy = 0$	This is...
$y'' + \cos(y) = e^x$	This is...

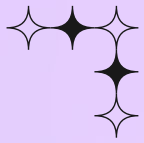




## Exercises: Linearity and Homogeneity

ODE	Linearity	Homogeneity
1. $(y'')^2 - 3y' + 2y = x^4$		
2. $y'' + [a + b \sin(2x)]y = 0$		
3. $y''' - 6(y'')^2 + 11y' + 6y = e^x$		
4. $y^4 = xy'' + y^2 = 0$		
5. $d(xy')/dx - xy = 0$		
6. $(x - y)dy = (x + y)dx$		





# Order and Degree

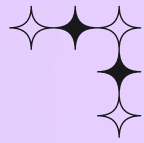
ODEs may consists of several order of derivatives (e.g.,  $y'$ ,  $y''$ ,  $y'''$ ) where such derivatives may have multiple degrees (e.g.,  $(y')^3$ ,  $(y'')^4$ ,  $(y''')^2$ )



**Order of ODE** is the highest order derivative it contains.

**Degree)** the degree of highest order derivative of the ODE.

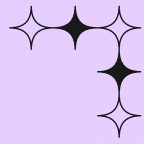




## Example: Order and Degree

ODE	Order	Degree
$y'' + 4xy' + 2y = 0$		
$(y''')^2 + 2x^2yy''' + x^2y = 0$		
$(y''')^2 + (y'')^3 + x^5y^8 = 0$		



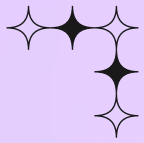


## Example: Order and Degree

ODE	Order	Degree
1. $(y'')^2 - 3y' + 2y = x^4$		
2. $y'' + [a + b \sin(2x)]y = 0$		
3. $y''' - 6(y'')^2 + 11y' + 6y = e^x$		
4. $y^4 = xy'' + y^2 = 0$		
5. $d(xy')/dx - xy = 0$		
6. $(x - y)dy = (x + y)dx$		







# Solution of ODEs

When we solve an ODE, the obtained result is called a **solution**.

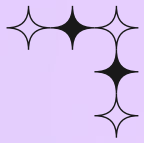
## Definition (Solution)

Solution is an expression for the dependent variable in terms of the independent one(s) which satisfies the relation

**Example** solutuin of the ODE :  $y' = \cos(x)$  is  $y = \sin(x)$  because

1.  $y = \sin(x)$  has no derivative,
2.  $y = \sin(x)$  corresponds to
$$y' = d \sin(x)/dx = \cos(x)$$





# Types of ODE solutions

## Solution of ODE

### General Solution

Solution with an arbitrary constant,  
e.g.,  $y = Ae^{-x}$

### Particular Solution

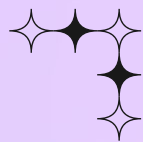
Obtained from a general solution, e.g.,  $y = 4e^{-x}$

### Complete Solution

Linear combination of general solutions, e.g.,  
 $y = Ae^{-x} + B\sin(x)$

### Homogeneous Solution

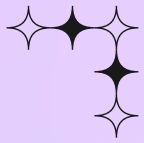
cannot be obtained from a general solution



## Example: Types of Solutions

ODE	Solution	Solution type
$y'' - y = 0$	$y = Ae^x, y = Be^{-x}$	general solutions ∴ has arbitrary constants
$y'' - y = 0$	$y = 3e^{-x}$	particular solution ∴ obtained by replacing B with 3
$y'' - y = 0$	$y = Ae^x + Be^{-x}$	complete solution ∴ combination of general solution
$(y')^2 - xy' + y = 0$	$y = cx - c^2, y = x^2/4$	homogeneous solution ∴ $y = x^2/4$ is not obtained from general solutions





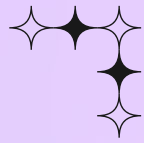
# Solution Verification

When we think a function is a solution of an ODE, we can **verify** whether the function is the solution or not. The verification procedure is stated below.



1. Find derivatives of the function we want to verify.
2. Substitute the derivatives on one side of the ODE.
3. If both sides of the ODE are equal, we can conclude that the function is a solution of the ODE.





**Example 1** Verify that  $y = ae^{-x} + be^{2x}$  is a solution of

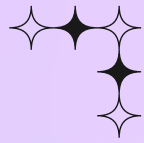
$$y'' - y' - 2y = 0$$

Answer Since there exist  $y'$  and  $y''$  in the equation, we have to derive the first- and second-derivatives of  $y = ae^{-x} + be^{2x}$ .

$$y' = \frac{d(ae^{-x} + be^{2x})}{dx} = \dots$$

$$y'' = \frac{dy'}{dx} = \dots$$





**Example 1** Verify that  $y = ae^{-x} + be^{2x}$  is a solution of

$$y'' - y' - 2y = 0$$

Answer (con.) Replacing  $y'$  and  $y''$  in the LHS of the ODE yields

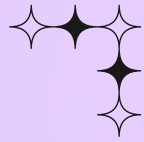
$$\text{LHS} = y'' - y' - 2y = \dots$$

**Note**

LHS : left-hand side

RHS : right-hand side





**Example 2** Verify that  $y = a\cos(2x) + b\sin(2x)$  is a solution of

$$y'' + 4y = 0$$

Answer Find the derivatives of  $y$ :

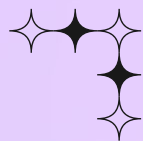
$$y''$$

$$y' =$$

$$y'' = dy'/dx =$$

$$\text{LHS} = y'' + 4y =$$





**Example 3** Verify that  $y^2 = ax - x \ln(x)$  is a solution of

$$2xyy' = (y^2 - x)$$

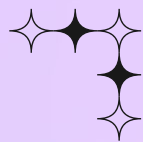
Answer Consider the LHS of the ODE.

$$\text{LHS} = 2xyy'$$

One can see that  $\text{LHS} = x(2yy')$  and we know that  $\frac{dy^2}{dx} = 2yy'$

Hence,  $\text{LHS} = x \frac{dy^2}{dx} = \dots$





**Example 3** Verify that  $y^2 = ax - x \ln(x)$  is a solution of

$$2xyy' = (y^2 - x)$$

Answer (con.) Consider the RHS of the ODE.

$$\text{RHS} = y^2 - x = \dots$$





# References



Adkins, William A., and Mark G. Davidson Ordinary differential equations. Springer Science & Business Media, 2012.

ศ. ดร.มงคล เดชนครินทร์ คณิตศาสตร์วิศวกรรมไฟฟ้า พิมพ์ครั้งที่ 4 สำนักพิมพ์ จุฬาลงกรณ์มหาวิทยาลัย 2558.

