History of Mathematics

MAC3314

Week 1

Timeline

Prehistoric era

| ① | ② | ③ | ④ | ⑤ | ⑥ | ⑦ | ⑧ | ⑨ | ⑩ |

3000 BC   1000 BC   300 BC   500 AD   1000   1500   1600   1700   1800

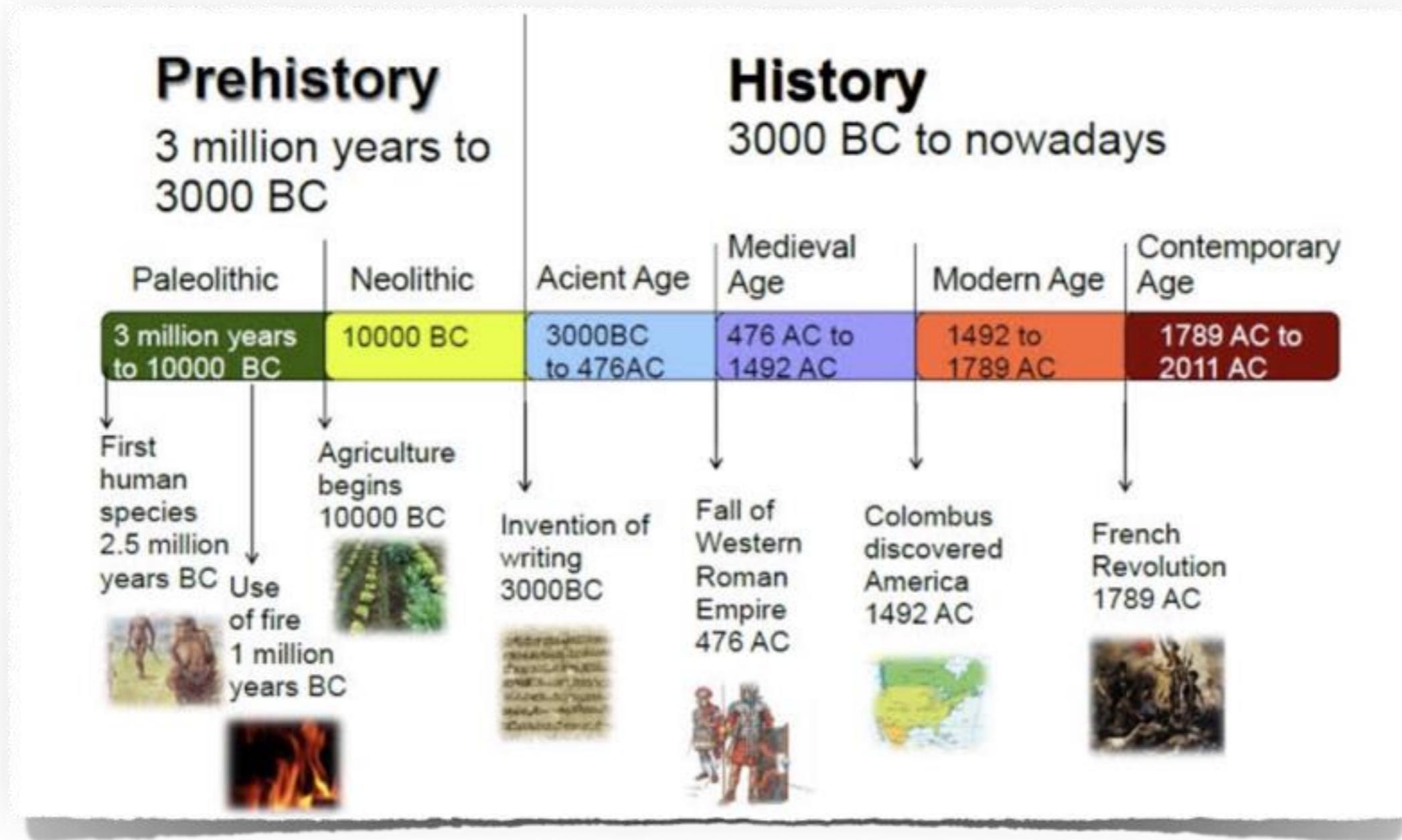History era
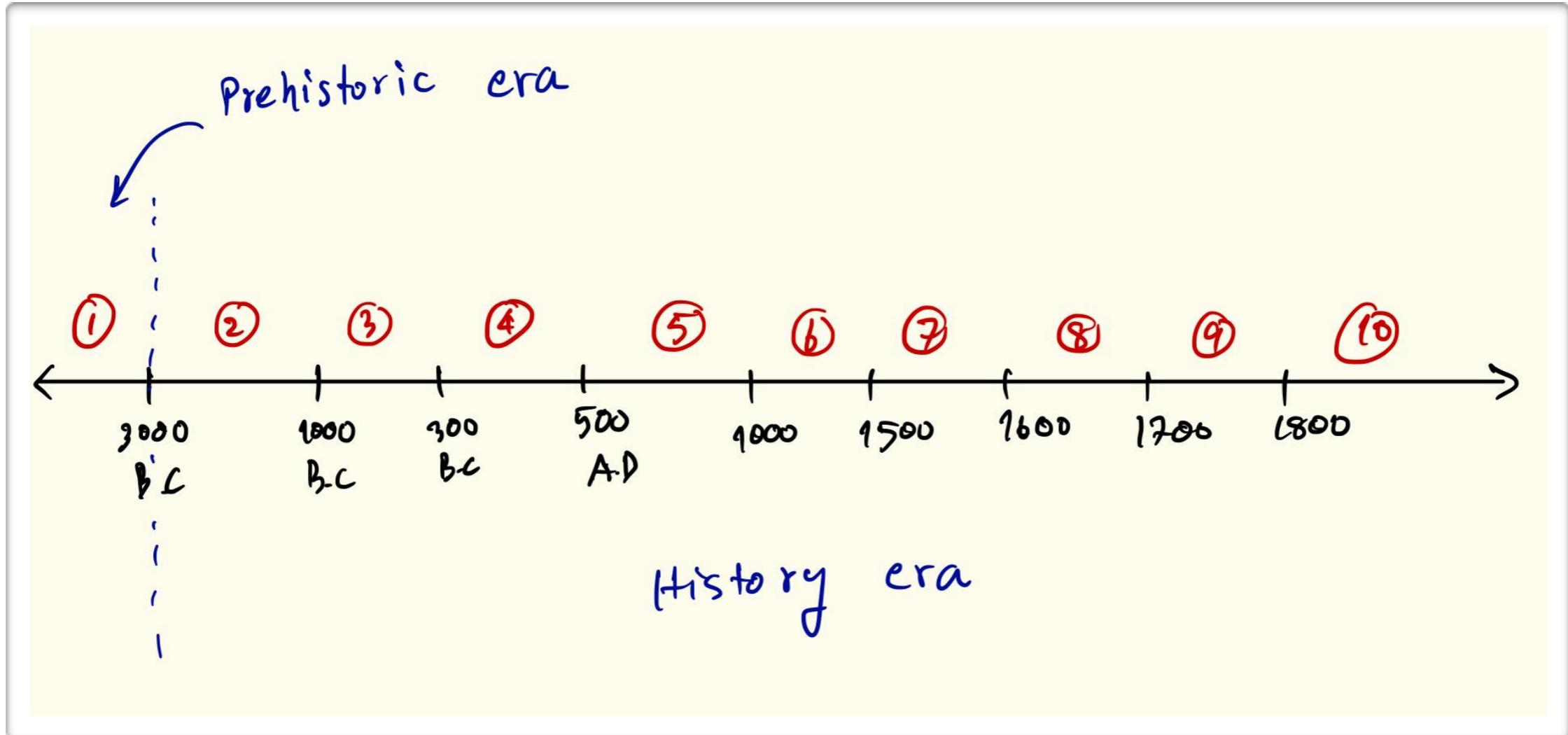
Chapter 1

Prehistoric Mathematics

Contents

- In the beginning
- Primitive counting
- Geometric ornament
- Mysticism

1. In the Beginning

In the Beginning

What do we mean by " history ", and
what does the term " mathematics " signify ?

Mathematics

Elementary mathematics

Abstract science which investigates deductively the conclusions implicit in the elementary conceptions of spatial and numerical relations.

Oxford Dictionary

It is well, therefore, to discard such niceties of definition and to take a broader view of the case, seeking to tell the story of the genesis of mathematics even before the period in which the science, as defined above, began to exist. Such a procedure will lead us back not only to the days when the human race was young, but to the ages immediately antedating its appearance upon the earth, and even farther. If one should wish to reach an absolute zero from which to begin his narrative, he would soon find himself lost in a maze of perplexities. For our purposes a brief statement of the mathematics in the Beginning will suffice.

Cosmic Figures

Solar system

Nebula

Mathematics —> No beginning   No end

discovery of existing laws in this science and of the invention of better symbols as needed from time to time for their expression.

Herbert Spencer once spoke of the properties of space as "eternal, uncreated"—as "anteceding either creation or evolution," adding these impressive words:

It is impossible to imagine how the marvellous space-relations discovered by the Geometry of Position came into existence. The consciousness that without origin or cause Infinite space has ever existed and must ever exist, produces in me a feeling from which I shrink.
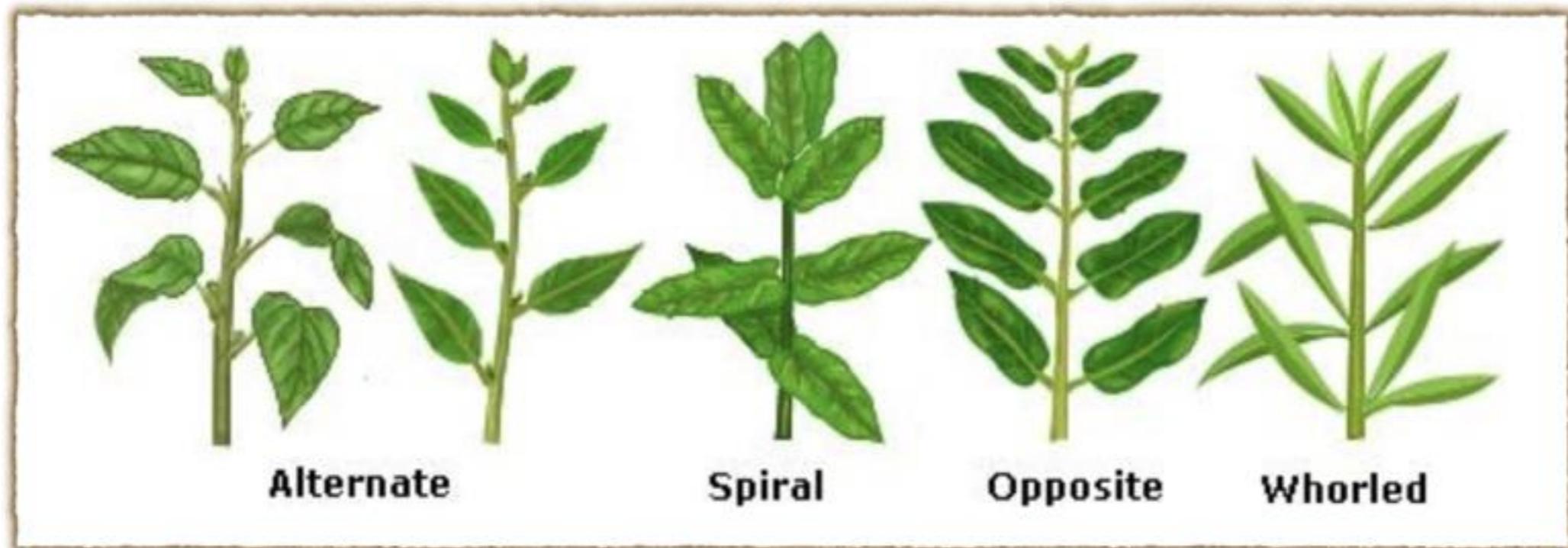

SNOW CRYSTAL

Photograph under a microscope below freezing temperature
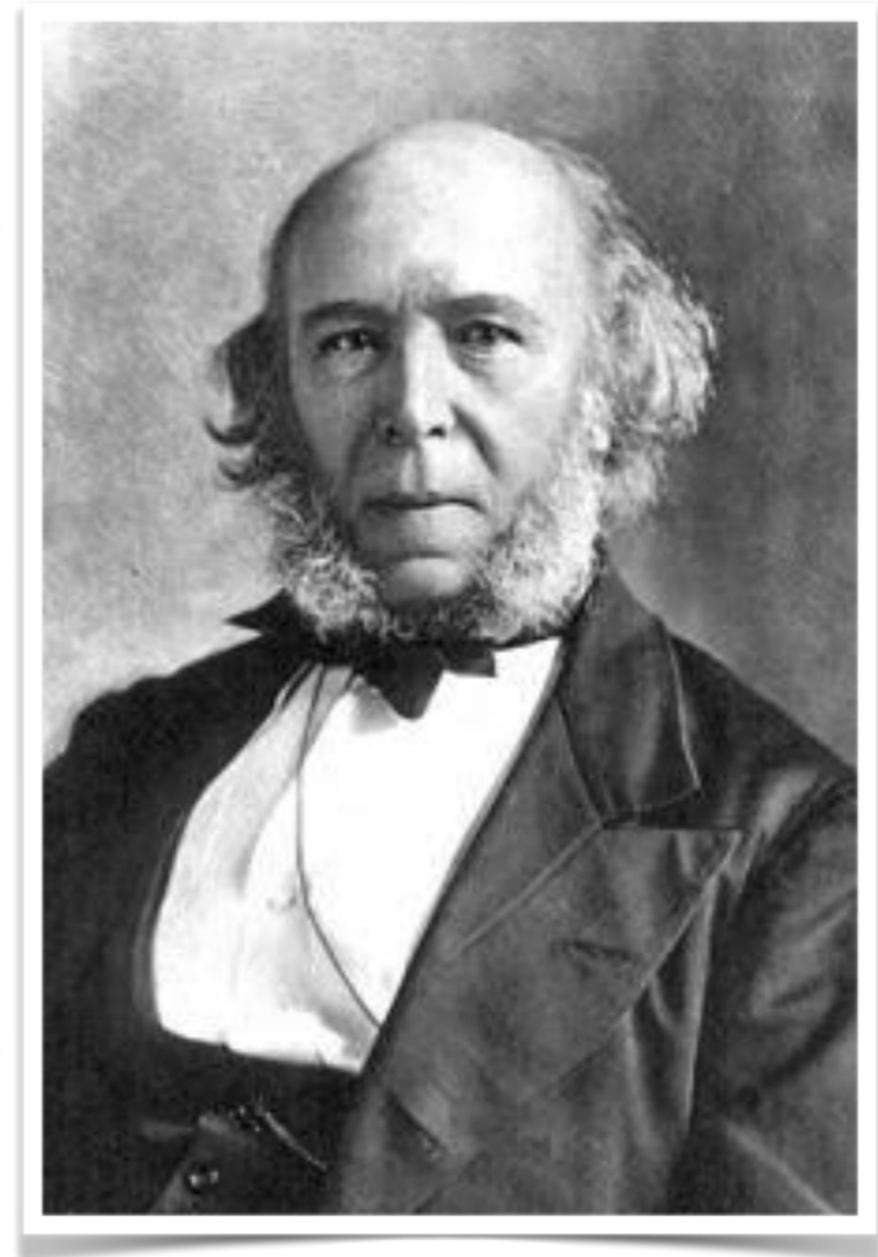
Herbert Spencer

Mathematical forms in plant life

Alternate   Spiral   Opposite   Whorled

Phyllotaxy

Regular polygons in the structure

Pineapple

Watermelon

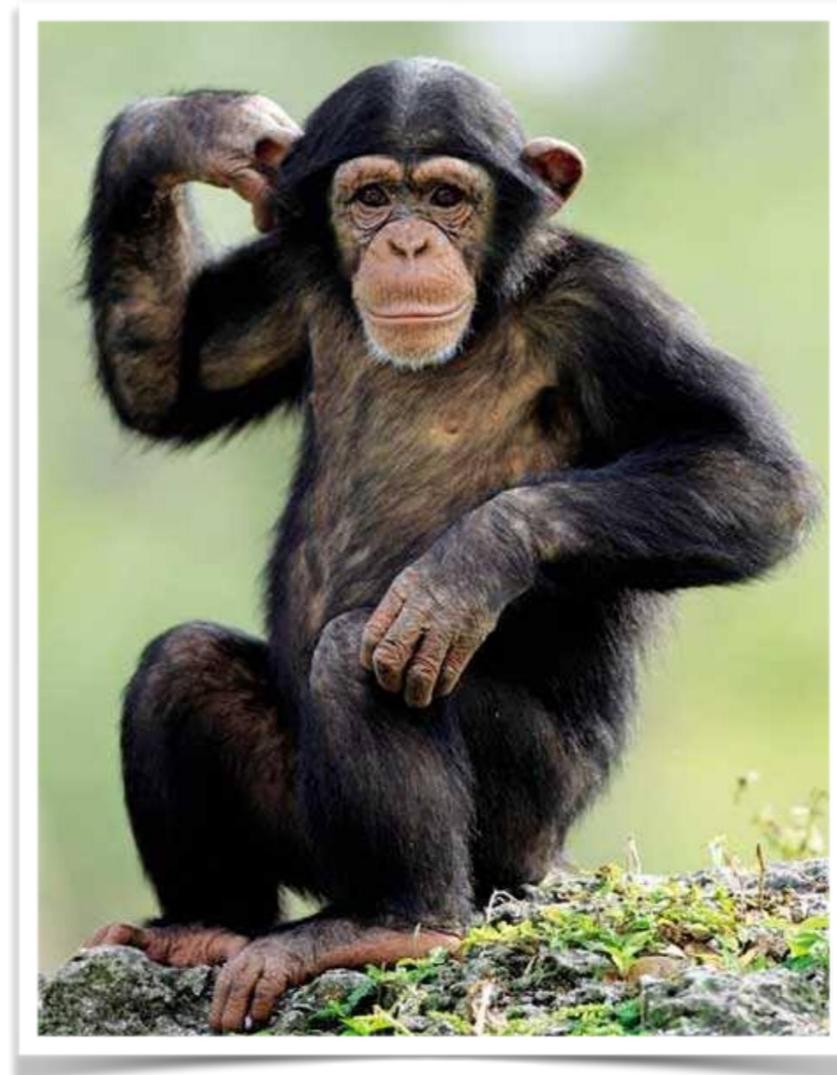# Recognition of Mathematical Concepts

Pseudo-counting

Chimpanzee

Magpies

Recognize the size of a group of five or
six objects

Knows five objects more than four

Spider web

Who can say when or where or under what influences the Epeira first learned to trace the logarithmic spiral in the weaving of her web ?

Honeycomb

The bee build up the hexagonal wax cells of the honeycomb.

Advent of the Human Race

Advent of the Human Race. With the advent of the human race there developed an opportunity for mathematics to show itself more consciously. This appeared in art, the world's universal language; in religious mysticism, the expression of our efforts to fathom the unfathomable; and, in a less degree, in commerce, war, and the needs of the pastoral life. Each of these human interests, but especially decorative art, contributed to an appreciation of geometry; each, but especially religious mysticism and commerce, contributed to the development of number; and each lent its aid to the creation of an interest in architecture and in the science of astronomy.
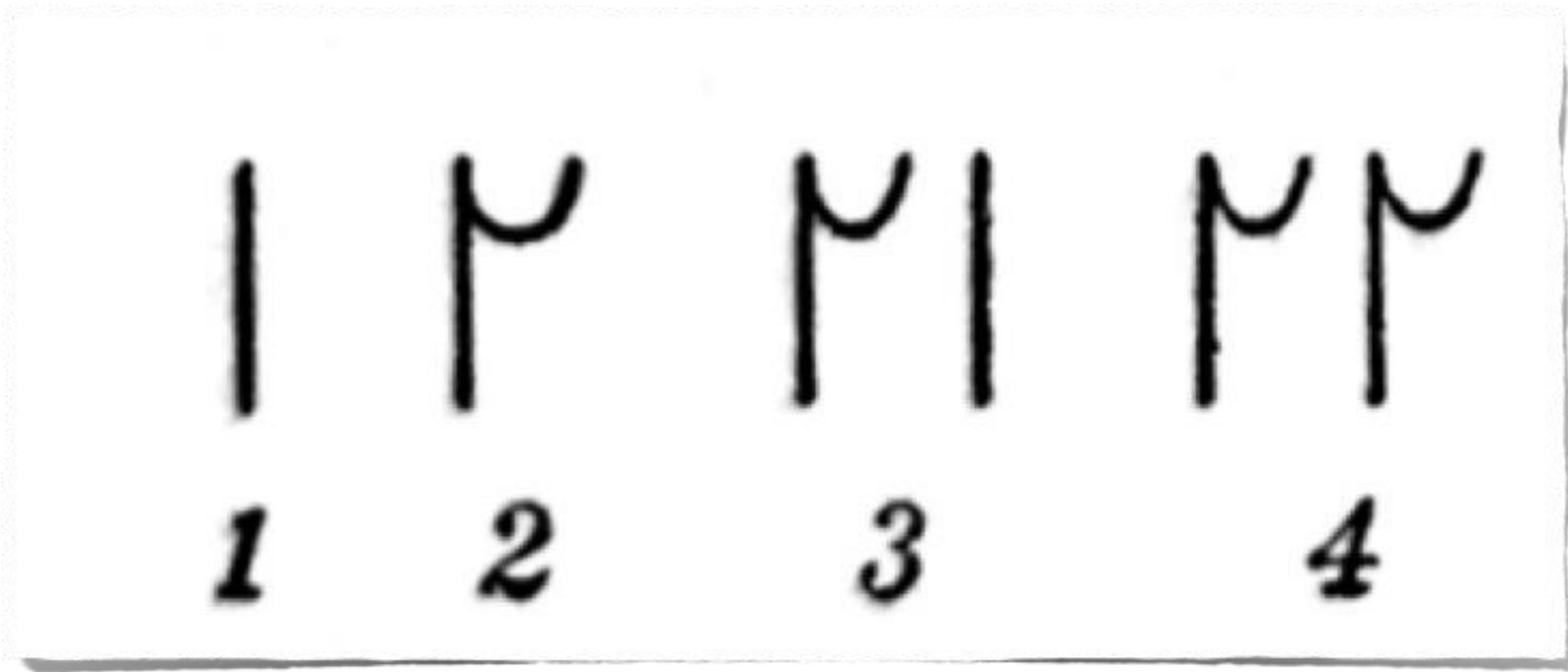
2. Primitive Counting

Early Efforts

Shepherd

Herdsman

A well-known instance of a deaf-and-dumb boy who acquired a knowledge of numbers from observing his fingers, even before he was taught to count, shows us that the idea of number did not have to await the development of spoken language, and so a savage may appreciate three without having a name for numbers beyond two.

- Develop number sense

- Number language

Australian Aborigines

The fingers
of the one hand

Cardinal numbers

studied within the last century. For example, the numerals of thirty selected Australian languages extend in no case beyond four, the tribes which use them not having even reached the point of recognizing, as a basis for counting, the fingers of one hand, and in most cases having number names for only one and two. In general, everything beyond two is called "much" or "many." So poor are these particular languages that only cardinal numbers exist, the ordinals being unknown. This paucity of vocabulary is not universal among the Australian

One and Two

names reach as far as fifteen or twenty. Furthermore, there is an element of doubt in the reports of anthropologists, first because they are not always correctly informed by the natives themselves, and also because the absence of number names does not generally mean that the primitive tribes had no names for groups of two or three. An illustration of the latter consideration is seen in the case of the Andamans, a tribe of Oceanic negritos. Their number names are limited to one and two, but they are able to reach ten by this process: the nose is tapped with the finger tips of either hand, beginning with one of the little fingers, the person saying "one" (*úbatúl*), "two" (*íkpór*), and then repeating with each successive tap the word "*anká*," which means "and this." When the second hand is finished, the two hands are brought together to signify 5 + 5, and the word "all" (*ardùru*) is spoken. To mention another example, the Pitta-Pitta, a tribe in Queensland, are able to count the fingers and toes without a system of numerals, but only by the aid of marks in the sand, and in various other parts of Australia the natives show habitual uncertainty as to the number of fingers they have on a single hand.

Objective Counting

Objective Counting. Judging by other races of low intelligence, the primitive man could count only by pointing to the objects counted, one by one. Here the object is all-important, as was the case with the early measures of all peoples. The habit is seen in the use of such units as the foot, ell,[1] thumb,[2] hand, span, barleycorn, and furlong (furrow long). In due time such terms lose their primitive meaning and we think of

longer think of a certain group of objects, nor do we demand such a group in order to count; we think of a word in an endless series, the word coming just after "six" in the series and just before "eight." In the Malay and Aztec tongues, however, the number names mean literally one stone, two stones, three stones, and so on; while the Niuès of the Southern Pacific use "one fruit, two fruits, three fruits," and the Javans use "one grain, two grains, three grains," all these being relics of the concrete stage of counting. When a Zulu wishes to express the number six, he says "taking the thumb" (*tatisitupa*), meaning that he has counted all the fingers of the left hand and has begun with the thumb of the right hand. For seven he says "he pointed" (*u kombile*), meaning that he has reached the finger used in pointing. After the world abandoned the

The Radix in Number Systems

Some number is selected as the radix ten being the general favorite for the reason that have ten fingers.

EARLY SYRIAC NUMERALS
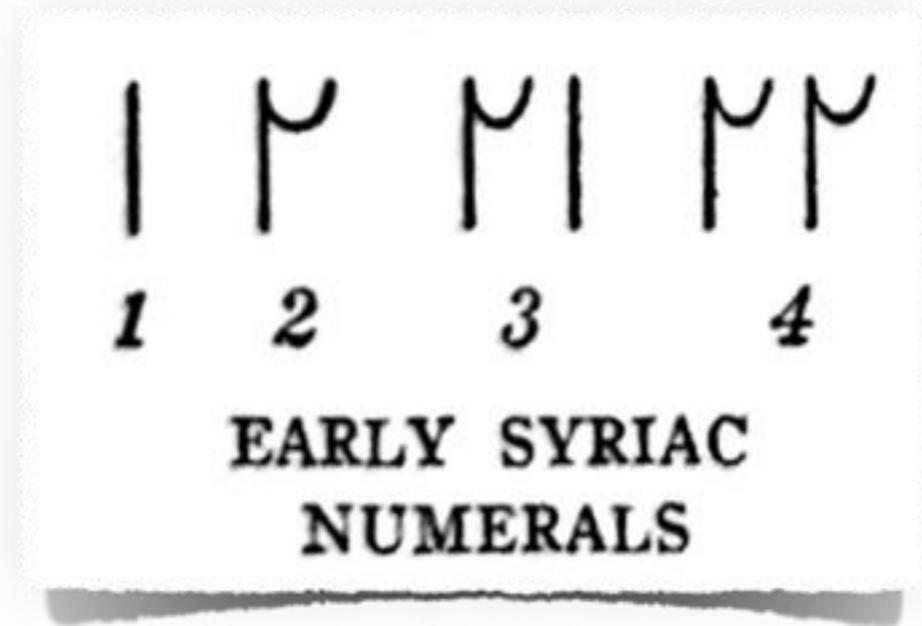
Scale of two

Natives of Queensland

One
Two
Two and One
Two Twos

Thrice

Native Tasmanians

One
Two
Plenty

> The ancient ternary system

Four

South American tribes

One
Two
Three
Four
Four and one
Four and two
…

Scales of Five

The five fingers of the hand
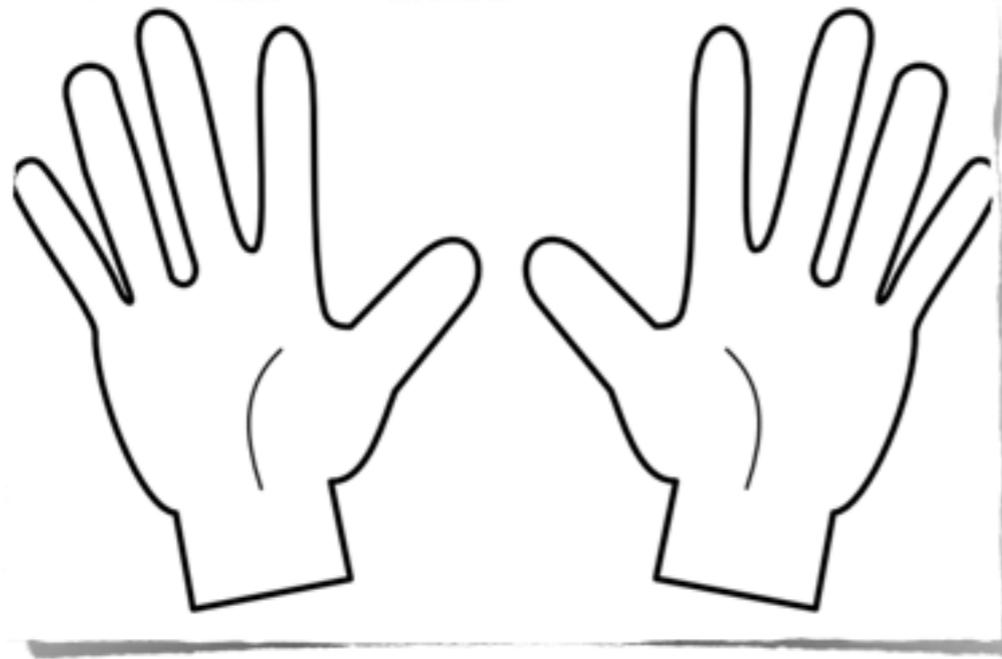
South American tribes	Yukaghirs of Siberia
One	One
Two	Two
Three	Three
Four	Three and one
Hand	Five
Hand and one	Two threes
Hand and two	one more
...	Two fours
	Ten

Scales of Ten

Scales of Twenty

The fingers of both hands

The fingers and toes

the race. An examination of more than seventy African languages showed that ten was used as a radix in every case, and although this may be due directly to the fingers, it is quite as likely to be due to a common linguistic source. There is an in-

3. Geometric Ornament

Geometric Forms

- painting of ruches

they are equally prominent in architecture in all parts of the world. They are found on the early monuments of Mexico, on the architectural remains of Peru, on the huts of the savage, and on the early buildings of the historic period in various parts of the Old World, especially on those devoted to the commemoration of the dead or to the worship of the gods.

The same instinct that leads to geometric decoration of religious structures shows itself in the decoration of personal ornaments and of articles intended for domestic use. This is seen in the handicraft of the Stone Age, it is found in the

rich gold work of early Egypt, and it is equally in evidence in most of the jewelry of modern times. It is not merely the instinct of symmetry that we find in these petrified thoughts of the race; it is quite as much a desire to fathom the mystery and grasp the meaning of the beauty of geometric form.

EGYPTIAN POTTERY OF THE PREDYNASTIC PERIOD

It shows the earliest stage of geometric ornament on pottery. The Predynastic Period extended from *c.* 4000 to *c.* 3400 B.C. From the Metropolitan Museum, New York

Early Pottery

CYPRUS JUG OF THE PERIOD
3000–2000 B.C.

Pottery of the Early Bronze Age, showing the second stage in geometric ornament. From the Metropolitan Museum, New York

Early Pottery. The early pottery of Egypt and Cyprus shows very clearly the progressive stages of geometric ornament, from rude figures involving parallels to more carefully drawn figures in which geometric design plays a more important part and in which such mystic symbols as the swastika are found. Art was preparing the way for geometry.
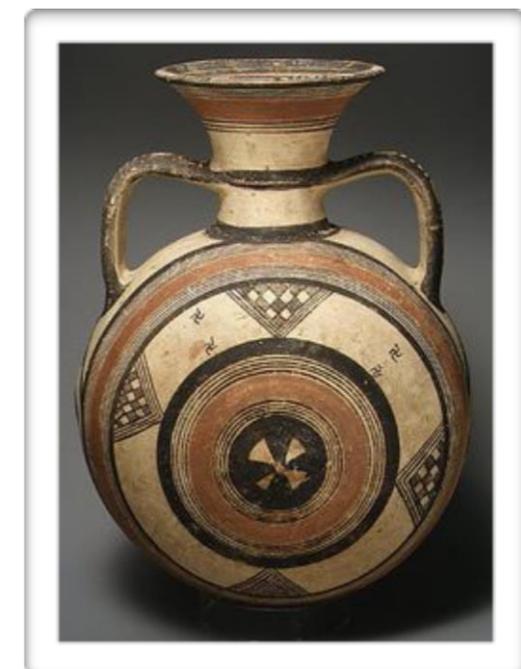
4. Mysticism

Religious Mysticism

Religious Mysticism. The beginning of an appreciation of the wonders of mathematics is closely connected with the beginning of religious mysticism. Man wondered at the heavens above him; he wondered at life and he wondered even more at death; all was a mystery. He

The number four

American Aborigines

A square man

Fourfold division of horizon

Four conners of the earth

Cruciform figures

Architecture

Geometric Forms

- Personal Ornaments

Architecture. Just as we find an instinctive appreciation of the beauties of geometric forms as applied to personal ornaments, so we find it as applied to architecture, not merely with respect to decoration as already mentioned, but in the general structure of temples, of altars, and of tombs. In early India, for example, there seems to have been no study of geometry as such except in connection with forms used in the temple, and this was probably the case in other parts of the earth. A desire to adapt symmetry to architecture is seen in the ter-

Observations of the Stars

Zodiac signs
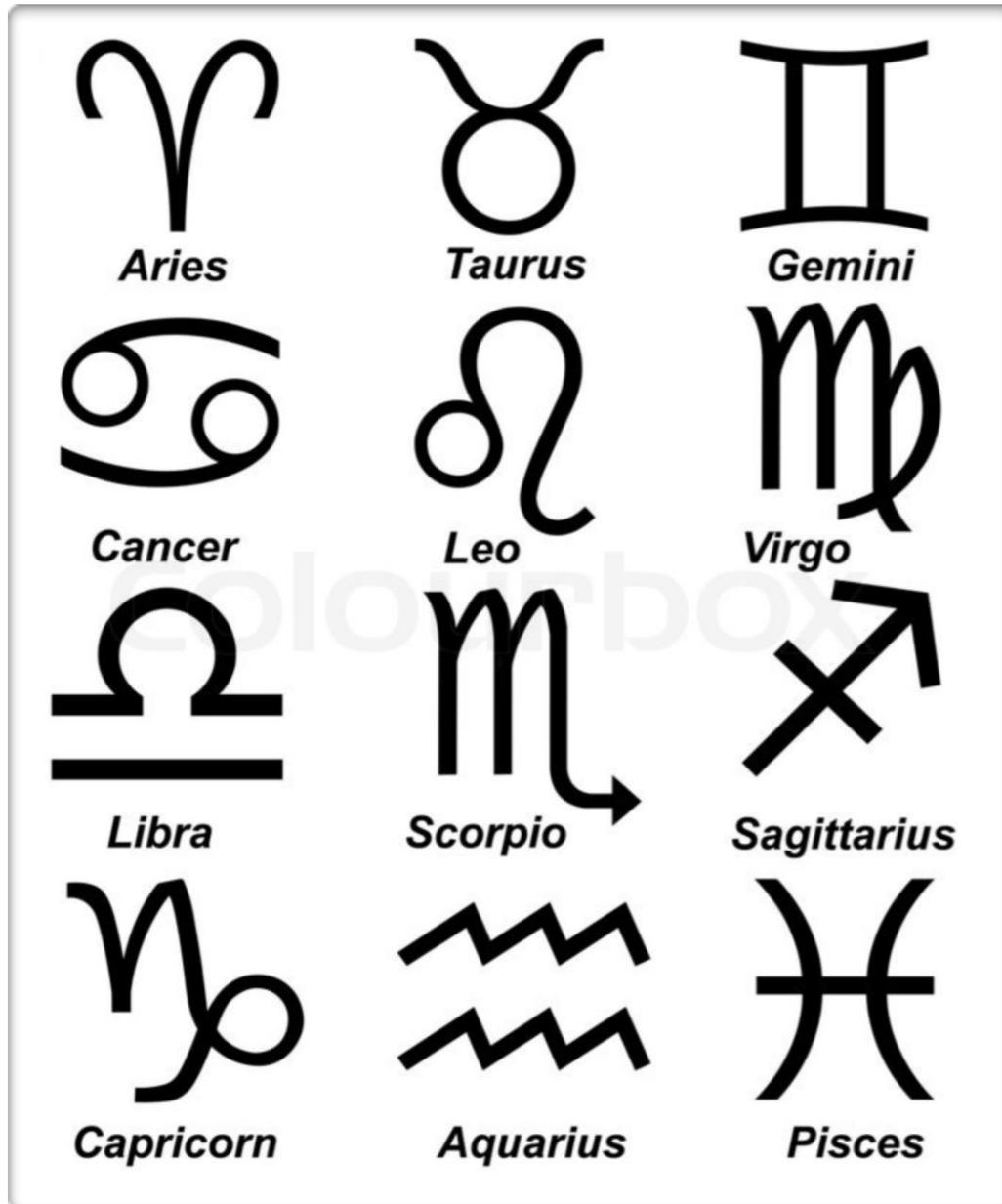
temples along the Nile and in the land of Mesopotamia. It was this, too, that led the early philosophers and poets to consider the stars as lighted lamps suspended in a vast material vault, or as golden nails fixed in a crystal sphere,—ideas perfectly suited to the childhood of the race. When it was that these observations of the heavens led to angle measure, to the recording of such celestial phenomena as eclipses, and to a naming of the signs of the zodiac and the constellations, we cannot say. One writer of prominence[1] places a recognition of the common constellations as early as 17000 B.C., and while this date seems to be very improbable, even though supported by certain historico-astronomical considerations, it is doubtless true that the period of this recognition and of the observ-

Discussion 1

1. Geometric forms that were in existence before the advent of life on the planet.

2. Laws of motion that entered into the formation and perpetuation of our solar system.

3. Geometric forms that appear prominently in the vegetable world and in the bodily structure of certain animals.

4. Geometric forms that appear prominently in the products of the labor of the lower animals, with the question of maximum efficiency in any of these cases.

5. The question of animal counting or pseudo-counting as discussed by psychologists.

6. Evidence of primitive counting without any scale.

7. The world's use of scales below five as shown by a study of our language and of savage tribes.

8. Reasons why the scales of five, ten, and twenty were the leading favorites.

9. Reasons why the scale of twelve would have been a particularly good one.

10. Reasons why three and seven have been particularly notable as mystic numbers, with several illustrations.

11. Circumstances which developed a high degree of skill in counting among certain peoples.

12. Reasons which led primitive peoples to the use of geometric forms in ornament.

13. The effect of religious mysticism upon primitive mathematics.

14. Various stages of geometric ornament in Cyprus, Crete, and the mainland of Greece.

15. Possible influence of geometric decoration upon the study of geometry as a science.

16. Geometric decoration that has persisted in all ages, with a study of the probable causes for this persistence.

17. Causes leading to an interest in astronomy among primitive peoples. Features of the ancient astronomy that are still found either in our present study of the science or in folklore.

18. Evidence of the antiquity of astronomical ideas, particularly in Mesopotamia, Egypt, and China, with probable evidence in the case of India and other parts of the East.